The Determination of the Best Fitting Geoid: A Case Study of Samsun

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Keywords: Geoid, Polynomial Geoid Determination, Outlier detection, Significance Test

ABSTRACT

The geoid determination is the model to determine the height of a point position is known. The importance of geoid determination problem has increased with GPS technologies. Many methods had been developed and applied for this problem. One of these is polynomial method. The polynomial method of the main purpose is expression of the work area data with a single function. In the polynomial geoid determination, at firstly the order of polynomial is determined. In order to determine the appropriate polynomial surface, coefficients of polynomial are obtained by adjustment method. The geodetic data has got measurement errors. The gross and systematic error can be deleted from measurements before the adjustment; the random errors can be modeled. But the outlier measurements closed to random measurement errors cannot be removed from the measurement group and affect negatively adjustment results. Therefore outlier test must be applying the results of adjustment. In this paper it is tried to determine identify the best fit polynomial geoid for Samsun. For this purpose, firstly the significance test for the parameters was made, then, the posteriori variances were obtained.

1. INTRODUCTION

The geoid determination is the most important problem for scientist interested in the earth. There are a lot of areas interested in geoid like geodesy, geophysics, geography etc. (Akçın, 2001). The geoid called the surface closed the average sea surface and formed by the combination of the points have got zero potential value. The geoid is a complex surface and it is not easy defined as mathematically. In the geodesy the measurements on the physical earth, but the calculation of measurements is done on the reference surface. Thus, the difference between the reference ellipsoid was called geoid ondulation. The geoid determination methods had been developed to obtained geoid ondulation values (Bolat ,2011)
The geoid determination methods can be given following,

1. The astro-geodetic method
2. The gravity values
   a. The stokes function
   b. The Fourier transformation
3. Geoid determination according to numerical density method
4. The geopotential approach
5. The combined methods (remove - restored)
6. The GPS/levelling methods
   a. The polynomials method
   b. The fuzzy logic method
   c. The Artificial neural network
   d. Etc.

The astro-geodetic method is first method using geoid determination. In the early 1970s, the geopotential models have been developed for geoid determination. 1980s the fast Fourier transformation has been used to obtained gravity data. In 1990s the satellites geodesy was started to spread rapidly. Then, the combination of the GPS and levelling data began to be used for geoid determination. Also, the gravity data were obtained the more accuracy with the help of satellites. In this case the accuracy of gravimetric geoid determination methods had increased and high grade geopotential model improved significantly (Arslan ve Yılmaz, 2005).
The most commonly used geoid determination method is the polynomial method. This method is defined a polynomial function by using the point 3D coordinates and can be applied regionally (Bolat , 2011). In this study, it is try to the best suitable geoid for Samsun were determined using the Samsun Levelling and GPS Network data using polynomial method. The data of study includes the 478 point coordinates.

2. GEOID DETERMINATION WITH POLYNOMIALS

This method is the most widely used surface fitting procedure. A surface is obtained from GPS ellipsoidal and orthometric heights of points. The function of surface is determined with basic definition of orthogonal polynomials: (Cakır, Yilmaz, 2014)

$$N_{(x,y)} = \sum_{i=0}^{m} \sum_{\substack{j=k-i \\ i=0}}^{k} a_{ij} x^i y^j \tag{1}$$

where (x, y) is the position coordinates of points, aij the constants of the polynomial and m the order of the chosen polynomial. 2nd order polynomial equation can be written for the polynomial:

$$N_{(x,y)} = a_{00} + a_{01}y + a_{10}x + a_{02}y^2 + a_{11}xy + a_{20}x^2 \tag{2}$$

Equ. 2 the measurement and unknown numbers are equal to the point and constants number. If the measurement number (n) is bigger than the unknown number (u), the solution must be realized by using adjustment procedure. When the Equ. (2) are designed according to indirect measurement adjustment mathematical model, the following equations are obtained:

$$V = AX - \ell \qquad P_{\ell\ell} = Q_{\ell\ell}^{-1} \qquad X = \begin{bmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{02} \\ a_{11} \\ a_{20} \end{bmatrix} \qquad A = \begin{bmatrix} 1 & y_1 & x_1 & y_1^2 & x_1 y_1 & x_1^2 \\ 1 & y_2 & x_2 & y_2^2 & x_2 y_2 & x_2^2 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 1 & y_n & x_n & y_n^2 & x_n y_n & x_n^2 \end{bmatrix} \qquad \ell = \begin{bmatrix} N_1 \\ N_2 \\ . \\ . \\ N_n \end{bmatrix} \tag{3}$$

This model can be solved by objective function of the least square adjustment method. The unknown parameters are obtained following equation (Sisman, 2014).

