The African Geoid Project and Its Relevance to the Unification of African Vertical Reference Frames

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Key words: geoid, vertical datum, leveling.

SUMMARY

The African Geoid Project (AGP) is an attempt to produce a uniform precise geoid model for Africa. In this paper we present a provisional geoid model and propose a strategy for further improving this model. Problems associated with linking disparate vertical reference frames are discussed, and a proposal is made to use an AGP geoid model together with GPS measurements made as part of the AFREF project to unify vertical datums in Africa.

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1. INTRODUCTION

A vertical reference frame or datum forms the basis for all development projects in which heights are used. In particular any project involving the impounding, transport and distribution of water is critically depend upon the appropriate vertical reference frame being used. Heights are generally considered to refer to mean sea level (MSL), and most vertical reference frames attempt to approximate MSL as the datum for heights. In principle the geoid (a level surface which globally best fits MSL) is the ideal datum. In practice both the geoid and MSL are approximated by taking tide gauge measurements at one or more sites over a limited time period. Different methods, differing tide gauge sites and variable departures of MSL from a single level surface will lead to adjacent countries effectively establishing different vertical reference frames. For some landlocked countries the vertical datum may be defined by using the height of a benchmark on or near the border with a neighbouring maritime state - this may also cause a departure from other established datums in the region.

Nowadays most control surveys are established using satellite positioning systems such as GPS. The reference frame for GPS is the WGS84, where heights are referred to the WGS84 ellipsoid, not to MSL. Consequently, in order to reference GPS-derived heights to the appropriate surface (geoid) the geoid-ellipsoid separation (geoidal height) must be known. A model for this separation can be derived using gravity data, and in recent years there has been much work carried out in deriving continental-wide geoid models (Roman and Smith, 2000; Featherstone et al., 2001). Africa has lagged behind somewhat and it was only in 2001 that the International Association of Geodesy (IAG) initiated a project to compute an African geoid - the African Geoid Project (AGP) (Merry and Blitzkow, 2001).

In this paper I outline the status and progress of the AGP, and describe how a precise African geoid may be used to convert GPS-derived heights to local vertical reference frames, and how this geoid may be used to establish the relationship between the disparate vertical reference frames in Africa.

2. THE AFRICAN GEOID PROJECT

The African Geoid Project was initially established as a project of the Committee for Developing Countries of the IAG. After the General Assembly of the IAG in Sapporo in July 2003, the project was taken over as a project of Commission II (Gravity Field Commission) of the IAG. The project is driven by a small working group of African geodesists, who collaborate in obtaining data and in investigating appropriate models. Ideally, there would be near continuous data coverage at a station spacing of the order of 5-10km. These data could then be used to interpolate gravity anomalies on a regular 5' (about 8km) grid, from which a geoid model can be calculated. Practically, the data coverage over Africa is a long way short of this ideal - see Figure 1. Continuous coverage over the oceans is available from satellite altimetry data - in this case from a data set compiled by the Danish National Survey & Cadastre (Andersen and Knudsen, 1998). On land the data set is based upon compilations produced by the Universities of Leeds and Cape Town (Fairhead et al., 1988, van Gysen and Merry, 1987). There are large blank areas (e.g. in the interior of Angola) and other areas where the coverage is sparse. A concerted effort is needed to fill these gaps, possibly with the aid of airborne gravimetry.

