Tidal Effects to Absolute Gravity Measurements on The Off - Shore Islands

Minh Hoa and Nguyen Ngoc Lau Vietnam

SUMMARY

Measuring high accuracy absolute gravity on offshore islands is used not only for developing national gravity system but also for estimating sea level increase due to global weather change. However to receive non-tidal absolute gravity, gravity measurements must be corrected by tidal effects of diurnal and semi-diurnal period. This paper presents necessary formulas for the purpose. Our calculating results at Con Dao show that tidal correction values are much larger than instrumental accuracy.

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1. INTRODUCTION

Research of increasing sea level due to global weather change is pertaining to research of changing gravity of sea area. This is because increasing of sea level changes water volume in sea and oceans and this again leads to change gravity in sea and oceans. Changing the Earth gravity is only estimated by using repeat measurements of absolute gravity on offshore islands. The principles of this method were recommended in [1]. At present, instrument of measuring absolute gravity can achieve accuracy of $\pm 2-5\mu Gal$, for example, gravimeter GABL-E made by the Russian Academy of Sciences. However, the above accuracy can only be obtained on land because the processing software in the machine only accounts for effects of solid earth tide from the sun and the moon. For ocean tide, only zonal component is corrected (Honcasalo correction). To achieve expected accuracy, we have to take into account effects of diurnal tidal variation (the tesseral contribution) and semi-diurnal tidal variation (the sectorial contribution). Because of neglecting these contributions, Russian experts failed to measure absolute gravity at Con Dao on 24/10/2008.

This paper considers theoretical principles of diurnal and semidiurnal tidal effects on absolute gravity and estimates their values at Con Dao.

2. DIURNAL AND SEMI-DIURNAL TIDAL EFFECTS ON ABSOLUTE GRAVITY

Potential at P on the earth surface caused by attractions from the sun and moon at a time is in second-order harmonic term [2, 3]:

$$W_t^{'} = \frac{fM \cdot R^2}{2 \cdot r_0^3} \cdot \left(3 \cdot Cos^2 Z_0 - 1\right), . \qquad (1)$$

Where fM – the earth gravitational constant, r_0 - geocentric vector of mass L (Moon, Sun), R - geocentric vector of P, Z_0 is zenith distance (angle) of L at P and expressed by following:

$$CosZ_0 = Sin\varphi \cdot Sin\,\delta + Cos\varphi \cdot Cos\delta \cdot Cost, \qquad (2)$$

where φ - latitude of point P; δ - declination of L on the equator; t – hour angle of L.

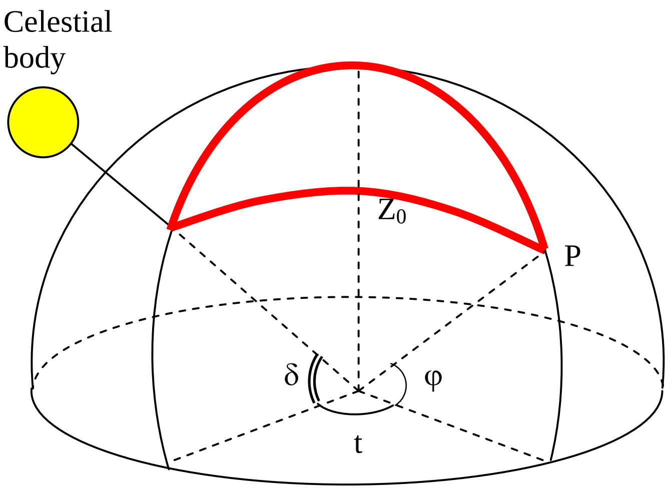

Figure 1. Relationship between latitude, declination and hour angle

Formulas (1), (2) are used to compute gravitational corrections of solid earth tide effects at point P (crust movement by lunar and sonar attractions).

Due to the sun and the moon, on sea and ocean area there are many kinds of tidal waves. They change mass distribution of sea and ocean and this leads to change the Earth gravity field in inshore areas. Therefore high accuracy gravity measurements at inshore areas and offshore islands must be corrected effects of solid Earth tide and Ocean tide loading.

