Evaluating Large Volume GPS Data for Deformation Studies

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SUMMARY

Software packages for high precision GNSS data analysis can handle large-volume data satisfactorily. However, new challenges will soon appear to test the software to their limit. The scale-up of networks and the increased number of satellites, as well as the ongoing need for extremely accurate and high resolution products may be considered as few of the imminent challenges.

Among other cases, deformation analyses demand, as a rule, high resolution products. In these studies, repeated high quality GPS geodetic observations allow the estimation of the free surface velocity field for a region. However, the amount of data gathered in such cases is often impressive, especially if the GPS network is a monitoring one with permanent stations. Data processing should be carried out carefully, since statistical assumptions and computational aspects could lead to setbacks in the adjustment procedure. Also, the large amount of data hinders the choice of realistic statistical criteria which are used in order to evaluate the statistical significance and reliability of the deformation model.

The experience gathered from handling a large volume of GPS data with a software package for high precision GNSS data analysis is discussed in the present paper. The data refer to a rather extensive GPS network, spread out over the region between the gulfs of Euboea and Corinth (Greece), and observed in two epochs. The purpose of the network is to study the long-term tectonic behaviour of the area.

Given the impressively large degree of freedom acquired in such cases, the question of estimating a realistic aposteriori variance factor and the respective parameter variances for the model are also discussed.

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1. INTRODUCTION

Repeated geodetic observations describe the velocity field for a region. This field is used in deformation analyses aiming at the generation of meaningful motion and deformation models. The observations quality and the analysis techniques, as well as the suitable consideration of all uncertainties, affect the final decisions.

In GPS analysis, station coordinates and, to a lesser extent, station velocities play a dominant role because GPS is used by the majority of users to estimate (high-accuracy) coordinates. GPS is usually used as an interferometric technique; that is station coordinates are estimated "differentially", relative to a reference station and with respect to the proper reference frame. Therefore, good a priori coordinates for at least one (reference) site have to be known [*Hugentobler U.,et al, 2001*]. The reference station should be considered stable in the space-time domain.

In the case of GPS stations (periodically re-occupied or permanent) for monitoring tectonic behaviour and the associated big number of observations to be processed, sequential processing methods should be preferred. The theory of combining sequential solutions is well-known in geodesy since F. R. Helmert first described it in 1872. An advantage of the sequential adjustment techniques is that they may be used regardless of the observation types of the individual solutions (results from different techniques, e.g., classical geodetic techniques or space techniques GPS, SLR, VLBI, DORIS) [*Hugentobler U.,et al, 2001*].

Since the early nineties a GPS network was established, piece-wise, over the regions of Northern Peloponnesus, Attica and Euboea, in Greece, in order to monitor the tectonic behaviour of the region. A large volume of data accumulated during two epochs of observations (in 1997 and 2005) is used in the present study. A high precision GNSS data analysis software package was used, namely BERNESE V.4.2, for the processing of the data. The purpose of the present paper is to analyse the pros and cons of the various adjustment possibilities offered by BERNESE, as well as the accuracies obtained in each case.

2. ANALYSIS OF DATA

2.1 Least Square Adjustment for large volume data

During the last two decades the Higher Geodesy Laboratory of NTUA participates in a European multi-disciplinary research program concerning the tectonic behaviour of the Northern

Peloponnesus, Attica and Euboea regions. The Laboratory contributes in the acquisition and analysis of the geodetic data gathered from a broad GPS network established gradually. In the present work GPS data collected during two campaigns, i.e. 1997 and 2005, are analysed, evaluated and discussed.

During the 1997 GPS campaign 150 network points were observed while 65 of them are part of the National Geodetic Network. Two sites (Arkitsa and Dionysos), being part of the permanent GPS network of the Higher Geodesy Laboratory, were occupied practically the whole length of time for both campaigns (*Figure 1*). The field work lasted 11 days (30th of September to 10th of October); 35 points were occupied at least 4 days, while several were observed during the whole period. Since a large amount of data was gathered it was decided to break the GPS network into sub-networks. The criterion was the length of time the points were occupied. Thus, the first order sub-network consists of the 35 points with the longer interval of occupation. The second order network includes 113 points; the time of observation in this case varies from 2 to 5 hours.

In 2005 the field work lasted 10 days (27th of September to 6th of October) and 71 points were observed with 7 of them being part of the National Geodetic Network (*Figure 2*). Arkitsa and Dionysos were also included in this case, as mentioned before. In order to have an equivalent schedule of analysis for both epochs the network was again distinguished into sub-networks. The first order comprised of 52 points with occupation intervals from 2 to 10 days. The second order network included 17 points observed from 2 to 7 hours.

Seven IGS stations, surrounding the network, were chosen to tie the network to the ITRF 2000 [*Brockmann E. and W. Gurtner, 1996*]. The IGS stations were ANKARA, GRAZ, MATERA, NICOSIA, PENC, SOFIA and WETTZELL (*Figure 3*). The stations were selected to conform to two criteria; all stations had to be active during both epochs, in order to provide the necessary data, while good network geometry had to be achieved.

Both epochs were analysed using the BERNESE GPS Software V4.2. Precise ephemeris orbits and Ionosphere and Troposphere corrections were used. For both epochs, the average percentage of ambiguities fixed in the network, using the Q.I.F. (Quasi- Ionosphere Free) algorithm, is of the order of 85%.

In the present work, the results of the 30 points of the first order network, common to both epochs (1997 and 2005), were chosen to be depicted and discussed (*Figure 5*).

The baseline approach was preferred in our analysis since the number of sites increases the computational load linearly. On the other hand, inter-baseline correlations known to exist are not taken into account. BERNESE may process observations with highest accuracy requirements via the GPSEST sub-programme taking into account correlations [*Hugentobler U.,et al, 2001, Brockmann E., 1996*]. However, for big networks this is often not feasible due to limited computer resources as in our case.

Due to the volume of the available data (for both epochs and for several sites there were more than 24 hours of observations), daily normal equation files were produced using the whole amount of parameters. The solution of the normal equations led to daily coordinate estimations (BERNESE GPS Software V4.2, sub-programme GPEST). A mean value for the (X, Y, Z or equivalently E, N, U) coordinates of every station was calculated considering the daily estimations equally weighted. For both epochs, variance – covariance matrices of all the parameters were computed and the coordinate sub-matrices were extracted. Finally, the corresponding standard deviations based on the coordinate repeatabilities were computed. These aposteriori RMS errors of the coordinates were considered as reference values to be compared with later results. This approach will be referred as: <u>*Solution using daily coordinate estimations (Solution A)*</u> (*Figure 4*) [*Papanikolaou X., 2009*].

