Mathematical Models and A Case Study of The Locata Deformation Monitoring System (LDMS)

Mazher CHOUDHURY, Bruce HARVEY, Chris RIZOS, Australia

Key words: Locata, deformation monitoring systems

SUMMARY

Locata Corporation’s positioning technology “Locata” is a terrestrial-based RF-ranging technology that provides high accuracy position solutions using a network (LocataNet) of time synchronised pseudolite-like transceivers (LocataLites). LocataLites can be installed in locations to ensure optimal network geometry for a certain coverage area. With batch least squares processing, millimetre precision is achievable and thus provides an opportunity to address shortcomings of current deformation monitoring systems. A prototype of a Locata Deformation Monitoring System (LDMS) has been developed, consisting of four fundamental parts, namely data acquisition, quality control and validation, coordinate solution, and deformation monitoring. This paper focuses on the mathematical models. To validate the feasibility of this system, simulation tests and a deformation monitoring field trial have been conducted.

Dam simulation studies have been conducted at The University of New South Wales using four LocataLites and six monitoring points for deformation analysis with the LDMS. An artificial deformation of 10 millimetres has been injected into three components of a monitoring point’s coordinates, and the LDMS identified the deformed point successfully as well as the amount of deformation.

A deformation monitoring trial has also been conducted at the Tumut Pond Dam in Cabramurra, NSW, Australia, where long term deformation history is available for comparison. After analysing the deformation network geometry suitable for millimetre-level precision of position components, and LocataLites’ line-of-sight visibility, four LocataLites, one receiver and a weather station have been installed. Measurement data, including pseudorange, carrier phase and atmospheric data, have been collected and post-processed for deformation analysis using the batch least squares method by the LDMS. After generating 3D coordinate solutions, statistically significant movement (deformation) was identified by standard deformation analysis. These results have been compared with previous total station based deformation surveys which show that Locata can continuously deliver high accuracy positioning.
1. INTRODUCTION

Deformation monitoring of man made engineering structures is sometimes necessary due to risk of catastrophic outcomes from any failure of such engineering structures. There are many different deformation monitoring systems based on a variety of technologies, however the most popular 24/7 deformation monitoring system is GPS. The main advantage for using GPS is the capabilities of continuous monitoring and the centimetre-level achievable accuracy using relative positioning and carrier phase-based methods. On the other hand, the accuracy, availability, reliability and integrity of GPS solutions depend on the number and geometric distribution of the visible satellites. To overcome these GPS deficiencies, Locata Corporation’s positioning technology “Locata” provides position solutions using a network (LocataNet) of time synchronised pseudolite transceivers (Locatalites). These Locatalites transmit ranging signals in the licence-free 2.4GHz Industry Scientific and Medical (ISM) band that can penetrate through different materials (Tambuwala et al., 2007). When a Locata receiver tracks four or more Locatalite signals, it can compute a high accuracy position entirely independent of GPS. An advantage of the Locata technology is that Locatalites can be placed almost anywhere for better network geometry. With a batch least square method millimetre precision can be achieved, which provides an advantage and a new opportunity to address shortcomings of current automated deformation monitoring systems (ADMS). Barnes et al. (2007a, 2007b, 2004, 2002) have demonstrated that the Locata technology can be used for deformation monitoring, with high accuracy, availability, reliability and integrity of the position solution, through a number of experiments. However, to the best of our knowledge, an ADMS has not been implemented for Locata. This paper describes a prototype ADMS for Locata technology.

This paper is organised as follows. In section 2, the ADMS software is briefly reviewed; in section 3, the functional principles of Locata measurements and Locata coordinate solution are presented; in section 4, the system architecture of the Locata Deformation Monitoring System (LDMS) is outlined. Section 5 presents the achievable precision of the resulting system. Finally, sections 6 and 7 present conclusions and discuss future research and development activities.

