# Technical Impact of Multiple GNSS on Applications

Professor Gethin Wyn Roberts
Dr. Lawrence Lau
Dr. Craig Hancock
University of Nottingham

# Overview

- Discrepancy among the GNSS Reference Frames
- Discrepancy among the GNSS Reference Time Systems
- Impact on the Mainstream Applications (i.e., SPP)
- Impact on the Land/Engineering Surveying Applications (i.e., RTK including NRTK)
- Impact on the Standalone/Remote Precise Positioning Applications (i.e., Precise Point Positioning - PPP)
- Summary and Discussion

# Discrepancy between ITRF and GNSS reference frames

- Discrepancy between ITRF2000 and WGS84 (G1150): "two reference frames are essentially identical with differences being statistically insignificant for most applications" (NGA, 2003), ~1-2cm (Slater, J.A. - NGIA) -> WGS84(G1674): sub-cm?
  - Diff. ITRF2000/ITRF2005 < diff. ITRF2000/WGS84
- Discrepancy between ITRF2005 and GTRF: sub-mm
- Discrepancy between ITRF2000 and PZ90.02 (introduced 20 Sept 2007): No rotation, dX:-36cm, dY: 8 cm, dZ:18cm (Ørpen, 2008)
- Discrepancy between ITRF2000 and CGCS 2000: +/- 3 cm or less (Cheng, et. al.,2009)?

US National Geospatial Intelligence Agency (NGA), Addendum to NIMA TR 8350.2: Implementation of the World Geodetic System 1984 (WGS 84) Reference Frame G1150, 2003.

O. Ørpen (Fugro Seastar), GNSS Status, Havbunnskartlegging og Inspeksjon, Geilo 6-8 Feb. 2008

CHENG P., et. al. (Chinese Academy of Surveying and Mapping), China Geodetic Coordinate System 2000, Eighteenth United Nations Regional Cartographic Conference for Asia and the Pacific Bangkok, 26-29 October 2009

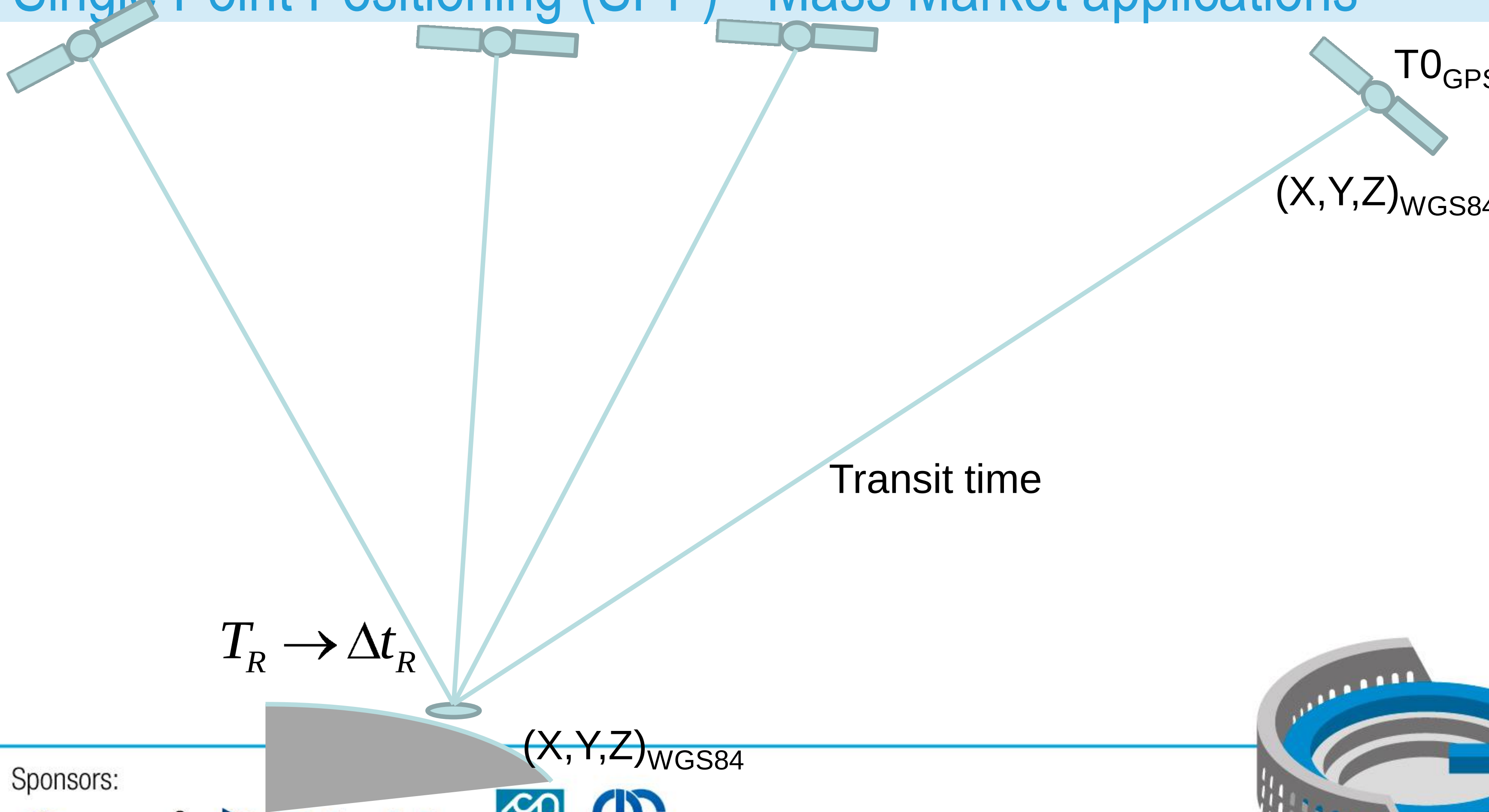
# GNSS Reference Time Systems

- GGTO specified requirement of 5 ns, 2 sigma over 24 hour (Hahn & Powers, 2005) –> too challenging
- "Error of determining GLONASS Time/GPS Time is no more than 20 ns" (RIRT)
- GPS Time & Beidou Time (BDT) -> GBTO: 20 ns or better?

J. Hahn & E. Powers, 2005, "Implementation of the GPS to Galileo Time Offset (GGTO)," in Proceedings of the 2005 Joint IEEE International Frequency Control Symposium and Precise Time and Time Interval (PTTI) Systems and Applications Meeting, 29-31 August 2005, Vancouver, Canada (IEEE 05CH37664C), pp. 33-212.