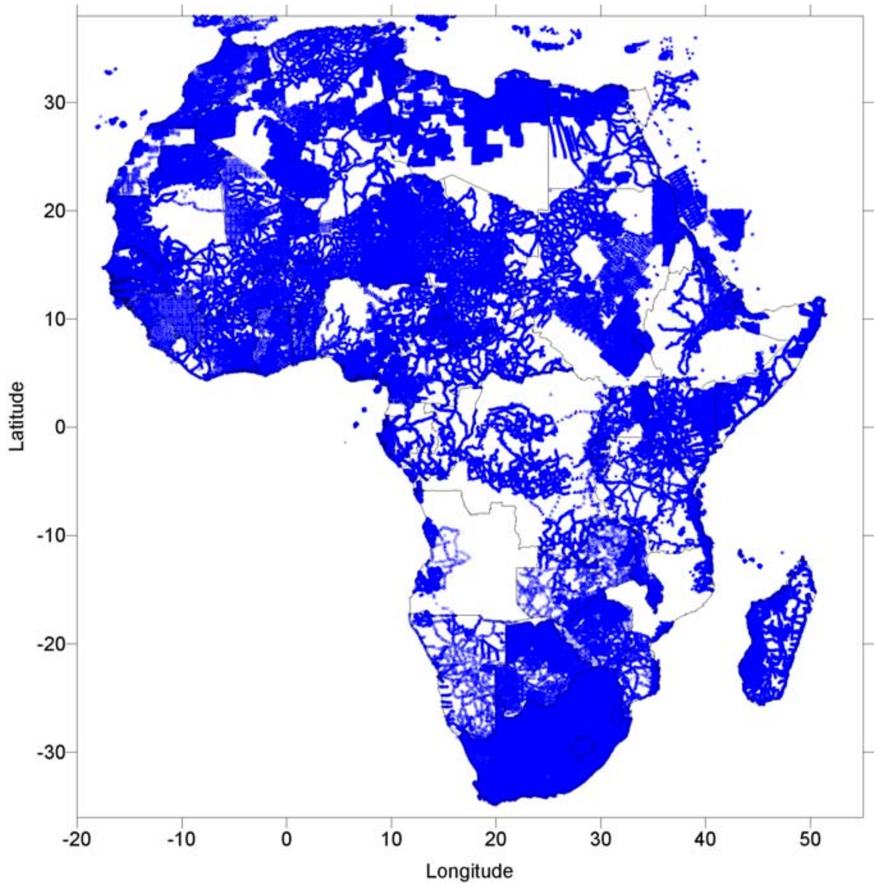

Figure 1: Gravity Data in Africa

A preliminary geoid model for Africa was computed in 2003 (Merry et al., 2003). In this model, the gaps in the 5' terrestrial data set were filled using the EGM96 geopotential model (Lemoine et al., 1996), and the geoid was computed in several steps:

- The long wavelength component of the height anomalies (quasi-geoid) was computed using the EGM96 geopotential model
- The short wavelength component was computed using reduced gravity anomalies in a two-dimensional convolution representation of the Stokes' integration
- The terrain effect (Molodensky term) was computed using the GLOBE (Hastings and Dunbar, 1998) digital elevation model (DEM)

- The height anomalies were converted to geoidal heights using Rapp's (1997) spherical harmonic representation of the separation between the two surfaces.