2. ADMS SOFTWARE

Software is a key element of an ADMS. An enormous amount of data can be collected from a number of rovers, processed and analysed, to detect deformation (change in position). There are several ADMS software packages, including ALERT (Wilkins et al., 2003), UGPS (Andersson, 2008), GOCA (Jäger, 2005), from different university and research groups. In
addition, vendor-based ADMS software is available, such as Leica’s GeoMoS. Fundamentally all of these software packages have similar functionality. Software components can be divided into data acquisition, quality control and validation, position solution (through carrier phase differential correction if GPS-based) and quality control, and statistical testing for deformation alarm. In the data acquisition component the measurements are collected from receivers via some form of network communications, e.g. by ethernet or wireless communications. These measurements are checked for random, systematic and gross errors, cycle slips (if GPS), and possibly against vendor-specified quality control criteria. Once all the checking is completed then the position solution module determines coordinates from “clean” measurements. Once again the position solutions are checked for outliers, gross errors, etc. Baarda data snooping, Pope’s Tau Test and Danish Method are common statistical tests for outlier/gross error detection (Caspary, 2000; Harvey, 2006).

The Locata Deformation Monitoring System (LDMS) is similar to other ADMS software modules. However, this prototype LDMS is developed using MATLAB. The LDMS is divided into four fundamental parts, namely data acquisition, quality control and validation, position solution through carrier phase differential correction, and statistical testing for deformation alarm.

3. LOCATA COORDINATE SOLUTION

3.1 Locata coordinate solution

The basic Locata carrier phase observation equation between receiver $A$ and LocataLite $t$ is

$$\rho^t_A = \phi^t_A \ast \lambda + \tau^t_\text{prop,}A + c \cdot \delta T_A + N^t_A \ast \lambda + \varepsilon^t_\phi$$  \hspace{1cm} (1)

where $\phi^t_A$ is the carrier phase observation in cycles, $\lambda$ the wavelength of the signal, $\tau^t_\text{prop,}A$ the tropospheric delay, $c \cdot \delta T_A$ the receiver clock error of the receiver $A$, $N^t_A$ the ambiguity and $\varepsilon^t_\phi$ unmodelled residual errors. Note that there is no transmitter clock error present in the observation equation due to the time synchronisation of the LocataLites.

To cancel out the receiver clock error and reduce systematic biases in the measurements, the carrier phase observations of the same frequency are single-differenced. When $i$ is chosen as the reference signal, and $t$ is another signal of the same frequency, the single-difference $\Delta \rho^t_i$ is defined as

$$\Delta \rho^t_i = \rho^t_A - \rho^t_i = (\phi^t_A \ast \lambda + \tau^t_\text{prop,}A + c \cdot \delta T_A + N^t_A \ast \lambda + \varepsilon^t_\phi) - (\phi^t_i \ast \lambda + \tau^t_\text{prop,}i + c \cdot \delta T_i + N^t_i \ast \lambda + \varepsilon^t_\phi)$$ \hspace{1cm} (2)

The unknown parameters are the receiver coordinates $(x_A, y_A, z_A)$ in the single-differenced range $\Delta \rho^t_i$. 
\[
\rho_{ij}^t = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} - \sqrt{(x_f - x_j)^2 + (y_f - y_j)^2 + (z_f - z_j)^2}
\]  

(3)

where \((x_i, y_i, z_i)\) and \((x_j, y_j, z_j)\) are the antenna coordinates of the LocataLites that transmit the signals \(i\) and \(j\). The tropospheric delays in \(\Delta t_{\text{trop}}\) must be estimated by an appropriate tropospheric model, and the receiver clock error has cancelled.

As a result, the model equation for least squares estimation is

\[
F(X) = F(x_i, y_i, z_i) = (\phi_{x_i}^t + \lambda + \tau_{\text{trop}}^t + c \cdot \delta T_i + N_{x_i} + \xi_{x_i}^t) - (\phi_{x_j}^t + \lambda + \tau_{\text{trop}}^t + c \cdot \delta T_j + N_{x_j} + \xi_{x_j}^t) - \Delta \rho_{ij}^t = 0
\]

(4)

The observations for equation (4) are \(\phi_{x_i}^t, \phi_{x_j}^t\) and the parameters are \(x: [x_i, y_i, z_i]\).