Russian Institute of Radionavigation and Time (RIRT), Complementarity of GNSS Regarding System Time Scales, www.oosa.unvienna.org/pdf/icg/2009/workgroupinterop2/08.pdf

# Single Point Positioning (SPP) - Mass Market applications



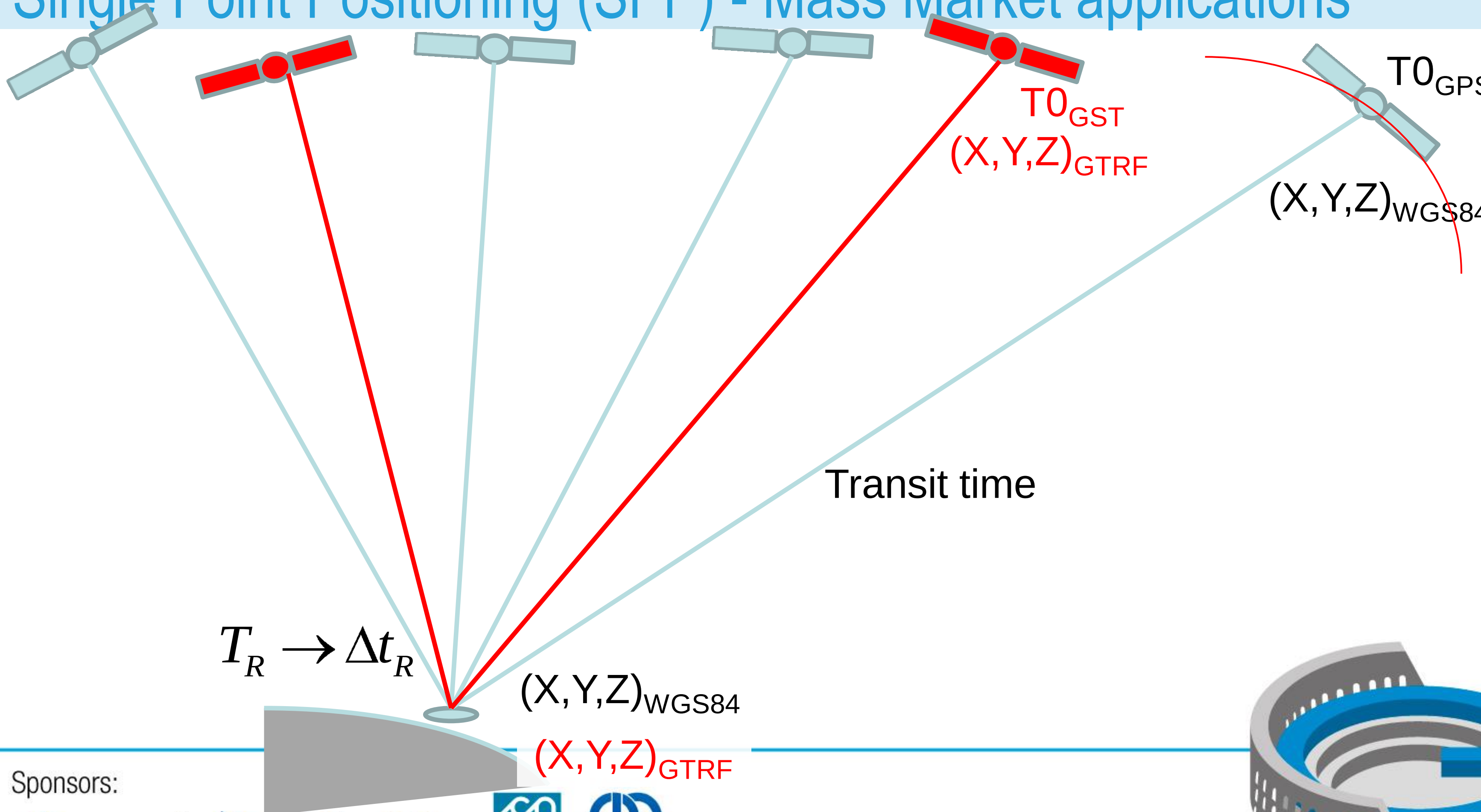
# Single Point Positioning/Navigation solution

$$
Ax = \begin{bmatrix}
-\dfrac{X^{S1}-X_R}{\rho_R^{S1}} & -\dfrac{Y^{S1}-Y_R}{\rho_R^{S1}} & -\dfrac{Z^{S1}-Z_R}{\rho_R^{S1}} & c \\
-\dfrac{X^{S2}-X_R}{\rho_R^{S2}} & -\dfrac{Y^{S2}-Y_R}{\rho_R^{S2}} & -\dfrac{Z^{S2}-Z_R}{\rho_R^{S2}} & c \\
-\dfrac{X^{S3}-X_R}{\rho_R^{S3}} & -\dfrac{Y^{S3}-Y_R}{\rho_R^{S3}} & -\dfrac{Z^{S3}-Z_R}{\rho_R^{S3}} & c \\
\vdots & \vdots & \vdots & \vdots \\
-\dfrac{X^{Sn}-X_R}{\rho_R^{Sn}} & -\dfrac{Y^{Sn}-Y_R}{\rho_R^{Sn}} & -\dfrac{Z^{Sn}-Z_R}{\rho_R^{Sn}} & c
\end{bmatrix}
\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \\ \Delta t \end{bmatrix} = l
$$

$$
l = P_i^k - (\rho_i^k + (\delta t_i - \delta t^k)c + I_i^k + T_i^k + \varepsilon_{1,i}^k)
$$

X n

# Single Point Positioning (SPP) - Mass Market applications



$$P_i^k = \rho_i^k + (\delta t_i - \delta t^k)c + I_i^k + T_i^k + \varepsilon_{1,i}^k$$