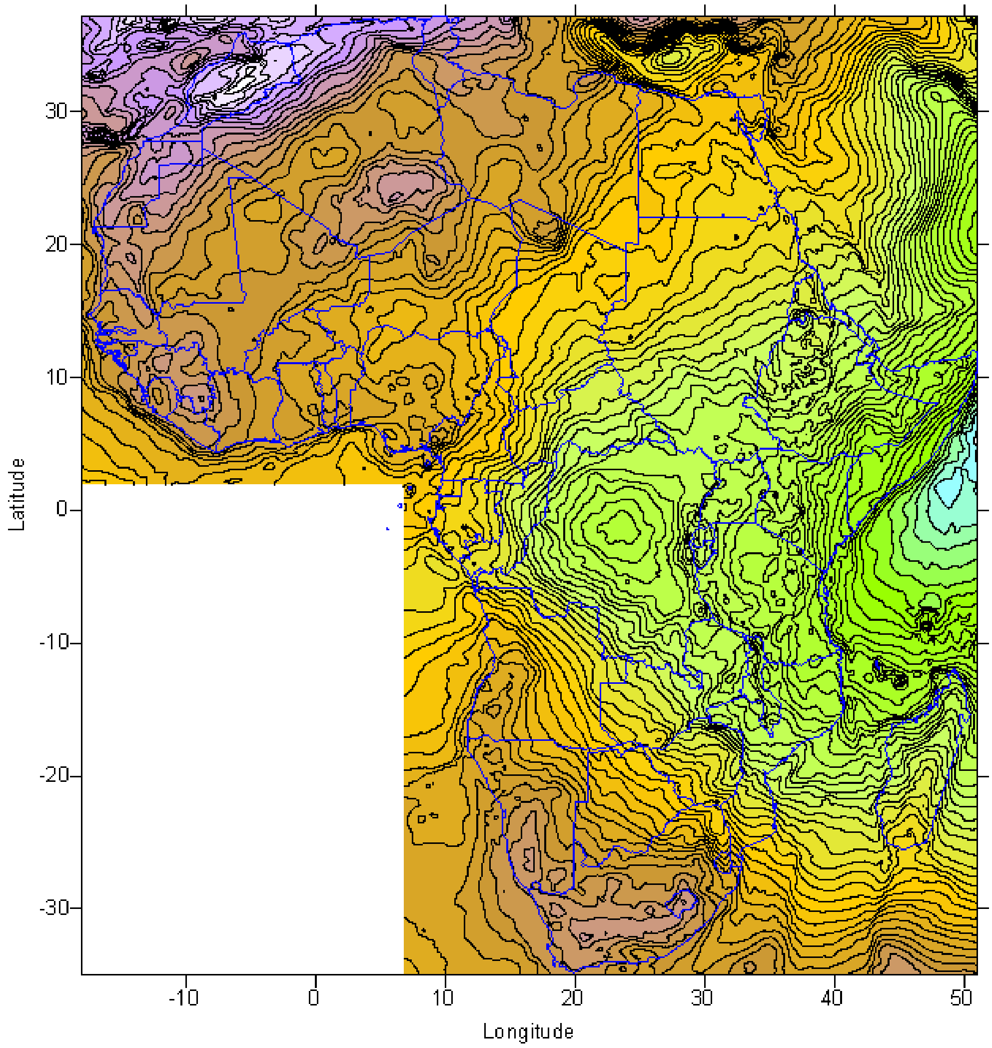

Figure 2: Geoid model for Africa - AGP2003 (contour interval: 2 metres)

The resultant geoid model (AGP2003) is shown in Figure 2. In order to validate this model attempts were made to obtain GPS/levelling data in Africa from which point estimates of the geoidal height could be deduced. Data were obtained for parts of Algeria, Egypt and South Africa (Figure 3). It was immediately apparent that significant biases existed between the three regions and the gravimetric and GPS/levelling data were separately compared for each region (Table 1). There is a multiplicity of potential sources for these biases:

- Errors in the long wavelength components of the EGM96 geopotential model
- Differences in the GPS reference frame used
- Biases in the vertical datums used in the different countries
- Cumulative systematic errors in the levelling networks.

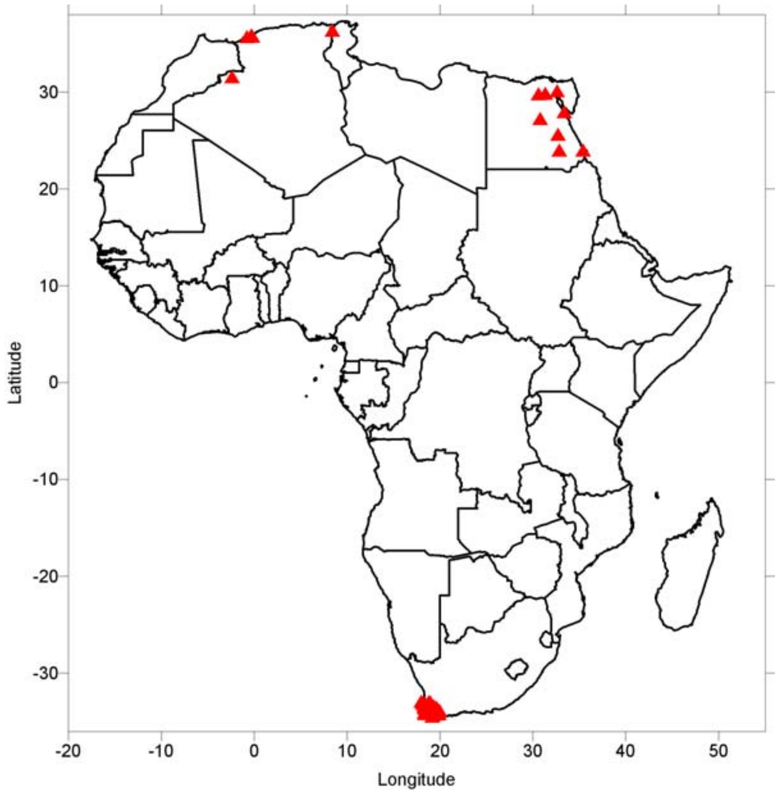

Figure 3: GPS/levelling data points

Region	No. Pts.	Bias (cm)	Std. Dev. (cm)
Algeria	13	-17	48
Egypt	8	+124	80
South Africa	42	-63	9

Table 1: Comparison GPS/levelling - AGP2003

The estimated accuracy of the EGM96 geoid model, which forms the basis of AGP2003, is of the order of 50cm, with potential zero and low frequency biases of 10-20cm (Lemoine et al., 1998). Recent gravity satellite missions such as CHAMP and Grace have focussed on determining low and medium frequency components of the Earth's gravity field with improved accuracy (Tapley and Reigber, 2001). Figure 4 shows the difference between the medium order (to degree 90) geoid model deduced from the EGM96 and a preliminary Grace model - GGM01, for southern Africa. There are significant (more than one metre) differences between the two models. A part of this difference could be an artifact, as the high order EGM96 coefficients have been neglected and there is a fair amount of correlation between these and the medium order coefficients. Nevertheless, this figure indicates that it would be well worth exploring the option of using GGM01 instead of EGM96 in computing the geoid for Africa.