We now consider (1) to determine disturbing potential due to tidal waves on sea and ocean. Noted (2), we have:

$$Cos^2 Z_0 = Sin^2\delta.Sin^2\varphi + 2.Sin\delta.Cos\delta.Sin\varphi.Cos\varphi.Cost + Cos^2\delta.Cos^2\varphi.Cos^2 t. \quad (3)$$

The second term of the right of (3):

$$2.Sin\delta.Cos\delta.Sin\varphi.Cos\varphi.Cost = \frac{1}{2}.Sin2\delta.Sin2\varphi.Cost; \quad (4)$$

Since $Cos^2 t = \frac{1}{2}.(Cos2t + 1)$,

$$Cos^2\delta.Cos^2\varphi.Cos^2 t = \frac{1}{2}.Cos^2\delta.Cos^2\varphi.Cos2t + \frac{1}{2}.Cos^2\delta.Cos^2\varphi. \quad (5)$$

Substituting (4) and (5) into (3) we have:

$$Cos^2 Z_0 = [Sin^2\delta.Sin^2\varphi + \frac{1}{2}.Cos^2\delta.Cos^2\varphi] + \frac{1}{2}.Sin2\delta.Sin2\varphi.Cost + \\ + \frac{1}{2}.Cos^2\delta.Cos^2\varphi.Cos2t. \quad (6)$$

In the other hand,

$$[Sin^2\delta.Sin^2\varphi + \frac{1}{2}.Cos^2\delta.Cos^2\varphi] = \frac{1}{2}[2.Sin^2\delta.Sin^2\varphi + Cos^2\delta.Cos^2\varphi] =$$

$$= \frac{1}{2}.\left[2.Sin^2\delta.Sin^2\varphi + (1 - Sin^2\delta).(1 - Sin^2\varphi)\right] = \frac{1}{2}.\left[3.\left(Sin^2\delta - \frac{1}{3}\right).\left(Sin^2\varphi - \frac{1}{3}\right) + \frac{2}{3}\right],$$

Hence (6) changes to:

$$Cos^2 Z_0 = \frac{1}{2}.\left[3.\left(Sin^2\delta - \frac{1}{3}\right).\left(Sin^2\varphi - \frac{1}{3}\right) + \frac{2}{3}\right] + \frac{1}{2}.Sin2\delta.Sin2\varphi.Cost +$$

$$+ \frac{1}{2}.Cos^2\delta.Cos^2\varphi.Cos2t,$$

or

$$Cos^2 Z_0 - \frac{1}{3} = \frac{1}{2}.\left[3.\left(Sin^2\delta - \frac{1}{3}\right).\left(Sin^2\varphi - \frac{1}{3}\right)\right] + \frac{1}{2}.Sin2\delta.Sin2\varphi.Cost +$$

$$+ \frac{1}{2}.Cos^2\delta.Cos^2\varphi.Cos2t, \qquad (7)$$

Replace (7) to (1), we get:

$$W_t^{'} = \frac{3}{4}.\frac{fM.R^2}{.r_0^3}\left\{\left[3.\left(Sin^2\delta - \frac{1}{3}\right).\left(Sin^2\varphi - \frac{1}{3}\right)\right] + Sin2\delta.Sin2\varphi.Cost +\right.$$

$$\left. + Cos^2\delta.Cos^2\varphi.Cos2t\right\}. \qquad (8)$$

The reference [3] provided (8) without any provident. Following [3], the first term on the right part of expression (8) does not depend on hour angle t of body L and is referred as **zonal contribution**. Term of $Sin^2\varphi - \frac{1}{3}$ equals zero at $\varphi = \pm 35^0 16'$. The period of $Sin^2\delta - \frac{1}{3}$ is 14 days for the moon and 6 months for the Sun. Therefore the zonal wave has long period. The effect of the zonal wave to the Earth gravity measurements are calculated by Honcasalo correction.

We denote M_0 and S_0 are zonal wave of the moon and the sun respectively. According to [3], the change of equipotential Δr and gravity under effect of M_0 are 8,9 cm and 27 μGal respectively, for S_0 are 4,1 cm and 13, 27 μGal.

The second term of (8) equals zero on the equator (φ = 0), the poles (φ = 90^0) and on meridian having t = 90^0. This term is called as **tesseral contribution**. It has maximum amplitude at $\varphi = \pm 45^0$ and zero amplitudes at the Earth equator and poles. Tesseral wave has a diurnal period, which is the same as function Cost. Tesseral waves are denoted as M_1 for the moon and S_1 for the sun.

The third term of (8) has the maximum value at the equator, zero value at the poles ($\varphi = 90^0$) and at meridians t = 45^0. This term is called sectorial wave. This wave has semi-diurnal period (the same as function Cos2t).