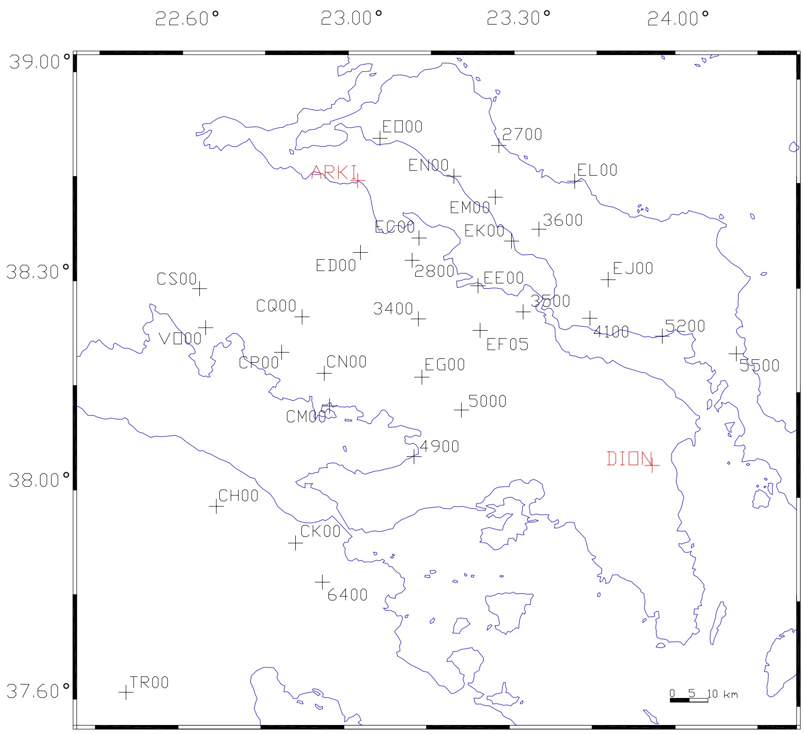

Figure 1 Geodetic Network observed at epoch 1997.76 [*Papanikolaou X., 2009*].

One of BERNESE sub-programmes, ADDNEQ (or its new version ADDNEQ2), is developed to compute multi-session solutions from the combination of a set of single-session solutions. Such cases are the creation of weekly or monthly solutions from daily solutions, the computation of "final" coordinates as a result of many days of continuous observations or several campaigns, the elimination of a type of parameters etc. Normal equations may be stored for a sequence of solutions including all possible types of unknown parameters (coordinates, troposphere, orbit parameters, Earth rotation parameters, nutation parameters, center of mass, satellite antenna offsets, etc.). Pseudo-observation equations are set up from each single-session estimation using the associated variance-covariance matrix as the corresponding weight matrix [*Hugentobler U.,et al, 2001, Brockmann E., 1996*]. In this way, combining pre- or post-elimination with sequential least squares adjustment of the single-session normal equations, a result is always possible even for very extensive and long campaigns.

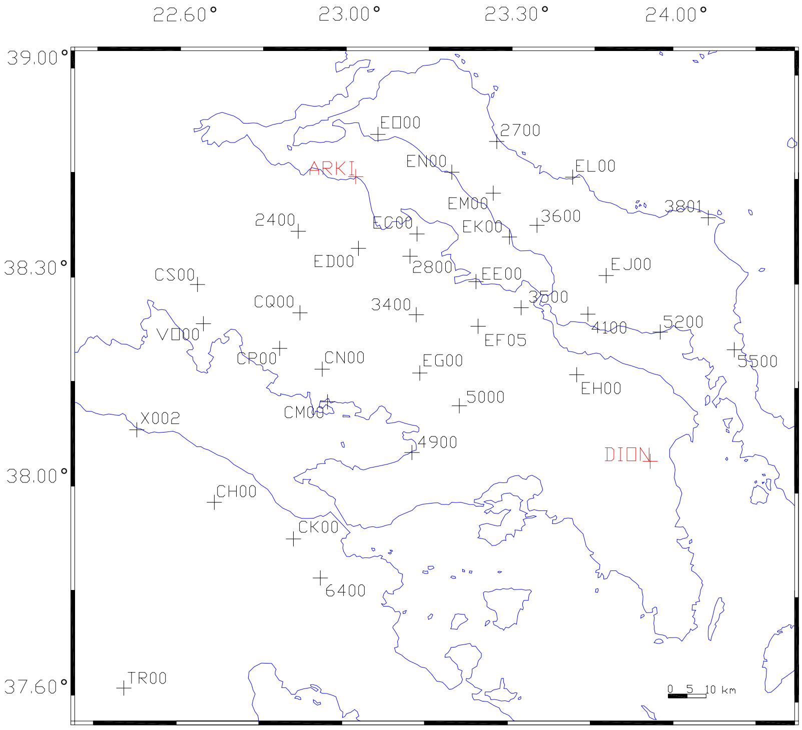

Figure 2 Geodetic Network observed at epoch 2005.76 [*Papanikolaou X., 2009*].

Daily normal equation files were reprocessed, by means of the ADDNEQ sub-programme, in order to eliminate troposphere parameters. The large size of the normal equation files (after the estimation of the phase ambiguities) is mainly due to these parameters, especially in high accuracy applications, making impossible a one-step solution for the whole epoch. The reduction in normal equation files is large, leading to a decrease of more than 90% (in our case from 495 kb to 40 kb after the elimination). It is worth mentioning, that the parameter elimination algorithm takes into consideration the effect of the parameters to be eliminated on the normal equation system, transforming it in a way that no information is lost. Besides, parameters eliminated at this point may be later re-established and solved for. In our case and for both epochs, the final solution and the respective variance-covariance matrices were estimated by sequential adjustment using all pertinent daily normal equation files mentioned above. The approach followed in this case will be referred as: *Combined adjustment with ADDNEQ from daily normal equations with parameter elimination (Solution B)*, (*Figure 4*) [*Papanikolaou X., 2009*].