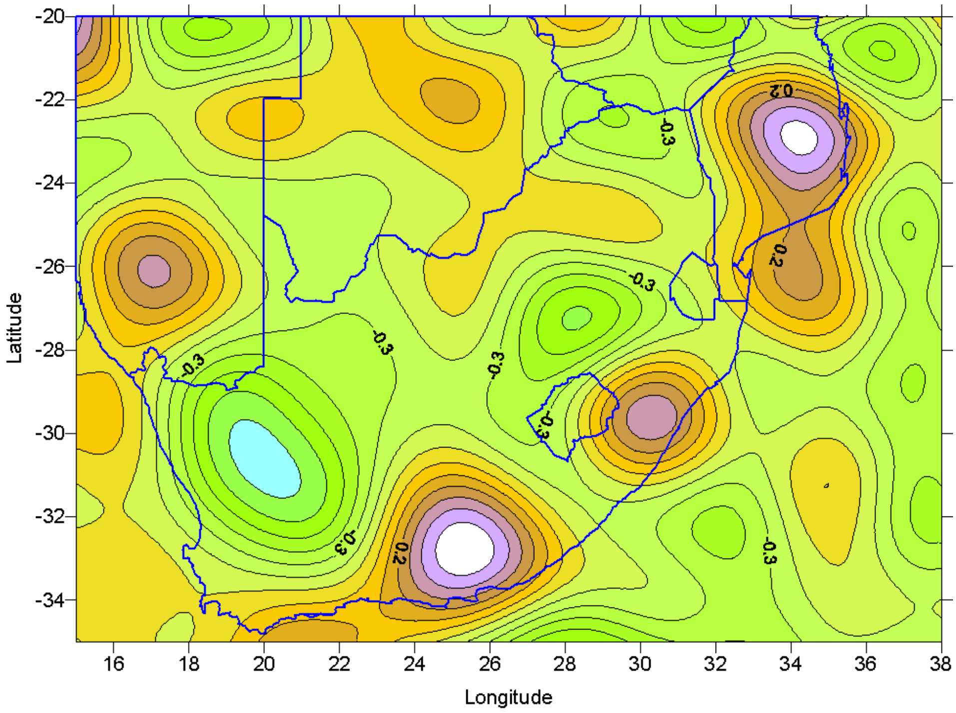

Figure 4: GGM01 - EGM96 Geoid (degree 90) for South Africa (contour interval: 10cm)

With regard to the GPS reference frame, it is not clear at this time whether all the supplied data are referenced to the same frame (e.g. ITRF2000). This can have a significant effect - for example, the South African data refer to the Hart94 datum, and it is known that the ellipsoidal heights for the GPS data used are in error with respect to ITRF2000 by up to 40cm (Chandler, 2001). If this were taken into account it would substantially reduce the bias shown in Table 1.

Likewise, the South African vertical datum is known to be some 15-20cm below current MSL (Merry, 1990). It is possible that biases of a similar magnitude exist for the Algerian and Egyptian vertical datums. Ways of modelling this variation in vertical datum bias using a precise gravimetric geoid and GPS will form the subject of the next section.

The impact of systematic errors in the levelling network (due to neglect of gravity effects, refraction, etc) must not be ignored. Most, if not all, national levelling networks in Africa were created at a time when dense gravity networks did not exist (they still do not in some regions), and no corrections were made for the influence of gravity variations. This could have an impact of several decimetres (Merry, 1985).

There are a number of ways in which the African geoid model can be improved:

- Filling in gaps in the terrestrial gravity data coverage. This will improve the accuracy in areas where data are sparse, but it will take many years to carry out the necessary surveys, even if airborne gravimetry is used.
- Replacing the EGM96 model with a more modern model, such as GGM01. This will reduce zero and low frequency errors and biases.

- Replacing the GLOBE DEM, which is known to have errors (Berry et al., 1999), with the more accurate SRTM DEM which has recently become available on a global 30" grid (NASA press release, 22 August 2003).