Partial derivatives matrix for observations from equation (3) are

\[
A_{ij}^n = \begin{bmatrix}
\frac{dF(X)}{dx_i} & \frac{dF(X)}{dy_i} & \frac{dF(X)}{dz_i} \\
\frac{dF(X)}{dx_j} & \frac{dF(X)}{dy_j} & \frac{dF(X)}{dz_j}
\end{bmatrix}
\]

where \(n\) is the number of observations.

Locata coordinate solution is obtained from an iterative least squares estimation process where

\[
\Delta x = (A' \cdot P \cdot A)^{-1} \cdot (A' \cdot P \cdot b)
\]

\(\Delta x = (N)^{-1} \cdot (W)\)

where

\(N = A' \cdot P \cdot A\)

\(W = A' \cdot P \cdot b\)

\(b = [\phi_{x_i}^t]_{\text{observed}} - [\phi_{x_i}^t]_{\text{calculated}}\)

The parameters are then updated

\[x = x + \Delta x\]

(8)

A is partial derivative matrix, \(P\) is inverse of input quality of observations matrix and \(b\) is a misclose vector, which is observed distance minus calculated distance from initial coordinates \(x\) (using equation (8)).
However, for the deformation monitoring application the precisions of monitoring points are also important. The precision of each position component can be generated using the following equation: $\sqrt{\text{Diagonal terms of } (N^{-1})}$

To achieve higher precision (i.e. millimetre), as well to reduce systematic biases in the measurements, multiple epochs are often used for generating solutions for a point. At each epoch the Locata deformation network is separately adjusted, and each adjustment is checked for outliers. Data definition for each epoch consists of five seconds of accumulated Locata observations. This epoch definition ensures sub-millimetre precision in horizontal position components and millimetre-level precision in the vertical component. To do so, the partial derivative matrix ($A$), the ‘weight’ matrix ($P$) and the misclose vector ($b$) will have data from multiple epochs.

$$A_{m \times n} = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mn} \end{pmatrix} \quad b_{m \times 1} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

where $m$ is the number of observations.

### 3.2 Deformation monitoring solution

This deformation monitoring solution provides the statistical testing for deformation testing. It uses the method described in Harvey (2006, pg, 293) for finding the stability of the monitoring point. Mathematically this unit follows the same principle as the previous section, however the difference is that instead of using one epoch at a time, two epochs ($1^{\text{st}}$ epoch as reference and latest epoch) of data are used for the least squares adjustment to ensure that systematic biases will be reduced. This is also necessary for developing the standard deviation as well as the relative error ellipse of the distance between the positions generated by the reference epoch and latest epoch datasets. Further details on deformation monitoring methods can be found in Harvey (2006), and flow charts of the least squares adjustment as well as deformation monitoring solution can be found in Choudhury et al. (2009).
4. SYSTEM ARCHITECTURE OF THE LOCATA DEFORMATION MONITORING SYSTEM (LDMS)

The prototype LDMS is comprised of four fundamental parts, namely data acquisition, quality control and validation, position solution through carrier phase differential correction, deformation monitoring unit and data management.

4.1 Data acquisition

The data acquisition part has the task of collecting data from all receivers in the network. The system consists of a number of small Locata receivers installed on the object to be monitored; a weather station for collecting temperature, humidity and pressure measurements; and a standard Locata network consisting of a minimum of four LocataLites. However, more Locatalites are preferred for high precision and better network geometry. These receivers are remotely accessible through a dedicated communication channel, including an internet dial-up connection. Every receiver is configured to collect Locata observation data for a sufficient amount of time, and the data will be transmitted (or collected manually) to the data acquisition unit for post-processing.

4.2 Quality control and validation

The quality control and validation part has the task of identifying gross errors, cycle slip detection and correction, monitoring Locata-defined quality control checking such as minimum signal-to-noise ratio, maximum pseudo-range and carrier phase difference, maximum carrier phase difference between successive epochs and cluster carrier phase differences.

4.3 Coordinate solution

The coordinate solution component provides the point coordinates using a batch least squares estimation process (described in section 3). This component also checks for outliers using Pope’s Tau test, checks the unit variance factor and the ‘weight’ matrix of the adjusted coordinates, checks for v/s (residuals / standard deviation of observation) for systematic trends, and lastly outputs dilution of precision and position accuracy.