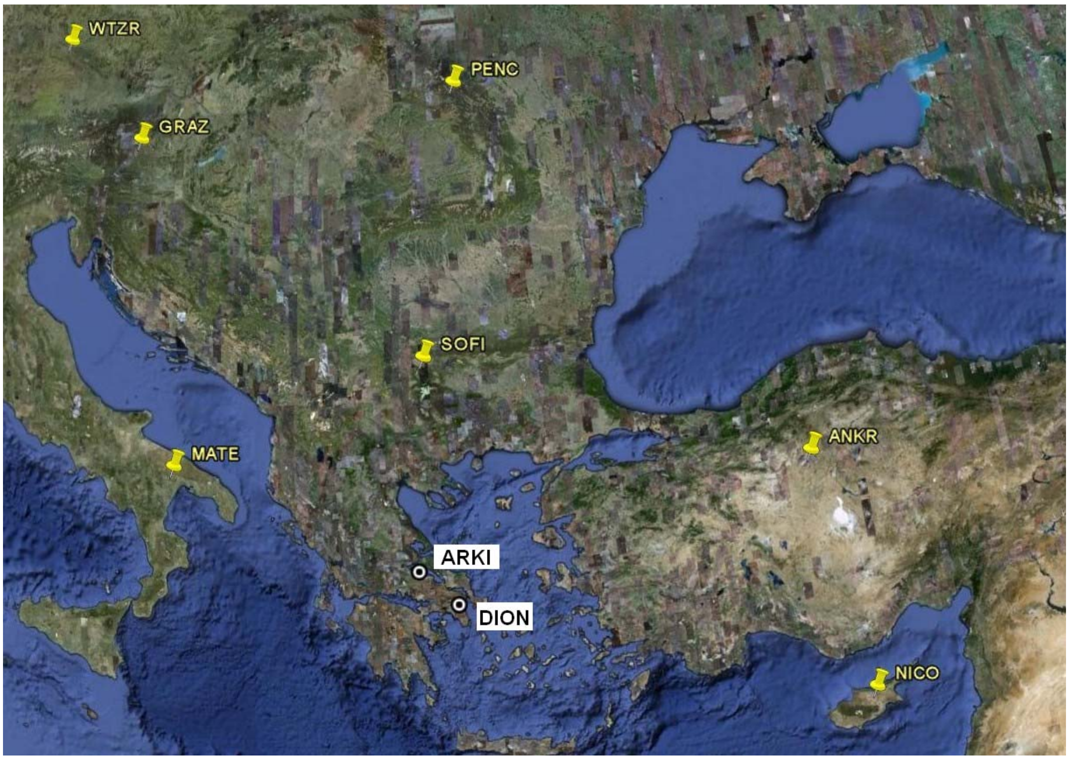

Figure 3 Stations of the International Geodetic Service (IGS) used for referring to the ITRF 2000.

Another sub-programme for combining solutions is COMPAR. ADDNEQ is based on normal equations while COMPAR is based on covariance information of coordinates, only. On the whole, it is equivalent to combine solutions based on either normal equations or on covariance information [*Hugentobler U.,et al, 2001*]. Instead of using the (often oversized) normal equation files, COMPAR offers an alternative way to reach a combined solution, using only coordinates and corresponding variance-covariance matrices, already calculated in previous processing steps. ADDNEQ is much more flexible, COMPAR is much simpler to use, faster in producing results. The selection of the tool depends on the user requirements.

However, with COMPAR the user does not have the flexibility to change the constraints specified previously, or to change the geodetic datum. Thus, it is not possible to combine coordinate sets computed using different fixed or heavily constrained sites [*Hugentobler U.,et al, 2001*]. The sub- programme is well-suited to study coordinate repeatabilities and baseline results being particularly useful in the case of geodetic applications.

As mentioned above daily solutions (coordinates) as well as the respective variance - covariance matrices were already evaluated (Solution A). Therefore the sub-programme COMPAR was used in order to compute a combined solution. The approach followed in this case will be referred as: *Combined adjustment using COMPAR from daily coordinates and variance-covariance estimations (Solution C)*, (*Figure 4*) [*Papanikolaou X., 2009*].

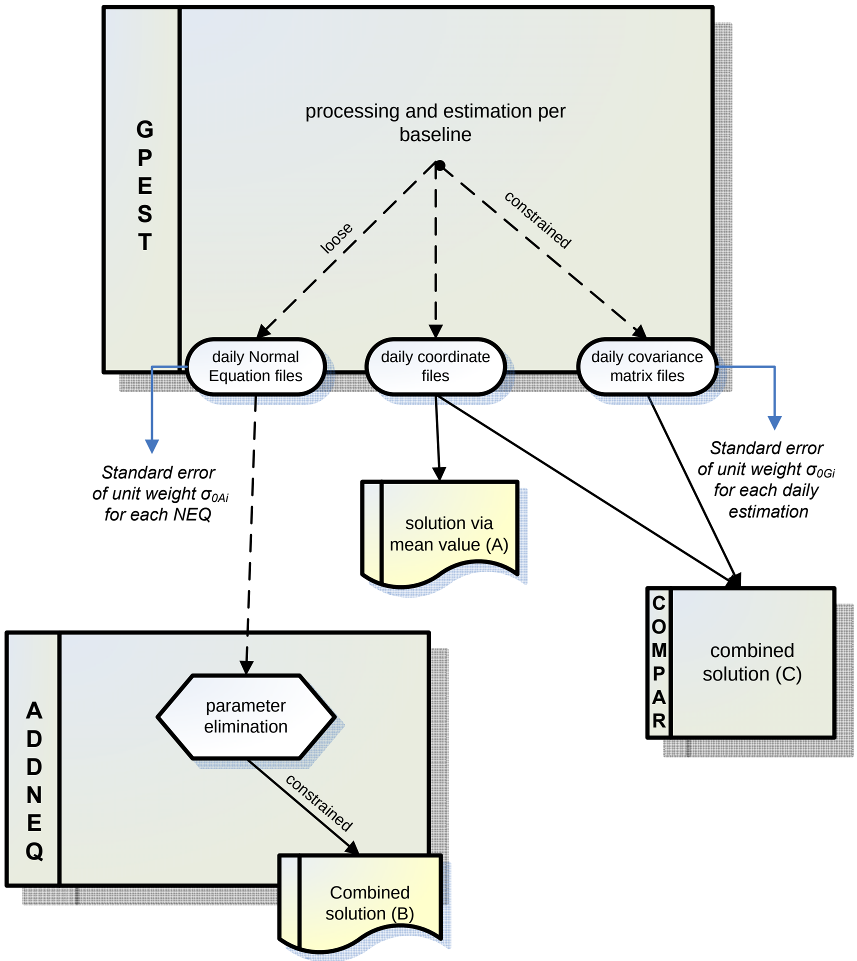

Figure 4 Structure of sub-programmes used in each solution

2.2 Comparison of the Solutions

A comparison between the coordinates computed from the various solutions was carried out between the 30 common points of the network (*Figure 5*). *Table 1* depicts the discrepancies, in mm, between pairs of the three solutions (A, B and C) computed for the common points' coordinates expressed in (ΔN, ΔE, ΔU). The differences for both epochs are presented.