# Single Point Positioning (SPP) - Mass Market applications

$$Ax = \begin{bmatrix}
-\frac{X^{G1}-X_R}{\rho_R^{G1}} & -\frac{Y^{G1}-Y_R}{\rho_R^{G1}} & -\frac{Z^{G1}-Z_R}{\rho_R^{G1}} & c & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-\frac{X^{Gn}-X_R}{\rho_R^{Gn}} & -\frac{Y^{Gn}-Y_R}{\rho_R^{Gn}} & -\frac{Z^{Gn}-Z_R}{\rho_R^{Gn}} & c & 0 & 0 \\
-\frac{X^{E1}-X_R}{\rho_R^{E1}} & -\frac{Y^{E1}-Y_R}{\rho_R^{E1}} & -\frac{Z^{E1}-Z_R}{\rho_R^{E1}} & 0 & c & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-\frac{X^{En}-X_R}{\rho_R^{En}} & -\frac{Y^{En}-Y_R}{\rho_R^{En}} & -\frac{Z^{En}-Z_R}{\rho_R^{En}} & 0 & c & 0 \\
-\frac{X^{B1}-X_R}{\rho_R^{B1}} & -\frac{Y^{B1}-Y_R}{\rho_R^{B1}} & -\frac{Z^{B1}-Z_R}{\rho_R^{B1}} & 0 & 0 & c \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-\frac{(X^{Bn}-X_R)}{\rho_R^{Bn}} & -\frac{(Y^{Bn}-Y_R)}{\rho_R^{Bn}} & -\frac{(Z^{Bn}-Z_R)}{\rho_R^{Bn}} & 0 & 0 & c
\end{bmatrix}
\begin{bmatrix}
\Delta X_R \\ \Delta Y_R \\ \Delta Z_R \\ \Delta t_{R,G} \\ \Delta t_{R,E} \\ \Delta t_{R,B}
\end{bmatrix}$$

One freq. only!, more..

## Single Point Positioning (SPP) - Mass Market applications

- GNSS time offsets
  - Can be “aligned” by applying the broadcast GxTO
  - Estimate in user level (Recommended!!)
- GNSS reference frames
  - Can be neglected
  - Can be mitigated by applying given transformation parameters
    - Satellite coordinates
    - User position

$$l = P_i^k - (\rho_i^k + (\delta t_i - \delta t^k)c + I_i^k + T_i^k + \varepsilon_{1,i}^k)$$

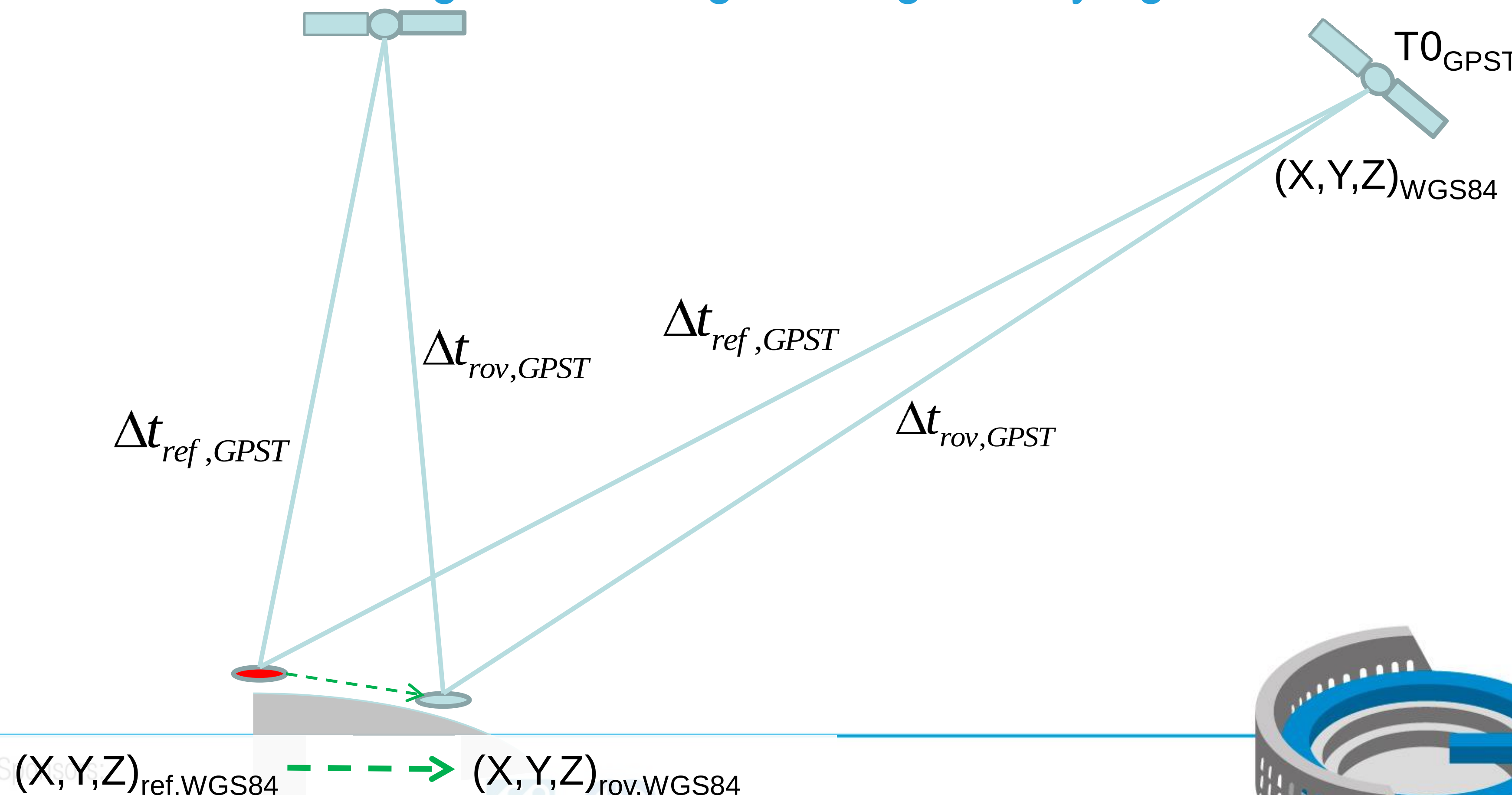
# Single Point Positioning (SPP) - Mass Market applications