The latter two steps will be carried out in the near future.

3. VERTICAL DATUM

Modern reference frames, such as ITRF2000 (Altamimi, 2001) use space-based techniques to provide a fully three-dimensional reference frame. In practice, separate horizontal and vertical datums are used. The horizontal datum may well make use of a three-dimensional frame, but only the horizontal components (latitude and longitude on a chosen ellipsoid) are used. The vertical reference frame is traditionally tied to the geoid, which is closely approximated by MSL. In concept at least, all national vertical datums use the same reference frame - the geoid. Difficulties arise in realising this datum. For a start, in its most broadly accepted definition, the geoid is defined as the equipotential surface of the Earth's gravity field that coincides, on average, with MSL. It is well established that global sea level is rising at the rate of 1-2mm per year - does this mean that the geoid is also rising? Should we define the geoid for a particular epoch? The "on average" could also give rise to differences in interpretation - is it a global average over the open ocean?, or is an average of MSL as realised at long-term tide gauges established at coastal sites (this would produce a bias in favour of the northern hemisphere)?

Because of these practical difficulties (and because auxiliary data, such as sea surface topography - SST - were lacking), many nations chose either the MSL record at a single tide gauge site, or the MSL record at several sites to define their vertical datum. If the latter, the datum was potentially distorted if MSL at the different sites was not on the same equipotential surface.

The end result is that national vertical datums differ from each other due to differences in SST at the tide gauge sites and to differences in the measuring techniques. The period over which MSL would be recorded may also vary from country to country, leading to a further bias between vertical datums. For landlocked countries the situation is further exacerbated by any accumulated systematic errors in the national levelling networks of its adjacent maritime neighbours. As an example, there is a discrepancy in height of the order of 2.5m at the South Africa/Zimbabwe border (C Matyukira, personal communication, 2002). The South African heights are based upon precise levelling tied to four tide gauges in South Africa, while the Zimbabwe heights are based upon a line of levelling carried from the port of Beira in neighbouring Mozambique. A large part of this discrepancy may be due to the neglect of systematic effects in the levelling.

Political and economic changes within Africa have led to greater regional co-operation in areas such as transportation, communication and the creation of water and electricity reticulation grids. These developments point towards the need to unify both horizontal and vertical reference frames on the continent (or at least to establish the precise relationship between the different datums). The argument for the unification of vertical datums is further strengthened by the increasing application of technologies such as GPS in establishing both horizontal and vertical control. GPS produces ellipsoidal heights - to convert these to the national vertical datum requires not just a geoid model but also a model for the conversion between the geoid and the local vertical datum.

4. AFREF, AGP AND A UNIFIED VERTICAL DATUM FOR AFRICA

The African Geodetic Reference Frame (AFREF) has been conceived as a unified reference frame for Africa which will be fully consistent with the ITRF reference frame (Anonymous, 2002). It is proposed that as a first step at least one continuously operating GPS reference station (CORS) be located in each African country and be operated according to IGS standards. As a follow on each national survey agency would establish a national GPS network based upon these CORS stations. This network would form the basis for linking the existing geodetic control network (and hence the horizontal datum) to the ITRF, and would also form the basis for future control surveys.

Although the main emphasis of AFREF is on uniting horizontal datums in Africa, the vertical datum has not been forgotten, and a key task of AFREF would be to place, in each country, some GPS control points at critical points of the national vertical network. Such critical points would include tide gauge benchmarks (TGBM's) and nodal points of the first order levelling networks. Having precise ITRF co-ordinates at these points, together with a precise geoid model, will enable the offset Z of the vertical datum with respect to the geoid to be determined, using the simple model (Figure 5):

$$Z = h - N - H \tag{1}$$

Here, h is the ellipsoidal height of the point (determined from its ITRF co-ordinates); N is the geoidal height at that point (from the geoid model); and H is the orthometric height of the point in the local vertical datum (determined from precise levelling). The same model can be used if the quasi-geoid is the reference surface, replacing the geoidal height with the height anomaly and the orthometric height with the normal height. If more than one such critical point is available, the redundant data could be used to estimate any potential tilt of the vertical datum.