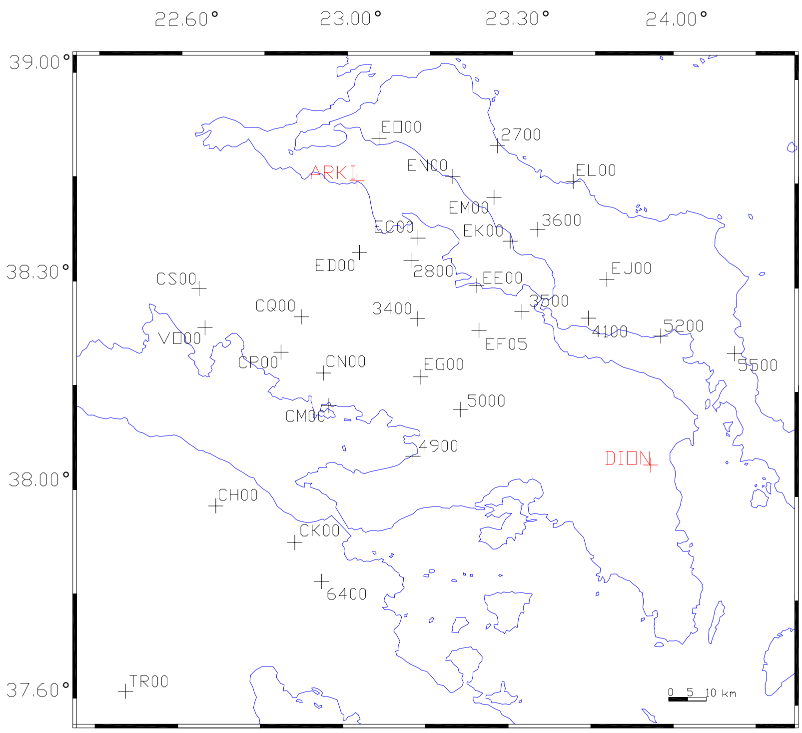

Figure 5 The sub - network used in the analysis consisting of the common for both epochs points [*Papanikolaou X., 2009*].

The mean and maximum discrepancies for each coordinate component and for both epochs are depicted below:

discrepancies between solutions	Epoch 1997.76			Epoch 2005.76		
	ΔN (mm)	ΔE (mm)	ΔU (mm)	ΔN (mm)	ΔE (mm)	ΔU (mm)
mean	0.1	-0.1	0.4	0.2	-0.5	0.6
max	1.8	5.7	13.5	2.5	2.9	17.9

The coordinate discrepancies vary up to 6 mm in the horizontal components while discrepancies of the order of 18 mm appear in the case of the height component. As expected, the discrepancies between the height estimations are larger than those for the horizontal components but their behaviour appears to be random (*Table 1*).

It is also obvious, that solutions (B) and (C) provide practically identical results. In most cases the coordinate differences appear smaller in value than the station's standard deviations, within observations noise. *Figure 6* depicts a similar behaviour between the pairs of the three solutions. Thus the third pair of diagrams (discrepancies between solutions B and C) clearly shows that both solutions give practically the same results.

		Discrepancies for epoch 1997.76 (mm)										Discrepancies for epoch 2005.76 (mm)								
Pairs of Solutions		A - B			A - C			B - C				A - B			A - C			B - C		
a/a	Point	ΔN	ΔE	ΔU	ΔN	ΔE	ΔU	ΔN	ΔE	ΔU		ΔN	ΔE	ΔU	ΔN	ΔE	ΔU	ΔN	ΔE	ΔU
1	*2700*	-0.1	1.0	13.5	-0.1	1.0	13.5	0.0	0.0	0.0		-0.5	0.5	0.2	-1.1	1.0	0.7	-0.6	0.4	0.4
2	*2800*	-0.1	-0.1	0.4	-0.1	-0.1	0.4	0.0	0.0	0.0		0.0	-0.4	0.3	-0.4	0.1	1.1	-0.5	0.5	0.7
3	*3400*	-0.1	0.0	-9.7	-0.1	0.0	-9.7	0.0	0.0	0.0		-0.1	-0.5	-0.9	-1.0	0.3	-0.6	-0.8	0.8	0.2
4	*3500*	0.2	0.7	0.1	0.2	0.6	0.1	0.0	-0.1	0.0		1.7	-2.9	9.0	1.3	-2.7	8.9	-0.4	0.1	-0.1
5	*3600*	-1.6	1.4	8.4	-1.6	1.5	8.5	0.0	0.1	0.0		0.1	-0.2	-1.6	-0.7	0.4	-1.3	-0.7	0.6	0.3
6	*4100*	-0.1	0.3	-0.4	0.0	0.3	-0.3	0.1	0.0	0.1		0.0	0.1	2.4	-0.2	0.1	2.7	-0.2	0.1	0.3
7	*4900*	0.3	0.2	-8.0	0.3	0.2	-8.0	0.0	0.0	0.0		-0.6	0.5	-4.2	-0.9	0.7	-3.1	-0.3	0.2	1.0
8	*5200*	-1.5	-0.2	1.3	-1.5	-0.2	1.3	0.0	0.0	0.0		1.9	0.6	9.6	1.7	0.7	9.3	-0.2	0.0	-0.3
9	*5500*	-1.8	5.7	1.1	-1.7	5.6	1.0	0.1	-0.1	-0.1		-0.2	0.2	-2.1	-0.4	0.2	-2.0	-0.2	0.0	0.1
10	*6400*	0.7	0.2	0.1	0.7	0.2	0.1	0.0	-0.1	0.0		0.4	-0.4	-7.3	0.1	-0.2	-6.3	-0.3	0.1	1.0
11	*CH00*	0.3	0.2	-0.2	0.3	0.2	-0.2	0.0	0.0	0.0		-1.5	0.7	-3.0	-1.7	1.0	-1.9	-0.2	0.2	1.1
12	*CK00*	-0.3	-1.3	0.9	-0.3	-1.3	0.9	0.0	0.0	0.0		-1.5	0.5	-3.6	-1.8	0.6	-2.6	-0.3	0.2	1.0
13	*CM00*	0.9	0.1	-0.9	0.9	0.1	-0.9	0.0	0.0	0.0		-1.6	0.2	-11.8	-1.8	0.2	-10.7	-0.2	0.0	1.1
14	*CN00*	1.0	-0.2	-6.0	0.9	-0.2	-6.0	-0.1	0.1	0.0		-0.5	0.3	-1.1	-0.7	0.4	-0.1	-0.3	0.1	1.0
15	*CP00*	1.2	-0.9	-2.6	1.2	-0.9	-2.6	0.0	0.0	0.0		-2.2	-1.0	-17.9	-2.4	-0.8	-16.9	-0.3	0.2	1.0
16	*CQ00*	-0.5	-1.8	5.1	-0.5	-1.8	5.1	0.0	0.0	0.0		2.2	1.3	0.8	1.9	1.4	1.9	-0.3	0.1	1.1
17	*CS00*	0.3	-1.2	-2.6	0.2	-1.2	-2.7	-0.1	0.0	-0.1		-1.0	0.4	-1.4	-1.2	0.5	-0.4	-0.2	0.2	1.0
18	*EC00*	0.2	-0.3	1.0	0.2	-0.3	1.0	0.0	0.0	0.0		-0.5	-0.2	-0.2	-1.1	0.2	0.4	-0.6	0.4	0.6
19	*ED00*	0.2	0.1	2.3	0.2	0.1	2.3	0.0	0.0	0.0		-0.6	0.3	-1.6	-1.0	0.7	-1.0	-0.5	0.4	0.6
20	*EE00*	0.2	0.3	2.0	0.1	0.3	2.0	-0.1	0.0	-0.1		1.5	-1.6	2.2	0.7	-0.8	2.6	-0.8	0.9	0.4
21	*EF05*	-0.6	0.6	-0.6	-0.6	0.6	-0.6	0.0	0.0	0.0		-0.1	-0.2	-0.6	-0.7	0.4	-0.2	-0.7	0.6	0.4
22	*EG00*	1.3	-0.2	0.0	1.2	-0.2	-0.1	-0.1	0.0	-0.1		-1.6	0.5	0.2	-1.8	0.7	1.3	-0.2	0.1	1.0
23	*EJ00*	-0.4	0.5	2.4	-0.3	0.5	2.3	0.1	0.0	-0.1		0.5	0.0	-0.4	0.3	0.0	-0.3	-0.2	0.0	0.1
24	*EK00*	0.0	-0.5	2.9	0.0	-0.5	2.8	0.1	0.0	-0.1		0.5	0.2	-2.2	0.1	0.5	-1.3	-0.4	0.3	0.8
25	*EL00*	0.6	-0.6	5.5	0.6	-0.6	5.5	0.0	0.0	0.0		-0.6	-1.5	-1.0	-1.5	-0.5	-0.6	-0.8	0.9	0.4
26	*EM00*	-1.3	0.3	1.1	-1.3	0.2	1.2	-0.1	0.0	0.1		-0.7	-0.2	-8.9	-1.3	0.4	-8.6	-0.6	0.6	0.3
27	*EN00*	-0.5	-0.4	-4.3	-0.5	-0.4	-4.3	0.0	0.0	0.0		-1.4	1.2	6.5	-1.7	1.3	7.6	-0.3	0.1	1.1
28	*EO00*	-1.7	3.5	2.5	-1.8	3.5	2.4	-0.1	0.0	-0.1		-1.1	0.8	-12.5	-1.3	0.9	-11.4	-0.3	0.1	1.1
29	*TR00*	-0.2	-0.7	-1.1	-0.3	-0.7	-1.0	-0.1	0.0	0.1		-1.6	0.0	-5.2	-1.8	0.2	-4.1	-0.2	0.2	1.0
30	*V000*	-0.1	-0.4	0.7	-0.1	-0.4	0.7	0.0	0.0	0.0		-2.3	1.2	12.5	-2.5	1.4	13.5	-0.2	0.2	1.0