$$Ax = \begin{bmatrix} -\frac{X^{G1}-X_R}{\rho_R^{G1}} & -\frac{Y^{G1}-Y_R}{\rho_R^{G1}} & -\frac{Z^{G1}-Z_R}{\rho_R^{G1}} & c & \\ \vdots & \vdots & \vdots & \vdots & \\ -\frac{X^{Gn}-X_R}{\rho_R^{Gn}} & -\frac{Y^{Gn}-Y_R}{\rho_R^{Gn}} & -\frac{Z^{Gn}-Z_R}{\rho_R^{Gn}} & c & \\ -\frac{X^{E1}-X_R}{\rho_R^{E1}} & -\frac{Y^{E1}-Y_R}{\rho_R^{E1}} & -\frac{Z^{E1}-Z_R}{\rho_R^{E1}} & 0 & \\ \vdots & \vdots & \vdots & \vdots & \\ -\frac{X^{En}-X_R}{\rho_R^{En}} & -\frac{Y^{En}-Y_R}{\rho_R^{En}} & -\frac{Z^{En}-Z_R}{\rho_R^{En}} & 0 & \\ -\frac{X^{B1}-X_R}{\rho_R^{B1}} & -\frac{Y^{B1}-Y_R}{\rho_R^{B1}} & -\frac{Z^{B1}-Z_R}{\rho_R^{B1}} & 0 & \\ \vdots & \vdots & \vdots & \vdots & \\ -\frac{X^{Bn}-X_R}{\rho_R^{Bn}} & -\frac{Y^{Bn}-Y_R}{\rho_R^{Bn}} & -\frac{Z^{Bn}-Z_R}{\rho_R^{Bn}} & 0 & \end{bmatrix} \begin{bmatrix} \Delta X_R \\ \Delta Y_R \\ \Delta Z_R \\ \Delta t_{R,G} \\ \\ \\ \end{bmatrix}$$

Use GxTO estimates

IFB/ICB?

# Relative Positioning - Land/Engineering surveying




Sponsors:

# Relative Positioning - Land/Engineering surveying

1 GNSS, 3 Freq.

$$\Phi_{1,ij}^{kl} = \rho_{ij}^{kl} - I_{ij}^{kl} + T_{ij}^{kl} + \lambda_1 N_{1,ij}^{kl} + \varepsilon_{1,ij}^{kl}$$

$$\Phi_{2,ij}^{kl} = \rho_{ij}^{kl} - (f_1 / f_2)^2 I_{ij}^{kl} + T_{ij}^{kl} + \lambda_2 N_{2,ij}^{kl} + \varepsilon_{2,ij}^{kl}$$

$$\Phi_{5,ij}^{kl} = \rho_{ij}^{kl} - (f_1 / f_5)^2 I_{ij}^{kl} + T_{ij}^{kl} + \lambda_5 N_{5,ij}^{kl} + \varepsilon_{5,ij}^{kl}$$

↓ Medium/Short baselines

$$\Phi_{1,ij}^{kl} = \rho_{ij}^{kl} + \lambda_1 N_{1,ij}^{kl} + \varepsilon_{1,ij}^{kl}$$

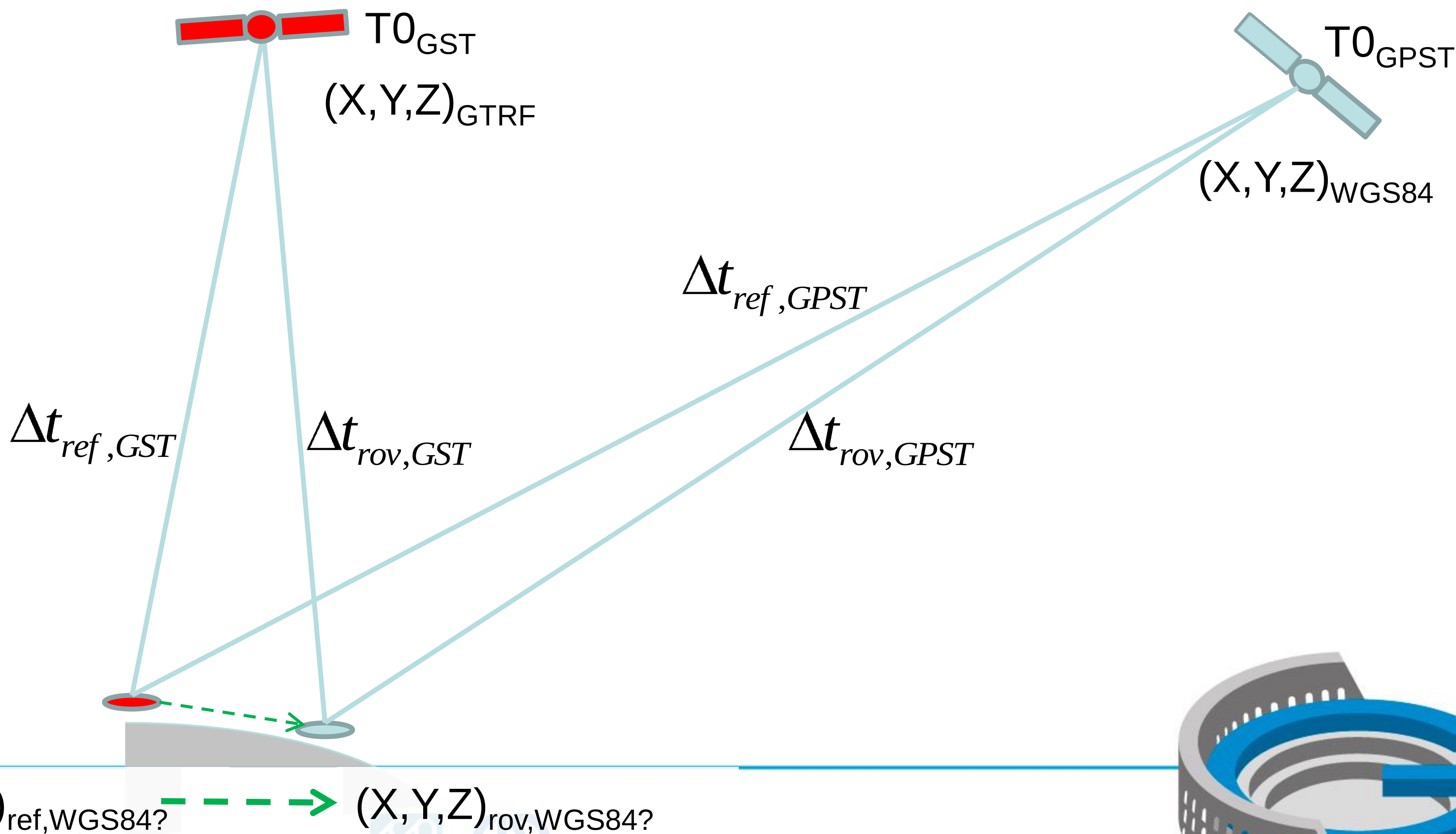
→

$$l = \Phi_{1,ij}^{kl} - (\rho_{ij}^{kl} + \lambda_1 N_{1,ij}^{kl} + \varepsilon_{1,ij}^{kl})$$