$$X = \left(A^T PA\right)^{-1} A^T P\ell \tag{4}$$

2.1. Outlier Detection

The measurement group has got the outliers inevitably. These outliers can adversely affect the adjustment. Therefore, the outlier detection test must be done to determine the outliers measurements, (Aksoy, 1984; Ayan, 1992; Uzun 2003; Bayrak, 2003; Teke and Yalçınkaya, 2005; Bektaş, 2005, Sisman et al. 2012). The outlier detection test is realized according to hypothesis m_0^2 is used for outlier detection. The test size is calculated by using the residuals of measurements V_i and their standard deviation m_{vivi}.

$$T_i = \frac{|V_i|}{m_{vivi}} = \frac{|V_i|}{m_0\sqrt{Q_{vivi}}} \tag{5}$$

This test value is compared with the q table values.

q= $T_{f,1-\alpha/2} = \sqrt{\frac{f t2f-1,1-\alpha/2}{f-1+t2f-1,1-\alpha/2}}$ (6)

If $T_i > T_{f,1-\alpha/2}$, this measurement is accepted as outlier measurements. The measurement has got a biggest value as outlier is removed the measurement group, then this procedure repeated until there are no outlier measurement (Bolat, 2011)

2.2. The Determination of The Best Suitable Polynomial Order

The significance test and the changing of a posteriori variance can be used to determine the best suitable polynomial order. The significance test of unknown parameters is realized by using the unknown value x_i and their standard deviation m_{xi}. For this procedure, the hypothesis test is established following.

$H_0 : E(xi) = 0$ zero hypothesis

$H_s : E(xi) \neq 0$ alternatively hypothesis

Then, the test value T_i is calculated.

$$T_i = \frac{|x_i|}{m_{xi}} \quad (7)$$

The test value is compared to the $q = t_{f,1-\alpha/2}$ table value. If T_i is less than q, it is accepted zero hypothesis. This unknown parameter is not significant statistically, (Wolf and Ghilani, 1997, Teke and Yalçınkaya, 2005).

The best suitable of the polynomial order can be determined by using changing of a posteriori variance. The order of the polynomial increases, the variance is reduced. This reduction will stop at one step. This step is determined the suitable polynomial order. (Bolat,2011)

3. NUMERICAL APPLICATION

The data of Samsun triangulation network was used for application. This data includes the ellipsoidal coordinate of 478 points as (X, Y, h) and orthometric height values as (H) (Figure 1).

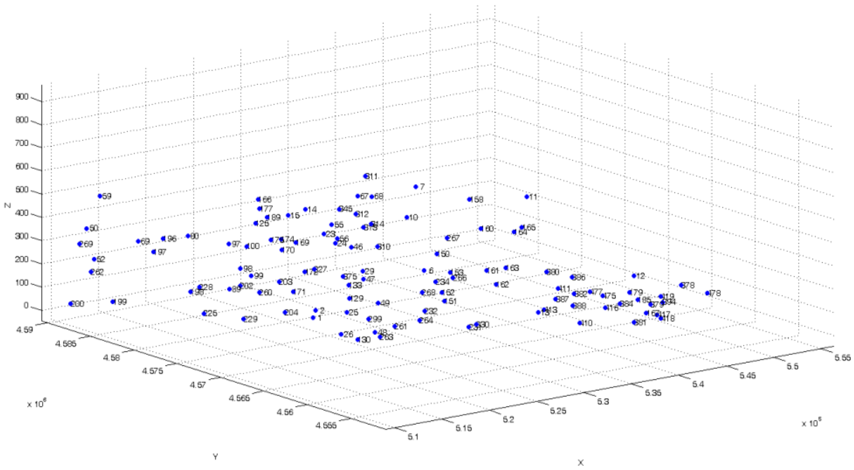

Figure 1. The Samsun triangulation network

The polynomial function was obtained from 1 to 6 order by using adjustment solution according to least square method. The outlier detection was realized in all solution until there were not outlier measurements in data. The outlier test results were given in Table 1.

Table 1 . The Outliers test results

1st order		2nd order		3rd order		4th order		5th order		6 orders	
Outlier Point		**Outlier Point**		**Outlier Point**		**Outlier Point**		**Outlier Point**		**Outlier Point**	
Num	**NN**	**Num**	**NN**	**Num**	**NN**	**Num**	**NN**	**Num**	**NN**	**Num**	**NN**
19	418, 417, 427, 428, 426, 187, 167, 429, 419, 178, 12, 186, 449, 430, 185, 450, 431, 179, 448	7	188, 418, 427, 417, 428, 426, 7	4	188, 378, 7, 25	9	188, 378, 25, 7, 10, 100, 274, 261, 275	19	188, 378, 25, 7, 10, 478, 100, 274, 261, 68, 275, 11, 49, 51, 412, 386, 380, 13, 6	16	188, 378, 25, 10, 7, 100, 274, 261, 68, 11, 275, 6, 49, 51, 13, 300

The significance test was performed on the unknown parameters after outlier detection test. It was found that only the first term was significant for each polynomial order. The changing of a posteriori variance was obtained using consistent measure group for each polynomial. (Table 2)

Table 2. The posteriori variance values of polynomial solution

The order of the polynomial	Standard deviation values (cm.)
1st order	22.428
2nd order	19.687
3rd order	9.643
4th order	4.020
5th order	2.996
6th orders	2.789

It was found that the changing of posteriori variance after 6th order term was decreased. In this case, it was decided that the best suitable value of polynomial function was obtained 6th order.