Table 1 Discrepancies in mm between the solutions (A, B and C), computed for the common network points coordinates expressed in (ΔE, ΔN, ΔU) for both epochs

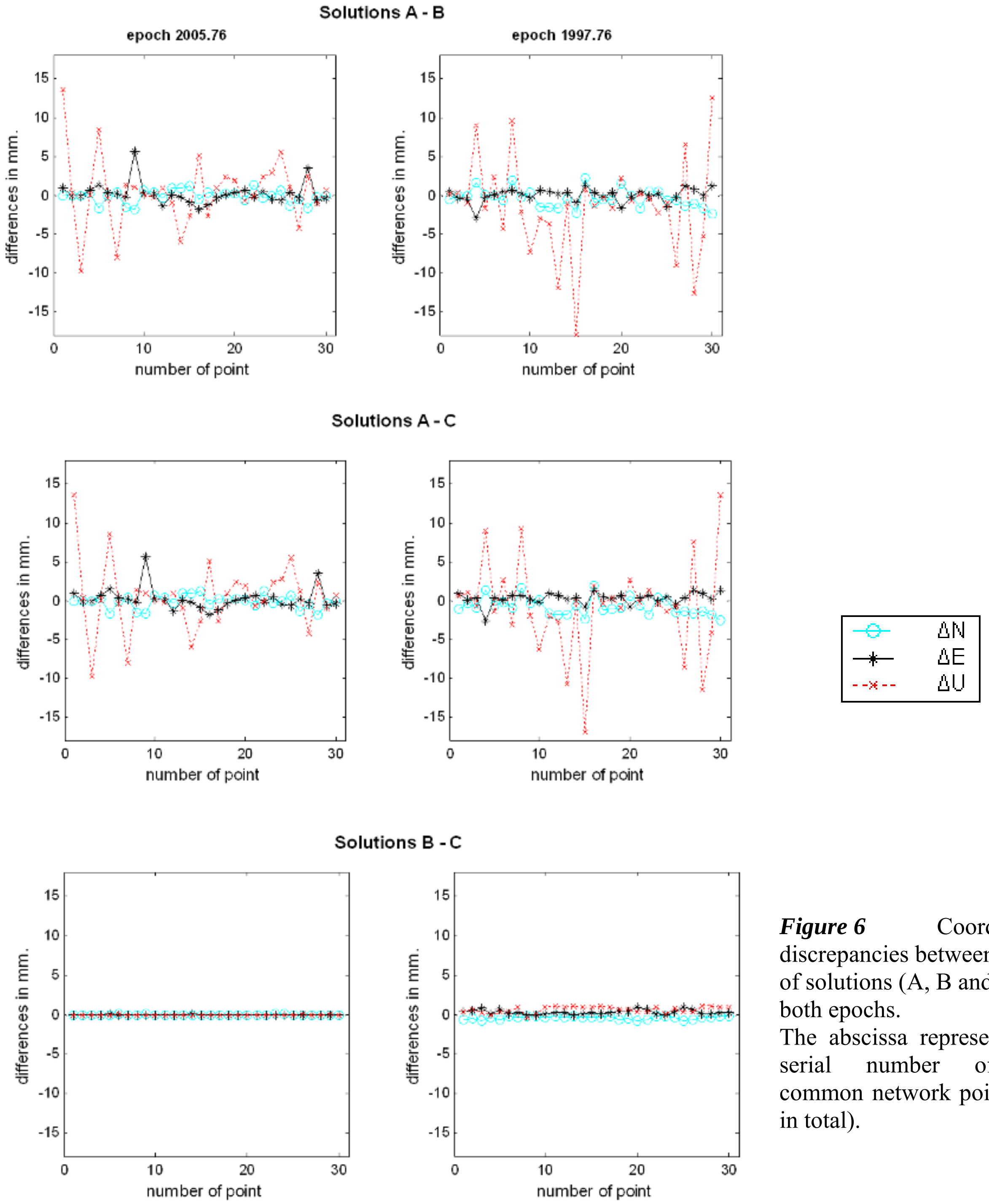

Figure 6 Coordinate discrepancies between pairs of solutions (A, B and C) for both epochs.
The abscissa represents the serial number of the common network points (30 in total).