In order to obtain a precise and reliable estimate of the offset, it is necessary that all components in equation 1 are both precise and free of bias. An organised well-structured campaign such as that proposed for AFREF should ensure that the co-ordinates of all AFREF points are precisely known in a single well-defined reference frame. The current provisional geoid model for Africa, AGP2003, does not meet the stated requirements. Due to lack of data in some regions it is of uneven quality and there are potential biases and tilts due to errors in the low frequency components. The Grace mission and the proposed GOCE mission should reduce these errors to negligible proportions - of the order of a few cms (Anonymous, 2003). However there is still a question mark concerning the resolution of the zero frequency term, and it is possible that this may limit the application of equation 1 to determining relative biases between adjacent vertical datums. The other measured quantity in the equation is the orthometric height - systematic biases are possible due to the neglect of the actual gravity field. As their effect accumulates with distance from the zero point (TGBM), this influence can be minimised by taking GPS measurements at or near the TGBM (or TGBM's, if more than one are used to define the datum).

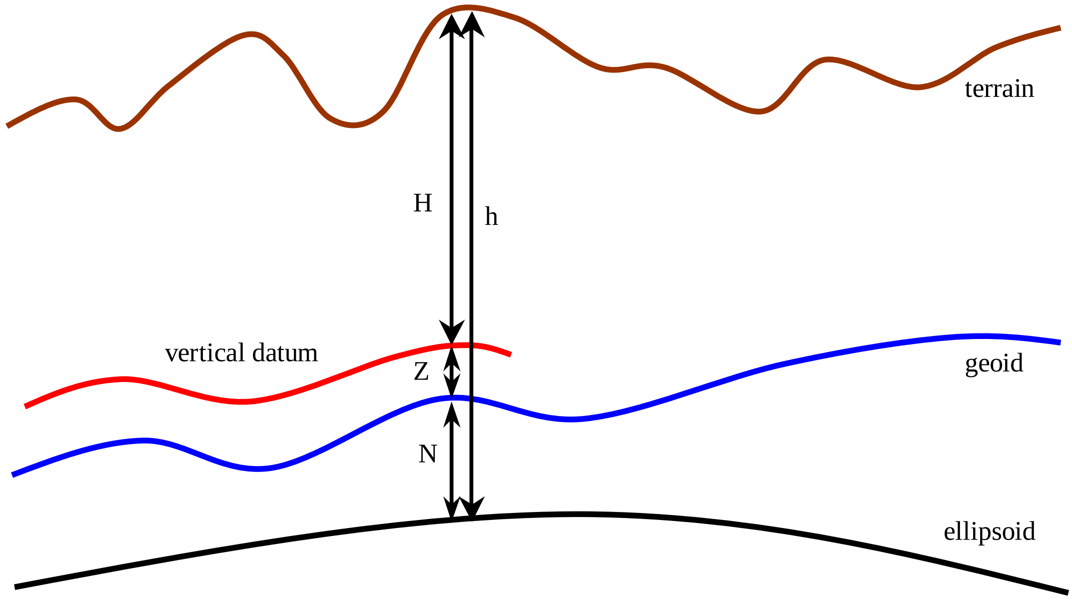

Figure 5: Vertical datum offset

The approach outlined above is not the only one. If the only source of bias in the local vertical datum is that due to SST, then a model for SST based upon satellite altimetry could be used to determine the offset Z for differing vertical datums (Bosch, 2001). Again, there is a potential zero frequency bias and it may only be possible to determine relative offsets between adjacent datums. A SST model could also be used just for the purpose of validating the offset determined using GPS measurements at TGBM's, as was done in Australia (Featherstone, 2001).

It may appear that the obvious method of determining the relative offsets between adjacent national vertical datums is to compare the heights at a common border benchmark, if such exists. This is of course the most practical approach, but it does require that there be a common benchmark, which is often not the case. More importantly, the common benchmark will generally be a long way from the zero points of both vertical datums and the intervening levelling networks will be burdened with systematic errors. As a consequence the comparison will generally not give the most reliable estimate of the offset between the two datums. This approach has been used to get provisional results for the offsets between Zimbabwe and South Africa (C Matyukira, personal communication, 2002), and between Brazil and Colombia and Venezuela (Hernandez et al., 2001).

5. CONCLUSIONS

There is a need for a unified vertical reference frame for Africa in support of infrastructure and development projects that require precise elevations. The AFREF and AGP projects provide a means for achieving this goal, provided they are structured to take into account the special requirements for unification of vertical datums. For AFREF, there is a need to ensure that sufficient precise GPS measurements are made at critical points of the national vertical networks, including the zero points and other TGBM's. For AGP, there is a need to fill gravity data gaps and to reduce errors in the zero and low frequencies of the geopotential spectrum.

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BIOGRAPHICAL NOTES

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