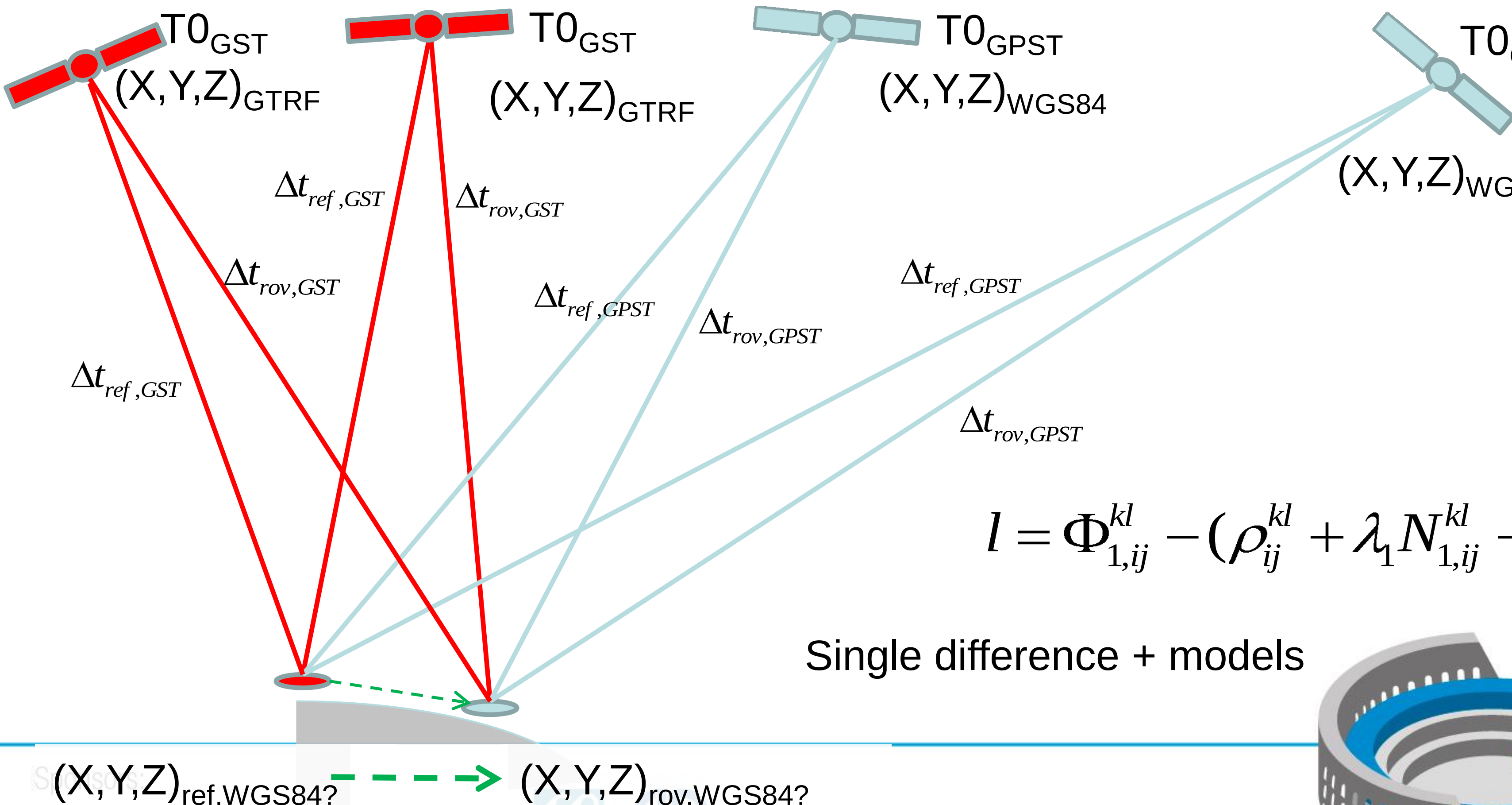
$$\Phi_{2,ij}^{kl} = \rho_{ij}^{kl} + \lambda_2 N_{2,ij}^{kl} + \varepsilon_{2,ij}^{kl}$$

$$\Phi_{5,ij}^{kl} = \rho_{ij}^{kl} + \lambda_5 N_{5,ij}^{kl} + \varepsilon_{5,ij}^{kl}$$