2.3 Error Analysis

After each day is processed two slightly different RMS error values may be computed from the BERNESE software (*Figure 4*) [*Hugentobler U.,et al, 2001*]. One is computed from the sub-programme GPEST, after estimating daily values for the coordinates which, afterwards, may be combined via the sub-programme COMPAR to compute the final geodetic coordinates. The second estimate is computed from sub-programme ADDNEQ after the formulation of the daily normal equation files which are later merged to provide the combined final solution. In our case, these two values were computed for both campaigns. Thus, σ_{0Gi} refers to the first case mentioned above (Solution C) and σ_{0Ai} to the second one (Solution B). The differences of the two RMS values observed are very small (< 0.8mm). The squares of these RMS errors may be considered as apriori variances of unit weight for each daily solution or daily normal equation file when weighting the pseudo-observations used for the combined solution.

Therefore the apriori variance of the combined solution may be calculated from:

$$\sigma_{0c}^2 = \frac{\sum\limits_{\text{sum over all days}} r_i \cdot \sigma_{0i}^2}{\sum\limits_{\text{sum over all days}} r_i} \qquad (1)$$

where:

σ_{0c}^2 : the apriori variance of unit weight of the combined solution,

r_i : the degrees of freedom for day i,

σ_{0i}^2 : the variance of unit weight computed from day i

The apriori variance of the combined solution was computed twice for each epoch, depending on the use of either σ_{0Gi}^2 or σ_{0Ai}^2 as variance of unit weight (σ_{0i}^2) in formula (1).

In the case of a large volume of available GPS data a challenging dilemma is the evaluation of reliable quality values. BERNESE, using a procedure approximating the variance component estimation method, allows the total RMS error estimation of the combined solution to be grouped into RMS values for various parameter groups. It is assumed that each sequential single-session (e.g., daily) solution is performed with a realistic weighting matrix. This assumption is due to the fact that at the combined normal equations level there in no longer any connection to the original observations [*Brockmann E., 1996*]. However, the pseudo-observation equations, at the normal equations level, may be split in various observation groups. Each group may consist of different types of parameters or of sets of different parameters [*Hugentobler U.,et al, 2001*].

The group RMS error for all coordinates of a GPS network is a good example of a group RMS value. It may represent the variance of unit weight of a "coordinate observation". Thus, it may be used as a more realistic aposteriori variance for the associated variance-covariance matrix of the coordinates instead of the formal aposteriori variance of unit weight derived from the original observations (in the case of GPS these are phase observations). It should be mentioned that it is possible to detect station problems by examining the group RMS errors of each coordinate component. High values of group RMS for the X, Y, Z components may indicate that particular days deviate significantly from the combined solution [*Brockmann E., 1996*].

The BERNESE v4.2 software provides several aposteriori estimates for the combined solution. In the present work, the apriori and aposteriori standard errors of unit weight of the two combined solutions as well as the respective coordinate group values, estimated for both epochs, are depicted in *Table 2*.

It is clear that the values of the aposteriori standard error of unit weight $\hat{\sigma}_0$ are very close - practically identical- for both solutions (*Table 2*). In contrast, the group values concerning the coordinate estimation ($\hat{\sigma}_{0AC}$ and $\hat{\sigma}_{0CC}$) are significantly higher than the formal $\hat{\sigma}_0$ values for both campaigns.

It should be mentioned that the group RMS error values of a parameter or a set of parameters may be considered comparable to the quality values derived from repeatabilities (Solution A) (*Table 3*). The latter are more realistic quality indicators than the formal errors of the combined solutions which are, usually, very optimistic [*Brockmann E., 1996*]. The aposteriori group standard error for the coordinates may provide a more realistic scaling of the aposteriori variance-covariance matrix for the coordinate group.

An empirical rule is often followed according to which the combined solution formal error estimates are multiplied by a factor of 3-5, or even larger, in order to acquire realistic quality estimates for the coordinates. Averaged discrepancies between the estimated formal errors and the group error value of each coordinate component offer, in most cases, a mean value close to this commonly preferred multiplying factor [*Brockmann E., 1996*]. This factor appears quite stable when data from IGS stations or permanent GPS networks for monitoring purposes are analysed. However, when campaign data are used, as in our case, the multiplying factor may be more erratic in value and larger in size. In our case the coordinate parameters standard errors were at least 5-10 times larger than the formal errors and occasionally as much as 30 times. However, such large scaling factors are met even when analysing permanent stations data in the case of the BERNESE software [*Kashani I., et als, 2004*].

Table 3 depicts the estimated standard deviations for the common network points for all types of solutions and for both epochs. Solution A provides estimated quality values derived from the repeatabilities that stem from the average daily solutions. Solutions B and C provide, among other, group aposteriori standard errors for each coordinate component (RMS2x, RMS2y, RMS2z).

	Solution B		Solution C		Epoch
a-priori (mm)	σ_{0Ai}	1.2	σ_{0Gi}	1.7	1997.76
		1.4		2.0	2005.76
a-posteriori (mm)	$\hat{\sigma}_0$	$\hat{\sigma}_{0AC}$	$\hat{\sigma}_0$	$\hat{\sigma}_{0CC}$	
	1.3	20.1	1.2	13.1	1997.76
	1.5	25.4	1.5	23.9	2005.76

Table 2 Apriori and aposteriori standard errors of unit weight for the combined solution and for both epochs of GPS observations where:

σ_{0Ai} : apriori standard error of unit weight computed from day ***i*** from the sub- programme ADDNEQ

σ_{0Gi} : apriori standard error of unit weight computed from day ***i*** from the sub- programme GPEST

$\hat{\sigma}_0$: aposteriori standard error of unit weight of the combined solution

$\hat{\sigma}_{0AC}$: aposteriori standard error of unit weight of the coordinate group computed from ADDNEQ (Solution B)

$\hat{\sigma}_{0CC}$: aposteriori standard error of unit weight for coordinate comparison computed from COMPAR (Solution C)

Since the daily standard errors derived from repeatabilities (Solution A) are generally acknowledged to be in close relation to the group errors for the coordinates (e.g., Solution C) a further investigation was considered of interest.

Sub-programme COMPAR, combines daily coordinate estimations and their respective variance-covariance matrices, to a final solution. Thus, its results are in close relation to the ones obtained from repeatabilities (Solution A) not only for the coordinate estimations but, also, for the calculation of the combined RMS error values for each point.

Such a relation was noticeable while analyzing the results for the common points of both epochs' networks. For each epoch, a linear model was fitted to all the available data (RMS2 from C Solution with respect to the standard error values from Solution A). The linear regression model residuals of each coordinate component versus the standard errors from the daily solutions (Solution A) were depicted in a diagram for both epochs (*Figure 7*).

