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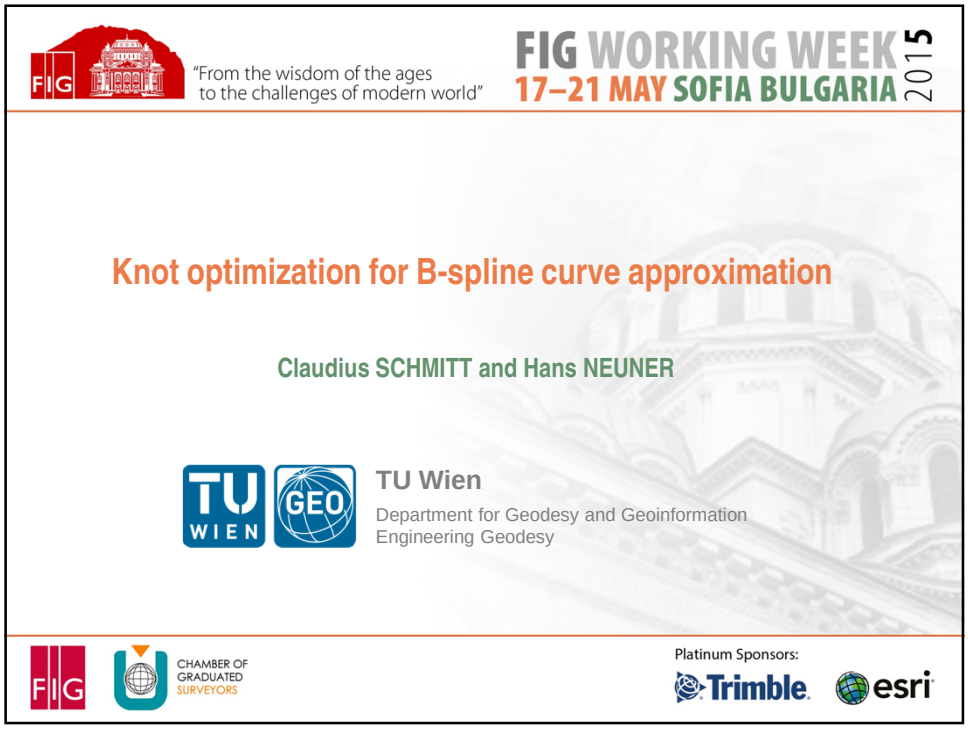
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Knot optimization for B-spline curve approximation
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Outline

- Motivation
- Basics of B-Spline curves
- Estimation of B-Spline knot position
- Results – simulated data & real data
- Summary/Outlook

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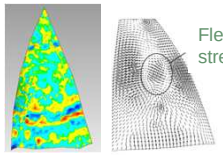
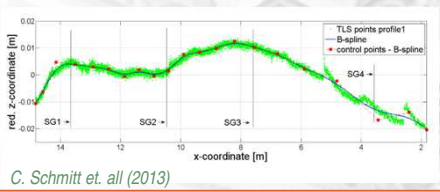
Motivation

- Point cloud (terrestrial laser scanning) approximation with B-Spline curves and surfaces
  - Global description
  - Local behavior
- Application of freeform curves and surfaces in Deformation analysis and structural mechanical calculations



C. Schmitt et. all (2014)



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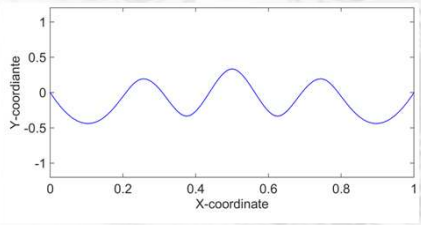
Basics of B-Spline curves

$$C_{X/Y}\,(obs_{Par.}) = \sum_{i=0}^{n} N_{i,p}(obs_{Par.}) * CP_{X/Y,i}$$

- $C_{X/Y}$ = curve point in 2D coordinates (3D also possible)
- $N_{i,p}$ = basis function
- $CP_{X/Y}$ = control point

- p = degree of the basis function
- $obs_{Par.}$ = position at the curve
- $n+1$ = number of basis functions