# Relative Positioning - Land/Engineering surveying



# Relative Positioning - Land/Engineering surveying



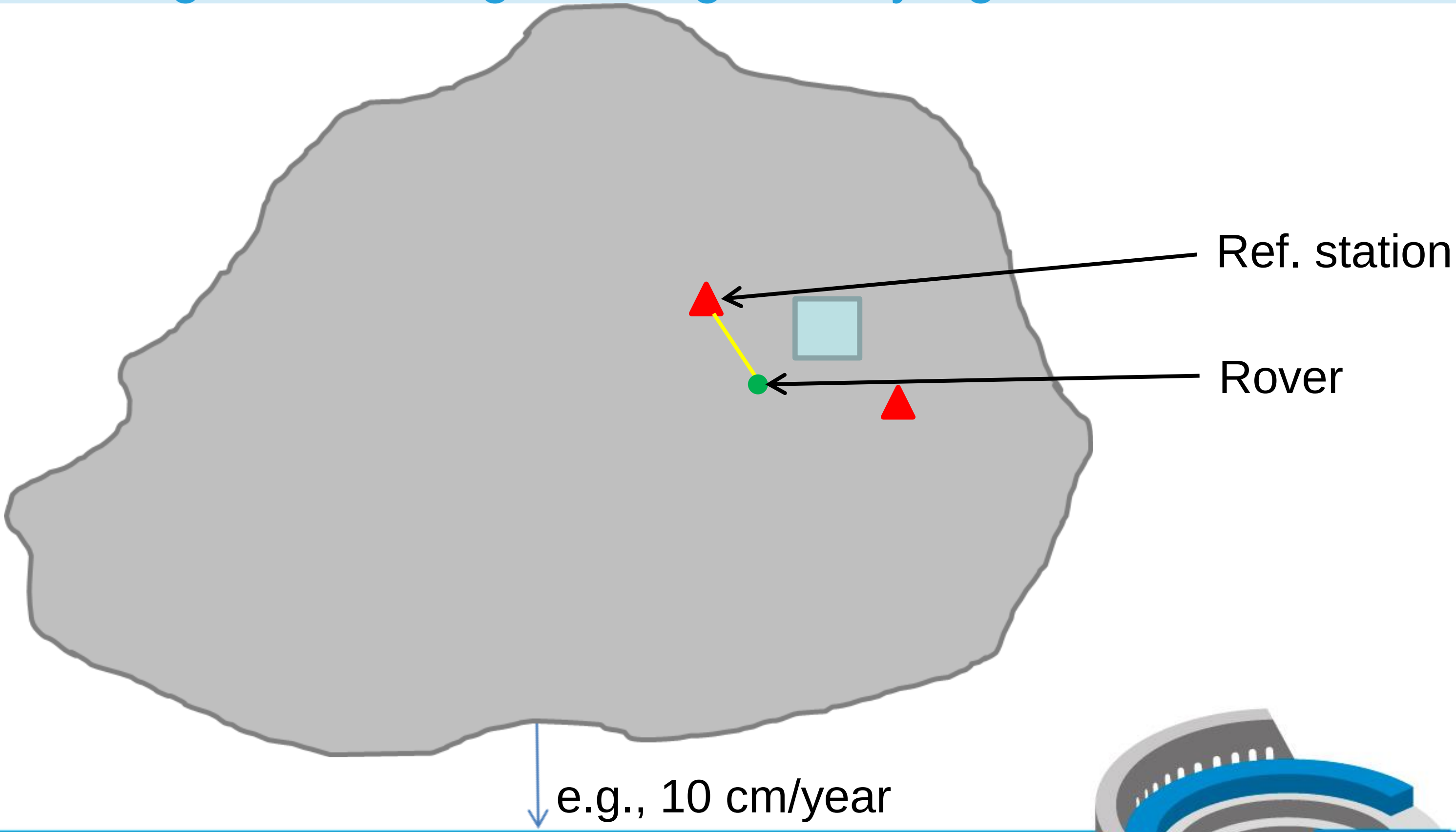

$$l = \Phi_{1,ij}^{kl} - (\rho_{ij}^{kl} + \lambda_1 N_{1,ij}^{kl} + \varepsilon_{1,ij}^{kl})$$

Single difference + models

# Relative Positioning - Land/Engineering surveying

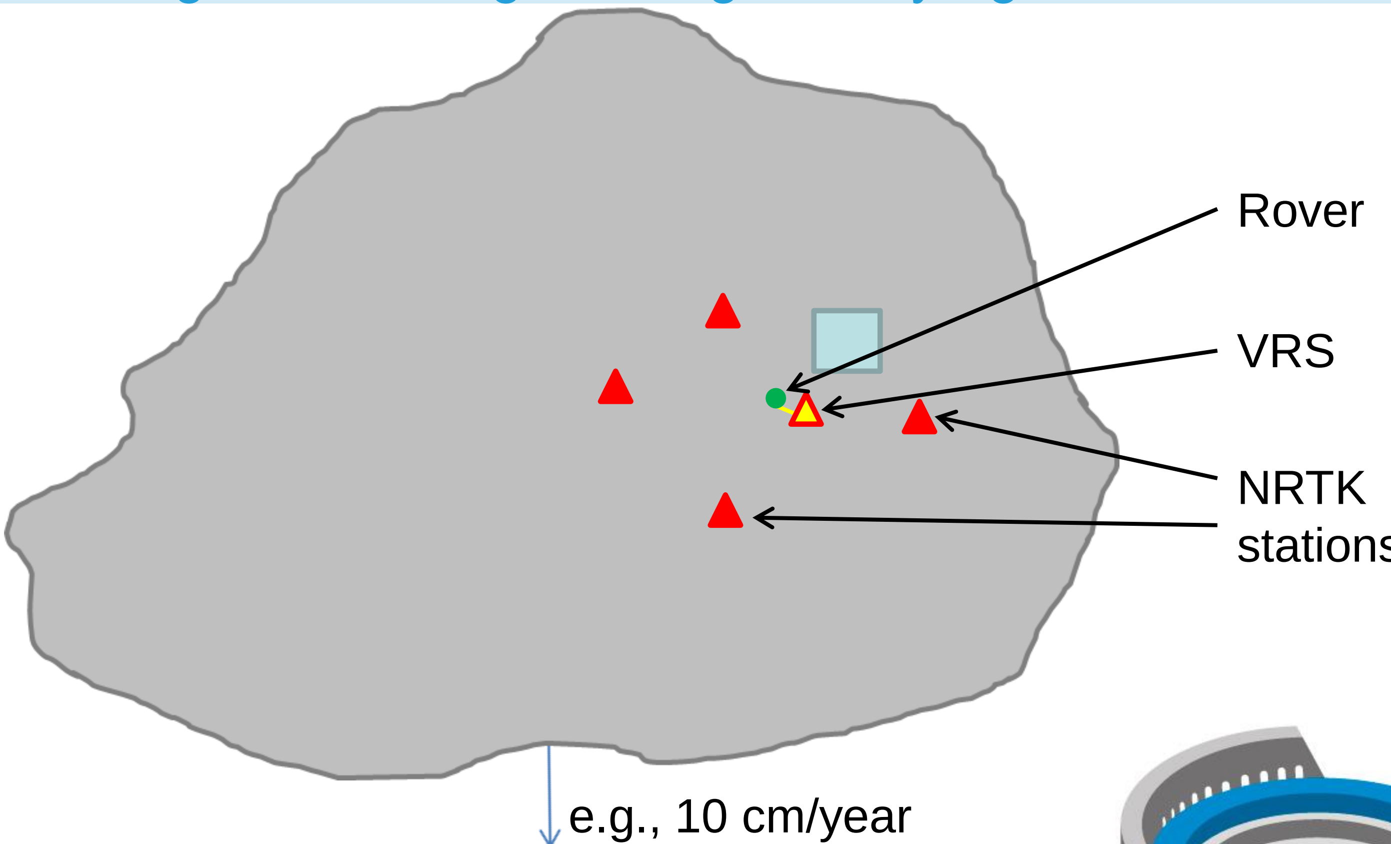
$$
Ax=\begin{bmatrix}
\frac{X_R-X^{G2}}{\rho_R^{G2}}-\frac{X_R-X^{G1}}{\rho_R^{G1}} & \frac{Y_R-Y^{G2}}{\rho_R^{G2}}-\frac{Y_R-Y^{G1}}{\rho_R^{G1}} & \frac{Z_R-Z^{G2}}{\rho_R^{G2}}-\frac{Z_R-Z^{G1}}{\rho_R^{G1}} & \lambda & 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & 0 & \ddots & 0 & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\frac{X_R-X^{Gn}}{\rho_R^{Gn}}-\frac{X_R-X^{G1}}{\rho_R^{G1}} & \frac{Y_R-Y^{Gn}}{\rho_R^{Gn}}-\frac{Y_R-Y^{G1}}{\rho_R^{G1}} & \frac{Z_R-Z^{Gn}}{\rho_R^{Gn}}-\frac{Z_R-Z^{G1}}{\rho_R^{G1}} & 0 & 0 & \lambda & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\frac{X_R-X^{E2}}{\rho_R^{E2}}-\frac{X_R-X^{E1}}{\rho_R^{E1}} & \frac{Y_R-Y^{E2}}{\rho_R^{E2}}-\frac{Y_R-Y^{E1}}{\rho_R^{E1}} & \frac{(Z_R-Z^{E2})}{\rho_R^{E2}}-\frac{(Z_R-Z^{E1})}{\rho_R^{E1}} & 0 & \cdots & 0 & \lambda & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & 0 & \ddots & 0 & \vdots & \ddots & \vdots \\