		Solution Statistics for epoch 1997.76 (mm)									Solution Statistics for epoch 2005.76 (mm)								
Solutions		(A)			(B)			(C)			(A)			(B)			(C)		
a/a	Point	$\pm\sigma_x$	$\pm\sigma_y$	$\pm\sigma_z$	$RMS2_X$	$RMS2_Y$	$RMS2_Z$	$RMS2_X$	$RMS2_Y$	$RMS2_Z$	$\pm\sigma_x$	$\pm\sigma_y$	$\pm\sigma_z$	$RMS2_X$	$RMS2_Y$	$RMS2_Z$	$RMS2_X$	$RMS2_Y$	$RMS2_Z$
1	2700	31.8	15.4	26.2	2.2	1.5	2.0	7.7	3.1	7.2	5.0	3.9	6.4	1.5	1.1	1.9	2.7	1.8	3.4
2	2800	8.2	3.6	3.1	5.6	2.5	3.1	3.2	1.2	1.2	10.2	3.7	13.4	17.5	11.4	25.5	4.7	1.7	6.4
3	3400	36.8	16.6	32.8	3.6	1.6	1.7	14.5	6.3	13.5	4.4	2.8	5.3	2.3	1.7	1.7	1.5	1.0	1.8
4	3500	8.4	4.5	7.1	6.9	3.7	7.6	3.6	2.0	3.6	5.8	3.2	0.1	69.8	29.1	62.2	3.8	2.1	0.0
5	3600	19.7	10.8	10.7	1.3	0.7	0.8	5.6	2.5	3.1	7.2	3.2	6.2	5.7	2.1	6.4	2.8	1.3	2.0
6	4100	4.0	3.5	3.0	3.2	1.3	1.2	1.5	1.3	1.3	8.3	5.4	8.0	3.4	0.4	4.6	2.8	1.9	2.6
7	4900	27.1	10.0	24.5	7.8	3.8	8.4	14.5	5.4	13.3	5.2	0.4	5.7	6.6	2.5	9.2	3.3	0.3	3.6
8	5200	12.9	5.4	6.2	6.7	2.5	5.8	3.6	1.6	1.7	7.1	1.2	5.9	11.5	8.3	13.0	3.5	0.7	3.0
9	5500	11.1	18.7	10.9	4.3	2.0	4.0	5.2	3.4	5.1	19.5	10.3	17.8	4.8	1.7	6.4	7.1	4.0	6.7
10	6400	6.4	5.0	11.0	1.1	0.7	1.3	2.7	2.0	5.0	6.7	7.8	1.6	3.6	0.9	2.7	3.4	3.7	0.7
11	CH00	4.2	2.2	2.8	3.7	2.0	3.6	1.3	0.7	0.8	8.8	2.7	5.9	7.1	4.0	6.7	5.6	1.7	3.8
12	CK00	2.8	1.4	2.7	8.7	5.7	9.6	1.1	0.6	1.3	6.4	1.8	4.4	6.3	2.1	7.0	2.5	0.7	1.8
13	CM00	3.8	2.0	5.0	4.1	2.5	3.4	1.9	0.9	2.3	29.4	12.6	23.1	3.7	4.2	3.9	16.4	6.9	12.7
14	CN00	23.8	10.8	21.6	7.8	3.2	7.2	12.0	5.4	11.0	6.7	10.1	8.1	15.1	5.7	13.4	3.3	4.1	3.6
15	CP00	14.1	9.5	14.2	14.5	6.3	13.6	4.7	3.2	5.0	43.3	17.6	39.8	1.6	0.6	2.0	14.8	5.6	13.0
16	CQ00	19.7	10.0	22.2	4.8	2.5	4.3	7.8	3.8	8.4	23.7	16.5	36.7	3.3	1.3	3.1	15.9	10.3	23.8
17	CS00	6.7	4.8	6.6	14.5	5.3	13.3	2.2	1.5	2.0	6.1	0.9	8.3	2.8	1.3	2.0	3.3	0.4	4.5
18	EC00	5.8	2.0	6.3	19.9	8.4	21.5	2.3	0.8	2.4	5.4	1.9	5.2	1.6	0.9	3.2	1.6	0.6	2.0
19	ED00	14.4	6.5	11.9	15.5	7.9	15.8	5.0	2.2	4.0	6.2	2.7	5.7	12.8	4.2	10.4	3.2	1.3	3.0
20	EE00	9.6	4.7	10.3	20.0	9.5	17.5	4.0	2.1	4.1	1.9	0.8	4.2	1.6	1.5	1.1	1.0	0.5	2.2
21	EF05	3.8	1.3	3.6	2.7	2.0	5.0	1.9	0.6	1.4	4.4	3.5	2.8	3.6	1.7	3.7	1.6	1.5	1.1
22	EG00	1.9	0.7	3.7	1.9	0.9	2.3	1.0	0.4	1.4	7.1	3.0	8.0	10.2	4.7	9.1	3.3	1.4	3.6
23	EJ00	11.6	8.3	9.3	4.8	3.2	5.1	5.4	3.8	4.3	32.2	14.1	28.2	4.9	1.3	2.2	10.2	4.7	9.1
24	EK00	10.3	5.0	7.6	2.3	0.8	2.5	4.0	2.0	2.9	13.2	2.9	5.8	30.3	14.1	26.1	4.9	1.2	2.2
25	EL00	39.7	17.3	43.3	4.0	2.1	4.1	19.9	8.4	21.5	2.4	2.4	6.8	14.5	7.4	12.0	1.2	1.3	2.6
26	EM00	18.9	8.4	17.9	1.8	0.7	1.4	6.9	3.7	7.6	22.4	16.0	24.1	7.0	1.8	6.1	11.4	8.3	12.9
27	EN00	37.4	17.8	37.0	1.0	0.5	1.4	15.5	7.9	15.8	31.3	15.5	25.4	19.7	8.7	17.0	14.2	7.4	11.9
28	EO00	59.8	35.7	53.2	2.9	1.3	1.3	20.0	9.5	17.5	34.3	15.4	29.4	38.7	15.0	35.6	19.6	8.7	17.0
29	TR00	6.3	4.3	2.7	5.4	3.8	4.3	2.7	1.9	1.3	2.8	6.2	7.8	1.8	1.5	3.1	1.3	2.7	3.8
30	V000	16.2	6.0	14.7	4.0	2.0	2.9	6.7	2.5	5.8	29.6	10.6	22.3	2.9	0.7	2.8	12.7	4.2	10.4

Table 3 Estimated standard deviations for the common network points according to the three solutions. Solutions B and C provide group standard deviations for each coordinate component (RMS2x, RMS2y, RMS2z).