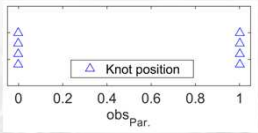


Knots

$$U = \{0_0, \dots, 0_p, 1_{p+1}, \dots, 1_m\}$$

$$N_{i,p}(obs_{Par.}) = \frac{obs_{Par.} - u_i}{u_{i+p} - u_i} N_{i,p-1}(obs_{Par.})$$

$$+ \frac{u_{i+p+1} - obs_{Par.}}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(obs_{Par.}), i = 0, \dots, n$$

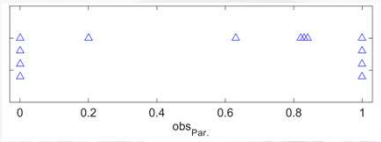


- recursive notation
- u_i = knot at the knot vector position i
- U = knot vector (minimal configuration)
- $m+1 = n+p+2$ = number of knots

Knots

Topics

- **Position of the knots inside the knot vector**
- Number of knots



Selected properties

- **Distance between the knots controls the locality of the curve**
- **Generate discontinuities by inserting p equal knots**
- **Approximation: highly nonlinear**

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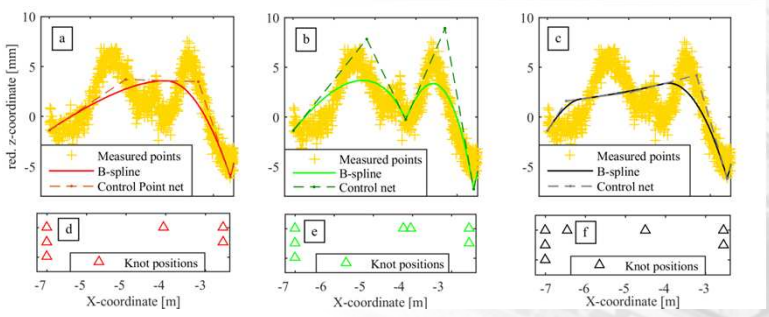
Influence of knot number and position



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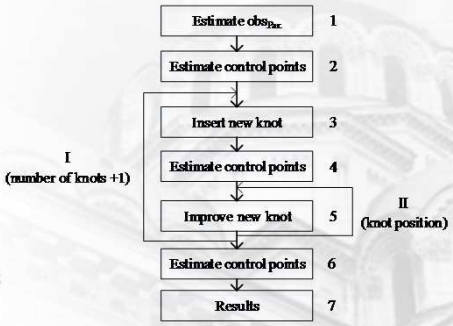
Estimation of the knot positions

- **Gauss-Markov model**
  - Linear (control points)
  - Nonlinear (knot positions)
- **Sequentially**
  - Two loops
- **Initial values (step 3)**
  - Cumulated sum of squared residuals



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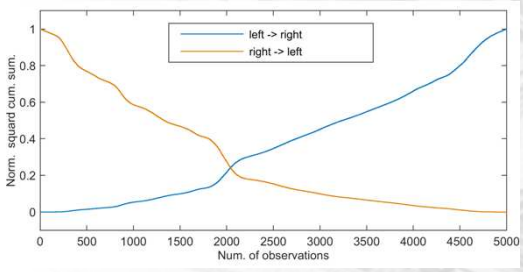
Initial values

Main idea

- **Unifying the variance of the residuals along the curve**

- **Cumulated sum of squared residuals**
  - Two functions
  - Normalized



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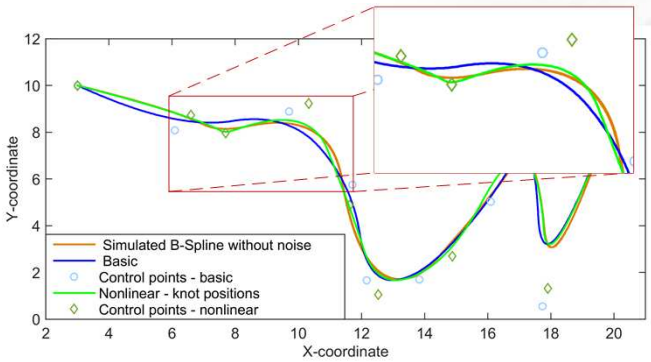
Results – simulated data