4.4 Deformation monitoring unit

The deformation monitoring unit undertakes the statistical testing. Point movement detection involves the LSE (described in section 3) process. However, this LSE uses both epochs’ measurements for adjustment and to derive the combined variance covariance matrix (which is required for computing the relative error ellipses), calculating the distances (displacements) between the two epoch’s positions of each point and to generate the related standard deviations. Statistically significant movement of a point is associated with the 95% confidence interval of the displacement of points between epochs. This will be referred as the ‘statistical threshold’ for the remainder of this paper. Formulas and procedures for identifying
the ‘statistical threshold’ are described in Harvey (2006) and USACE (2002). This unit also outputs the error ellipse and deformation vector in the local coordinate system (east, north and height).

### 4.5 Data management

The data management component stores the data for future reference as well as deformation analysis. Figure 1 is a system diagram of the LDMS system.

![System Diagram of LDMS System](image)

**Figure 1: Components of the LDMS system**

### 5. SIMULATION AND RESULTS

#### 5.1 Simulation setup

The primary objective of a slow structural deformation monitoring application is to detect millimetre to centimetre-level movement. Before starting real life experiments, simulated movement experiments were conducted at The University of New South Wales (UNSW), Sydney, Australia, using four LocatalLites and one monitoring point. After data collection, the LDMS was used for deformation analysis. An artificial total movement of 1 centimetre was injected into the three components of the monitoring point’s coordinate. The LDMS software was used for detecting simulated deformation in a Locata network. This experiment was conducted in August 2009 at UNSW. Figure 2 is the Locata network at UNSW. In this experiment four LocataLites were used together with one Locata receivers on known pre-surveyed positions for one hour. The receiver was configured to output data at 2Hz and the data definition (described in section 3.1) for each epoch defined as five seconds of accumulated Locata observations.
5.2 Simulation results

Position solutions for the monitoring point are plotted in Figure 3 and the precision of the point is presented in Figure 4. It can be easily observed that the horizontal component has sub-millimetre precision whereas precision of height component is between 2 to 3.5 millimetre. Simulated deformation of 1 cm has been introduced into point 1. Figure 5 shows the standard situation when no simulated deformation is added, whereas Figure 6 presents the effect of simulated deformation, which is movement above the statistical threshold.
6. EXPERIMENT AND RESULTS

6.1 Experiment setup

A deformation monitoring trial using Locata as well as total station were also conducted at the Tumut Pond Dam (Figure 7) near Cabramurra, NSW, Australia, where long term deformation history is available for comparison purposes. The Tumut Pond Dam, as part of the Snowy Mountains Scheme, collects the inflow from the Tumut River to form the Tumut Pond Reservoir. Water is diverted through the Tooma-Tumut and the Eucumbene-Tumut tunnels to provide the head pondage for the Tumut 1 Power Station. The dam is 86.3m high and 217.9m long. The crest and base width are 3.7m and 29.6m respectively (Snowy Hydro Limited, 2003).

After analysing the deformation network geometry suitable for millimetre-level precision of position components, and the LocataLites’ line-of-sight visibility, four LocataLites, one receiver and a weather station were installed (Figure 8). Measurement data, including pseudo-range, carrier phase and atmospheric data, were collected and post-processed for deformation analysis by the LDMS. After generating 3D coordinate solutions, statistically significant movement (i.e. deformation) was identified using the standard deformation analysis described above.
Before measuring the deformation monitoring trial, a simulated Locata network was designed using the FIXIT 3 software tool (Harvey, 2006). During this simulation the Locata network was analysed for redundancy number, error ellipse of the monitoring points and geometric strength of the Locata network. The points are all near one plane and analysis found that heights of the monitoring point can not be reliably determined. Figure 9 shows the geometry of the Locata network in 2D view and the error ellipse for the monitoring point with respect to the fixed LocataLite transmitter sites. Simulation has shown that the monitoring point’s standard deviation would be 1.3 millimetre, and 0.8 millimetre for east and north components respectively. The semi-major, semi-minor and azimuth of the error ellipse would be 1.3mm, 0.7mm and 89° respectively. For this simulation it has been assumed that each Locata observation’s standard deviation is 5 millimetre, and that observations are uncorrelated and not affected by systematic or gross error. Results from this simulation show that millimetre-level precision is achievable for an experiment at the Tumut Pond Dam.
6.2 Results and analysis