Sectorial wave due to lunar attraction is denoted as M_2, due to sonar attraction as S_2. Following [3], the change of equipotential Δr and gravity under effect of M_2 are 26,7 cm and 82 μGal, for S_2 are 12,3 cm and 38 μGal.

As a result, if we neglect effects of semi-diurnal and diurnal waves, there will be large errors in absolute gravity measurements at inshore areas and offshore islands. In the previous time, measuring absolute gravity with high accuracy ($\pm 2 - 5\mu Gal$) on offshore islands was not interested. However, when one requires to estimate sea level increase by the way of absolute gravity on offshore islands, the addition of corrections due to effect of diurnal and semi-diurnal tide becomes more urgent.

2.1. Building formulae for correction of gravity due to effects of diurnal tide

The disturbing potential at point P due to diurnal wave on sea and ocean can be determined as following formulae (directly from (8))

$$W_t^{'} = \frac{3}{4} \cdot \frac{fM.R^2}{r_0^3}.Sin2\delta.Sin2\varphi.Cost \qquad (9)$$

Change of gravity under effect of diurnal tide is:

$$\Delta g = -\frac{\partial W_t}{\partial R} = -\frac{3.fM \cdot R}{2.r_0^3} \cdot Sin2\delta.Sin2\varphi.Cost.$$

We can determine correction of gravity under effect of diurnal tide as:

$$\delta g = -\Delta g = \frac{3.fM \cdot R}{2.r_0^3} \cdot Sin2\delta.Sin2\varphi.Cost.$$

When accounting for Love's numbers:

$$\delta g = 1{,}2.\frac{3.fM \cdot R}{2.r_0^3} \cdot Sin2\delta.Sin2\varphi.Cost.$$

For the moon $fM_D = 4903 km^3.s^{-2}, (r_0)_D = 384395 km$ and for the sun $fM_\oplus = 132496.10^6 km^3.s^{-2}, (r_0)_\oplus = 149{,}5.10^6 km$ (see [3]). Assume R = 6371024 m and Love's number k = 1,2, we can get coefficients $1{,}2.\frac{3.fM_D \cdot R}{2.(.r_0^3)_D} = 98{,}773$ and $1{,}2.\frac{3.fM_\oplus \cdot R}{2.(.r_0^3)_\oplus} = 45{,}372$ for the moon and the sun respectively.

The correction of gravity due to total effects of the sun and the moon is:

$$\delta g_{tes} = \left(98{,}773.Sin2\delta_D.Cost_D + 45{,}372.Sin2\delta_\oplus.Cost_\oplus\right).Sin2\varphi \quad < \mu Gal >, \ (10)$$

where δ_D, t_D - declination and hour angle of the moon at interest time, $\delta_\oplus$, $t_\oplus$ - declination and hour angle of the sun at interest time.

2.2. Building formulae for correction of gravity due to effects of semi-diurnal tide

The disturbing potential at point P due to semi-diurnal wave on sea and ocean can be determined as following formulae (directly from (8)):

$$W_t^{'} = \frac{3}{4} \cdot \frac{fM.R^2}{r_0^3} .Cos^2 \delta .Cos^2 \varphi .Cos 2t$$

Change of gravity under effect of semi-diurnal tide is:

$$\Delta g = -\frac{\partial W_t}{\partial R} = -\frac{3.fM \cdot R}{2.r_0^3} \cdot Cos^2 \delta .Cos^2 \varphi .Cos 2t.$$

We can determine correction of gravity under effect of semi-diurnal tide as:

$$\delta g_{\sec} = -\Delta g = \frac{3.fM \cdot R}{2.r_0^3} \cdot Cos^2 \delta .Cos^2 \varphi .Cos 2t.$$

When accounting for Love's numbers:

$$\delta g_{\sec} = k.\frac{3.fM \cdot R}{2.r_0^3} \cdot Cos^2 \delta .Cos^2 \varphi .Cos 2t.$$

Similarly to 2.1, we can derive formulae for semi-diurnal tide correction of gravity due to the moon and the sun:

$$\delta g_{tes} = \left(98{,}773.Cos^2 \delta_D .Cos 2t_D + 45{,}372.Cos^2 \delta_{\oplus} .Cos 2t_{\oplus} \right) .Cos^2 \varphi \quad < \mu Gal >, \ (11)$$

3. EXPERIMENTS

In order to check corrections of gravity due to effects of diurnal tidal wave (10) and semi-diurnal tidal wave (11), we calculate these corrections on 24th October 2008 at Con Dao island. Lunar and sonar ephemeredes are downloaded from Website: fttp:/navigator.jpl.nasa/govephem/export .