\frac{(X_R-X^{En})}{\rho_R^{En}}-\frac{(X_R-X^{E1})}{\rho_R^{E1}} & \frac{(Y_R-Y^{En})}{\rho_R^{En}}-\frac{(Y_R-Y^{E1})}{\rho_R^{E1}} & \frac{(Z_R-Z^{En})}{\rho_R^{En}}-\frac{(Z_R-Z^{E1})}{\rho_R^{E1}} & 0 & \cdots & 0 & 0 & 0 & \lambda & 0 & \cdots & 0 \\
\frac{(X_R-X^{B2})}{\rho_R^{B2}}-\frac{(X_R-X^{B1})}{\rho_R^{B1}} & \frac{(Y_R-Y^{B2})}{\rho_R^{B2}}-\frac{(Y_R-Y^{B1})}{\rho_R^{B1}} & \frac{(Z_R-Z^{B2})}{\rho_R^{B2}}-\frac{(Z_R-Z^{B1})}{\rho_R^{B1}} & 0 & \cdots & 0 & 0 & \cdots & 0 & \lambda & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & 0 & \ddots & 0 \\
\frac{(X_R-X^{Bn})}{\rho_R^{Bn}}-\frac{(X_R-X^{B1})}{\rho_R^{B1}} & \frac{(Y_R-Y^{Bn})}{\rho_R^{Bn}}-\frac{(Y_R-Y^{B1})}{\rho_R^{B1}} & \frac{(Z_R-Z^{Bn})}{\rho_R^{Bn}}-\frac{(Z_R-Z^{B1})}{\rho_R^{B1}} & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & 0 & \lambda
\end{bmatrix}
\begin{bmatrix}
\Delta X_R \\ \Delta Y_R \\ \Delta Z_R \\ N_R^{G2G1} \\ \vdots \\ N_R^{GnG1} \\ N_R^{E2E1} \\ \vdots \\ N_R^{EnE1} \\ N_R^{B2B1} \\ \vdots \\ N_R^{BnB1}
\end{bmatrix}
$$