4. CONCLUSION

The polynomial geoid determination is the most widely used method. In this study the best suitable geoid was determined for the Samsun triangulation network. For this application 478 points were used. The first step of application was to determine the outlier measurements group of data by using the outliers test. Most outlier points have been found in 1 and 5 order polynomial function. It was found in at least outlier points in the 3 degree polynomial function. Then, the significance test of unknown parameters and the changing of a posteriori variance were made to determine the best suitable order of polynomial function. The significance testing was found the only first term was significant in all polynomial function. The result of this procedure it can be said that the zero-order polynomial best fit polynomials function. The changing of a posteriori variance was not observed from 6th to 7th order polynomial function. In this case, it is decided that the best suitable geoid determination function was the 6^{th} order polynomial function for the Samsun triangulation network.

REFERENCES

Akçin, H., (2001) , Jeoit Kavramı ve Belirleme Teknikleri Üzerine Bir İnceleme , *Niğde Üniversitesi Mühendislik Bilimleri Dergisi*, Cilt 4 Sayı 1, 37-50.

Aksoy, A., (1984), Uyuşumsuz Ölçüler Testi, *Harita Dergisi*, 93, 15-24.

Arslan E., Yilmaz M., (2005) , Geoit Belirleme Yöntemleri , *TMMOB Harita ve Kadastro Mühendisleri Odası 10. Türkiye Harita Bilimsel ve Teknik Kurultayı*, Ankara.

Ayan, T., (1992), Uyuşumsuz Ölçüler Testi, *Harita ve Kadastro Mühendisliği Dergisi*, 72, 38-46.

Bayrak, T., (2003), *Heyelanlar için bir Dinamik Deformasyon ve Bir Dinamik Hareket Yüzeyi Modellerinin Oluşturulması*, Doktora Tezi, Karadeniz Teknik Üniversitesi, fen Bilimleri Enstitüsü, Trabzon.

Bektaş S., (2005) , *Dengeleme Hesabı* , Samsun:Ondokuz Mayıs Üniversitesi Yayınları, Samsun

Bolat S. (2011*), Lokal Jeoid Belirlemede Kullanılan Enterpolasyon Yöntemleri Samsun İli Örneği* , Yüksek Lisans Tezi , Ondokuzmayıs Üniversitesi, Samsun.

Cakıl L., Yılmaz N., 2014 , Polynomials, Radial Basis Functions And Multilayer Perceptron Neural Network Methods In Local Geoid Determination With GPS/Levelling

Sisman Y., Dilaver A., Bektas S. (2012), Outlier Detection in 3D Coordinate Transformation with Fuzzy Logic, *Acta Montanistica Slovaca*, 17(1), 1-8.

Sisman Y., (2014), Coordinate transformation of cadastral maps using different adjustment methods, *Journal of the Chinese Institute of Engineers*, Volume 37, Issue 7, 869-882.

Teke, K., Yalçinkaya M.(2005), Yerel Jeoid Yüzeyinin Belirlenmesinde Kullanılan Enterpolasyon Yöntemleri , *TUJK Düşey Datum Çalıştayı*, Karadeniz Teknik Üniveristesi, Trabzon.

Uzun, Y. (2003), *Üç Boyutlu Astrojeodezik Dik Koordinat Sistemlerinde Dönüşüm Modelleri ve Uyuşumsuz Ölçülerin Belirlenmesi Yöntemlerinin Karşılaştırılması*, Doktora Tezi, Karadeniz Teknik Üniversitesi, Fen Bilimleri Enstitüsü, Trabzon.

Wolf, H., Ghilani, C.D. (1997), *Adjustment Computation: Statistics and Least Squares in Surveying and GI*S, John Wiley and Sons Inc., New York, 1997.

BIOGRAPHICAL NOTES

I was born in 1991 in Samsun. I finished Ondokuz Mayıs University Geomatics Engineering in (2013). I began graduate education in Ondokuz Mayıs University Geomatics Engineering in (2013). I have been working as a research assistant in Ondokuz Mayıs University Geomatics Engineering since January 2014.

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