The graph of diurnal corrections of gravity in time is given in figure 2

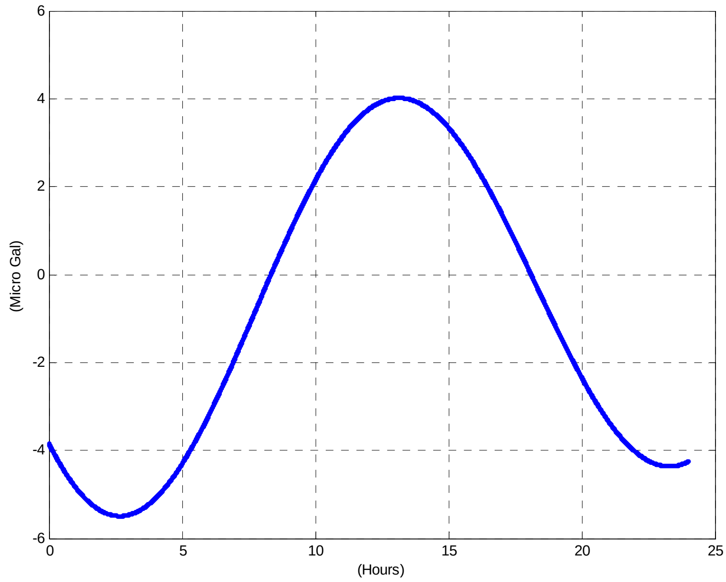

Figure 2. Diurnal corrections of gravity

Figure 2 shows that the maximum value of the diurnal correction of gravity is about ± 5 μGal, approximately allowable accuracy of the instrument (± 5 μGal).

The graph of semi-diurnal corrections of gravity in time is given in figure 3

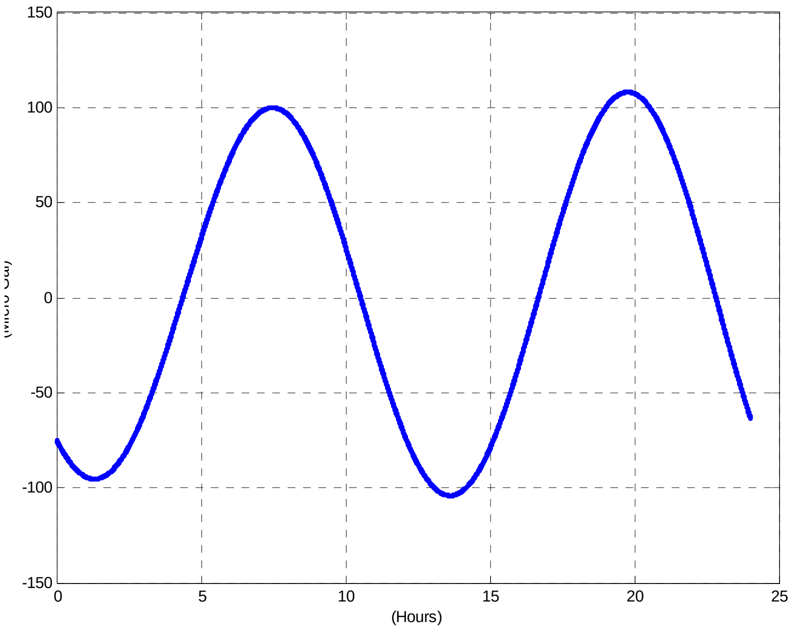

Figure 3. Semi-diurnal corrections of gravity

Figure 3 shows that the maximum value of the semi-diurnal corrections of gravity is about ± 100 μGal, approximately 20 times larger than allowable accuracy of the instrument (± 5 μGal).

These results show that why the absolute gravity measurement campaign at Con Dao island on 24/10/2008 was failed. It requires that the technique for measuring absolute gravity with high accuracy at inshore areas and offshore islands must account for diurnal and semi-diurnal corrections of gravity.

4. CONCLUSIONS

Research of diurnal and semi-diurnal tidal effects to absolute gravity measurements at inshore areas and offshore islands is important for developing national gravitational network, also for research of changing global gravity. Especially it is meaningful for solving sea level increasing problem. In this paper, we review some formulas for calculating corrections of gravity due to effects of diurnal and semi-diurnal tide. Our calculating results of these corrections at Con Dao showed that their values are much larger than accuracy of absolute gravimeter. This is the main reason why the campaign was not successful and requires further improvements on the firmware installed in these gravimeters.

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