# Relative Positioning - Land/Engineering surveying



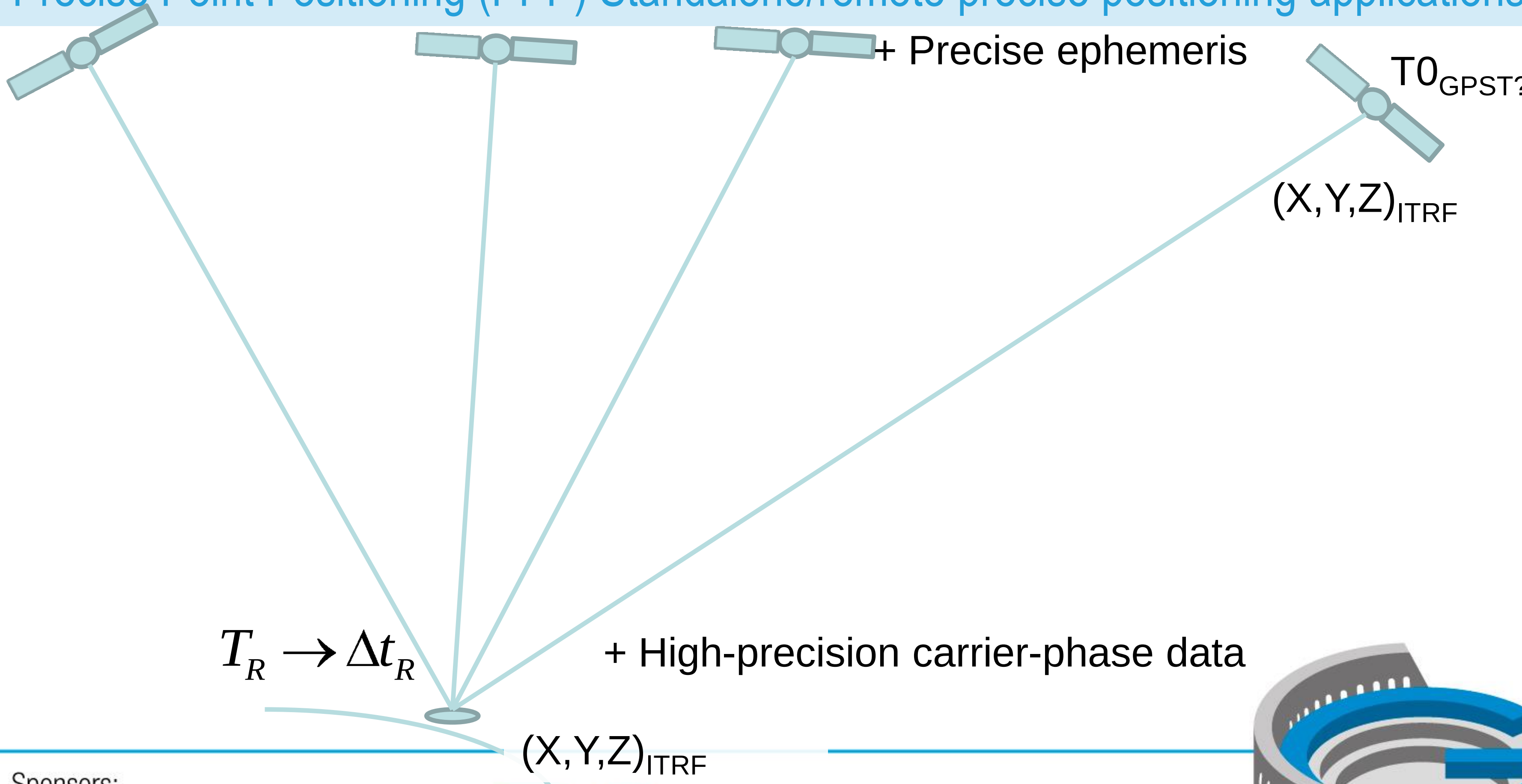

Intraplate earthquake and other local deformation such as subsidence and landslides!!

# Relative Positioning - Land/Engineering surveying




Intraplate earthquake and other local deformation such as subsidence and landslides!!

# Precise Point Positioning (PPP) Standalone/remote precise positioning applications



## Precise Point Positioning (PPP) Standalone/remote precise positioning applications

$$P_i^k = \rho_i^k + (\delta t_i - \delta t^k)c + I_i^k + T_i^k + \varepsilon_{i,P}^k$$

Iono-free

$$f_P = \rho_i^k + c\delta t_i + MF \cdot ZTD + \varepsilon_{i,P}^k - P_i^k = 0$$

$$\Phi_i^k = \rho_i^k + (\delta t_i - \delta t^k)c - I_i^k + T_i^k + \lambda N_i^k + \varepsilon_{i,\Phi}^k$$

Iono-free

$$f_\Phi = \rho_i^k + c\delta t_i + MF \cdot ZTD + \lambda N_i^k + \varepsilon_{i,\Phi}^k - \Phi_i^k = 0$$

## Precise Point Positioning (PPP) Standalone/remote precise positioning applications

Phase

PR

$$
Ax=\begin{bmatrix}
-\frac{X^{G1}-X_R}{\rho_R^{G1}} & -\frac{Y^{G1}-Y_R}{\rho_R^{G1}} & -\frac{Z^{G1}-Z_R}{\rho_R^{G1}} & c & 0 & MF_R^{G1} & \lambda_{L1} & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & \ddots & 0 & \vdots & \ddots & \vdots \\
-\frac{X^{Gn}-X_R}{\rho_R^{Gn}} & -\frac{Y^{Gn}-Y_R}{\rho_R^{Gn}} & -\frac{Z^{Gn}-Z_R}{\rho_R^{Gn}} & c & 0 & MF_R^{Gn} & 0 & 0 & \lambda_{L1} & 0 & \cdots & 0 \\
-\frac{X^{E1}-X_R}{\rho_R^{E1}} & -\frac{Y^{E1}-Y_R}{\rho_R^{E1}} & -\frac{Z^{E1}-Z_R}{\rho_R^{E1}} & 0 & c & MF_R^{E1} & 0 & \cdots & 0 & \lambda_{L1} & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & 0 & \ddots & 0 \\
-\frac{X^{En}-X_R}{\rho_R^{En}} & -\frac{Y^{En}-Y_R}{\rho_R^{En}} & -\frac{Z^{En}-Z_R}{\rho_R^{En}} & 0 & c & MF_R^{En} & 0 & \cdots & 0 & 0 & 0 & \lambda_{L1} \\
-\frac{\left(X^{G1}-X_R\right)}{\rho_R^{G1}} & -\frac{\left(Y^{G1}-Y_R\right)}{\rho_R^{G1}} & -\frac{\left(Z^{G1}-Z_R\right)}{\rho_R^{G1}} & c & 0 & MF_R^{G1} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
-\frac{\left(X^{Gn}-X_R\right)}{\rho_R^{Gn}} & -\frac{\left(Y^{Gn}-Y_R\right)}{\rho_R^{Gn}} & -\frac{\left(Z^{Gn}-Z_R\right)}{\rho_R^{Gn}} & c & 0 & MF_R^{Gn} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
-\frac{\left(X^{E1}-X_R\right)}{\rho_R^{E1}} & -\frac{\left(Y^{E1}-Y_R\right)}{\rho_R^{E1}} & -\frac{\left(Z^{E1}-Z_R\right)}{\rho_R^{E1}} & 0 & c & MF_R^{E1} & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
-\frac{\left(X^{En}-X_R\right)}{\rho_R^{En}} & -\frac{\left(Y^{En}-Y_R\right)}{\rho_R^{En}} & -\frac{\left(Z^{En}-Z_R\right)}{\rho_R^{En}} & 0 & c & MF_R^{En} & 0 & \cdots & 0 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
\Delta X \\ \Delta Y \\ \Delta Z \\ \Delta t_{R,G} \\ \Delta t_{R,E} \\ ZTD \\ N_R^{G1} \\ \vdots \\ N_R^{Gn} \\ N_R^{E1} \\ \vdots \\ N_R^{En}
\end{bmatrix}
=l
$$

## Precise Point Positioning (PPP) Standalone/remote precise positioning applications

- Site displacement effects
  - Solid Earth tides
  - Ocean loading
  - Sub-daily Earth Rotation Parameters (i.e. Pole position Xp, Yp and UT1-UTC)
- Satellite attitude effects
  - Satellite antenna offset
  - Phase wind-up correction
- IFB/ICB

## Summary and Discussion

- Transform all GNSS reference frames to a reference frame
- Align all GNSS reference time to a reference time
- Multiple GNSS Benefits and Residuals
  - SPP
    - Reference frames
    - Reference time
  - Relative positioning
    - Reference frames
    - Reference time
  - PPP
    - Reference frames
    - Reference time

## END

# Thank you for your attention

# Local Datum

- Difference to Local Datum
  - depends on the time of synchronization with ITRF or other GNSS reference frames
- Tectonic movement of the corresponding plate
- Preferably there is a set of transformation parameters between the national datum and the selected GNSS reference frame