Method	$\sigma_{apost.}$
Nonlinear	0.11
Basic	0.18

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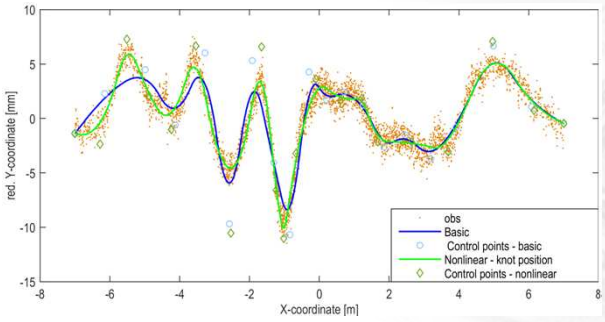
Results – real data



Num. points	Method	$\sigma_{apost.}$ [mm]
5000	Nonlinear	0.68
	Basic	0.97
12293	Nonlinear	0.70
	Basic	0.94

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Summary/Outlook

- Application of freeform element modeling
- Structure of B-Spline curves
- Estimation of the knot positions
- Procedure for initial values of the knot positions
- Improvements of the method shown on simulated and real data
- Expand the method to a non-sequential estimation

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References

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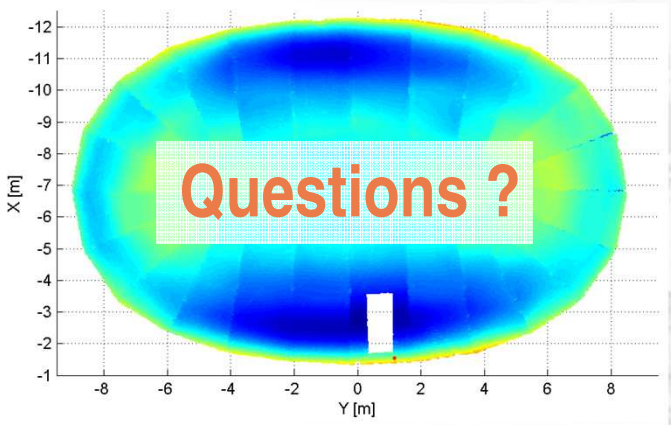

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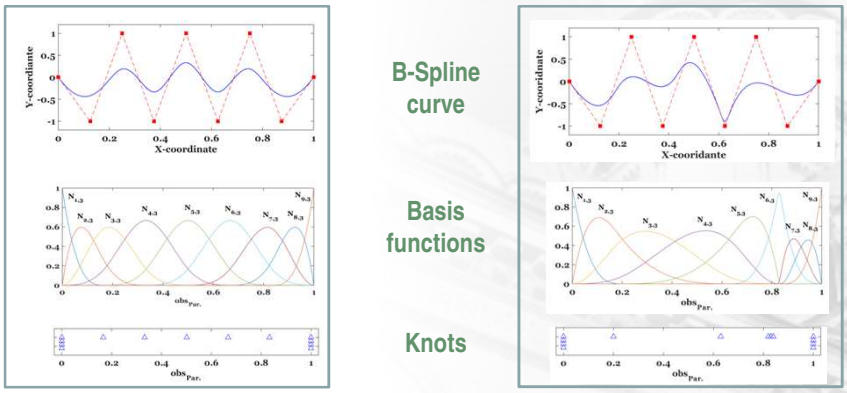
B-Spline parameters

Uniform | Nonuniform

B-Spline curve

Basis functions

Knots



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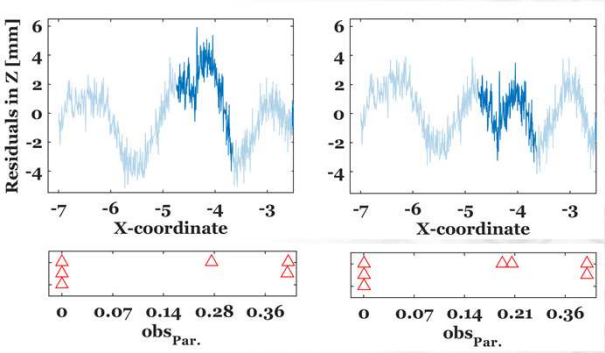
B-Spline parameters

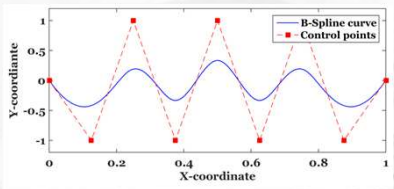
Influence of one additional knot



Control points

Topic

- Position of the control points



Selected properties

- Scaling factor of the basis functions
- Responsible for the change of curvature
  - All control points with the same value -> straight line
- Responsible for the position of the curve at the coordinate system
- Approximation: linear