The Locata network at the Tumut Pond Dam was configured to operate for 24 hours. However, battery power only lasted for 22 hours. To detect point movement at higher precision, data definition for each epoch has been defined as five seconds of accumulated Locata observations. The coordinates of the LocataLite transmitter sites and the height of the monitored point were held fixed at values determined by reliable total station surveying. The monitoring point’s position solutions are shown in Figures 10(a) and 10(b), where the standard deviation of the east and north components are 0.4 millimetre and 0.2 millimetre respectively. The corresponding 95% confidence levels for these values are 0.8mm and 0.4mm.

Figure 10(a): Position solution (3D)  Figure 10(b): Position solution (2D) remove the height figure

Figure 11 presents observable movements of the monitoring point. The maximum horizontal movement detected by LDMS was 2.5 millimetres. This horizontal movement of 2.5 mm for this structure over a 24 hr period is reasonable. There was only a very 0.04% (i.e. 606 out of 15665 epochs) epochs with higher movement than the statistical threshold and most importantly these are not continuous. These movements might be due to vehicle movement on the dam. In the case of continuous movements (like Figure 6), a deformation flag would be set for further investigation. However, this experiment was only conducted for one day, and different trends of movements might be observed from a longer experiment (i.e. weeks, months or years).
A survey robot (Leica TCRP1201) was used for comparison with the Locata system. The instrument measured horizontal and vertical angles and distances to a 25mm prism near the Locatalite. It was used in "lock" mode from a single setup, recording in 10 minute intervals at a distance of 176m. Only four hours of data were available for comparison with the Locata system. Although the system showed horizontal movement at about the 5mm level, the standard deviation of these coordinates was at +/- 6mm and hence was difficult to compare with Locata.

7. CONCLUDING REMARKS

LDMS has been developed with the help of the mathematical models discussed in this paper. To validate the usability of this system, simulation tests and a deformation monitoring field trial have been conducted. Simulation test confirm the validity of the mathematical models for Locata, and the field trial is the real life implementation of the system/software. LDMS works as post-processing software at this stage. In future versions, it is planned that the LDMS will be converted to a real time deformation monitoring system.

8. ACKNOWLEDGEMENT

The authors are grateful to John Browne and John Bartell from Snowy Hydro Limited for providing opportunities for setting up the Locata network, providing surveying expertise as well as supporting this study. Authors are also grateful to Nonie Politi and Aire Olesk from The University of New South Wales for organising this experiment.
REFERENCES


**BIOGRAPHICAL NOTES**

Mohammad Mazher-ul Alam Choudhury is currently a Ph.D. student at the School of Surveying & Spatial Information Systems, University of New South Wales (UNSW), Sydney, Australia. His current research area is integrating Locata technology into an automated deformation monitoring application. He holds a Masters in Computer Science Degree from UNSW, and B.E. (Computer Science) degree from North South University, Bangladesh.

Bruce Harvey is a Senior Lecturer at the School of Surveying & Spatial Information Systems, UNSW. Bruce has been researching least squares survey data analysis and alternatives, high precision industrial survey measurements, deformation monitoring surveys and 3D terrestrial laser scanning techniques since 1985.

Chris Rizos is currently Professor and Head of the School of Surveying & Spatial Information Systems, UNSW. Chris has been researching the technology and high precision applications of GPS since 1985, and has published over 400 journal and conference papers. He is a Fellow of the Australian Institute of Navigation and a Fellow of the International Association of Geodesy (IAG). He is currently the Vice President of the IAG and a member of the Governing Board of the International GNSS Service.

**CONTACTS**

Mr. Mazher Choudhury
School of Surveying & Spatial Information Systems,
Faculty of Engineering,
The University of New South Wales
Sydney, NSW 2052,
Australia
Tel. +6142335034 Fax + 61293137493
Email: mohammad.choudhury@student.unsw.edu.au
Web site: [www.gmat.unsw.edu.au](http://www.gmat.unsw.edu.au)