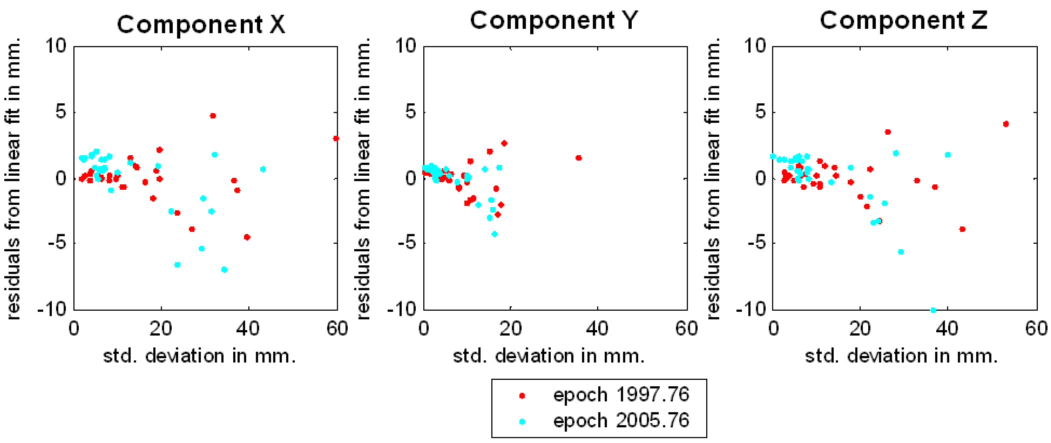

Figure 7 Coordinate component residuals of the linear fit versus daily standard errors for both epochs

It appears that the residuals disperse significantly for standard error values larger than 20mm. Since the sample of the 30 points was not a large one it was decided that only points with RMS error values larger than 30mm and/or points with large residuals were rejected from the next step.

The linear model was re-applied to the remaining data providing a rather clear and stable (for both epochs) relation between the two standard error values (*Figure 8*). It is interesting to notice that the two linear model parameters are almost identical for each coordinate component and for both epochs, despite the relatively different network geometry observed in each campaign (*Table 4*).

Component	**X**		**Y**		**Z**	
	a	**b**	**a**	**b**	**a**	**b**
Epoch 1997.76	0.36	0.7	0.39	0.2	0.37	0.7
Epoch 2005.76	0.42	-0.2	0.34	0.3	0.44	-0.3

Table 4 Parameters of the linear model **y=ax+b** for each coordinate component and both epochs.

However, it is early, yet, to conclude that this linear relationship may be considered as valid due to the sample size and the fact that both networks expand over the same region. All the same, since the two quality values are acknowledged as being related, establishing such a simple model between them could provide an alternative way of estimating realistic quality measures instead of following the rather time-consuming Solution A.

Although a similar attempt was carried out in the case of the group quality estimates of the sub-programme ADDNEQ no relation, linear or otherwise, was noticeable. In this case, the coordinate group RMS2 values appear to have no correlation when compared with the standard error values from Solution A.

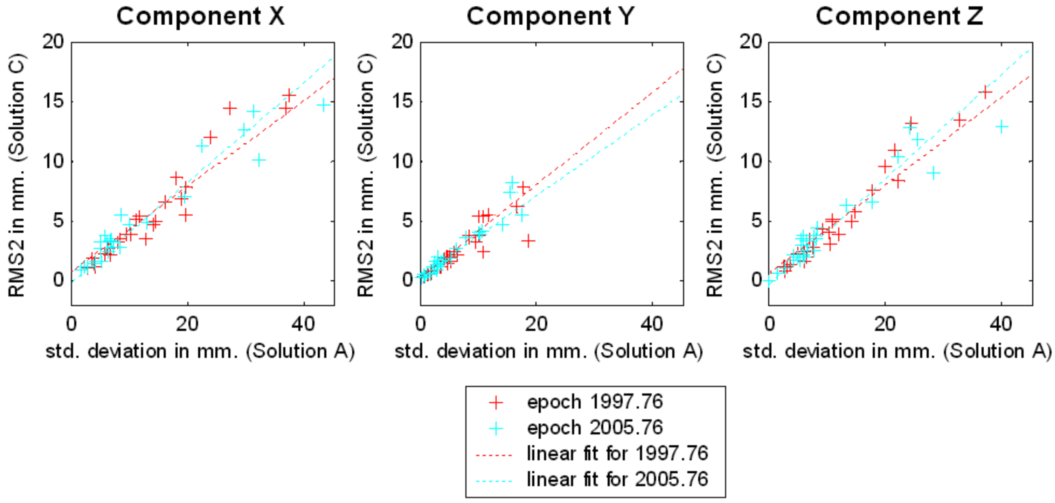

Figure 8 Linear regression models between the standard error of Solution A and the group RMS2 of Solution C for the three coordinate components and for both epochs.

3. DISCUSSION – CONCLUSIONS

The aim of this paper was to study the various adjustment techniques available in the BERNESE software when large volume GPS data are available. The compatibility of the derived parameters (geodetic coordinates) and their respective aposteriori quality estimates were examined.

It is important to bear in mind that the quality of the adjustments and, consequently, the differences between the coordinates estimated may be affected by various factors, among them the quality of the GPS observations, the size of the network and the processing mode. In our case the observation data may be considered of high quality, since they are gathered from static GPS observations with sessions of at least 4 hours duration. The network is a local one for both epochs; with maximum baseline length 150km (the majority of the baselines were less than 100km). Finally, the baseline processing was carried out instead of adjusting the network.

The algorithm of parameter elimination, applied to the troposphere parameters (Solution B), does not influence the coordinate estimation procedure even for the height component. At the same time it reduces significantly the amount of parameters to be estimated and the size of the normal equation files, a useful feature in the case of large and dense networks of many points. On the other hand, it requires further reprocessing of the daily normal equation files which increases the computing time.

The easy to use sub-programme COMPAR leads to precise coordinate estimation (Solution C). Its input variables are familiar to geodesists, while the demanding sub-programme ADDNEQ can be circumvented. However, it is applicable only to pure "geodetic" applications, as it is only capable of estimating coordinates and the respective aposteriori variance-covariance information.

It appears that the discrepancies between the approaches used are very small and can be neglected even for monitoring crustal deformations networks, since they are markedly smaller than the (realistic) standard errors calculated by method A.

Last but not least, as in most GPS applications, one of the most demanding problems is the estimations of realistic RMS error values. Both sub-programmes (ADDNEQ and COMPAR) overestimate the aposteriori variance of unit weight and the respective formal standard errors, as mentioned by many before. In our case, this is clearly observed when the formal errors are compared with the standard errors derived from daily solutions (Solution A). Both sub-programmes provide aposteriori coordinate quality values for each component (RMS2 values). However, ADDNEQ estimates appear to be rather optimistic.

On the other hand, the RMS2 values computed from Solution C, appear to be linearly related with the RMS error values calculated from Solution A, indicating a correlation of the computing algorithm used in the case of sub-programme COMPAR to repeatabilities. The linear dependence observed is rather stable for all coordinate components and for both epochs, although the geometry of the network observed during each campaign differed significantly. In the future, data from GPS networks established in other areas are planned to be analysed in a similar way in order to verify whether this linear dependence is as stable as it appears to be.

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