Building a Unified Height System with Various Initial Points for the ASEAN Countries

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Key words: unified height system, ASEAN, GPS levelling, ellipsoid

SUMMARY

In the paper there is proposed to build a unified height system with various initial points for the South East Asian countries. There is pointed out to a neccesity of the normal height according to the Molodenski's original interpretation and an advisability of using GPS – levelling for this task. There are also recommended concrete problems to be solved for building such a height system in the region and discussed an expected accuracy of the normal height values at the initial points in the unified height system.

Tóm tắt

Tiến tới thiết lập hệ thống độ cao thống nhất cho các nước Đông Nam Á

Bài báo đề xuất xây dựng một hệ thống độ cao thống nhất với nhiều điểm khởi tính cho các nước Đông Nam Á. Đã chỉ ra sự cần thiết của việc sử dụng độ cao chuẩn theo cách hiểu nguyên căn của Molodenski và tính hợp lí của việc áp dụng đo cao GPS vào mục đích này. Cũng đã kiến nghị các vấn đề cụ thể cần giải quyết để thiết lập một hệ thống độ cao như thế ở khu vực và xem xét độ chính xác có thể đạt tới cho các giá trị độ cao chuẩn tại các điểm khởi tính của hệ thống độ cao thống nhất.

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1. INTRODUCTION

The partnership and relationship between countries of the ASEAN community is becoming closer and closer day by day. In order to promote and benefit more from this relationship, a cooperative infrastructure is needed. For that reason, early in 1996 the PCGIAP (Permanent Committee of GIS Infrastructure for Asia and the Pacific) is called, responsible to establish a GIS-infrastructure for the region. According to the PCGIAP first step working plan, a precise geodetic network had been successfully built, using precise GPS-measurement techniques. As a result the planimetric connection measurement between ASEAN countries become available with high precision, while the altimetric connection remains unpractical due to the lack of a unified height system in the region. Beeing deeply concerned that elevation information is very necessary for many scientific-technical and practical cooperative undertakings between ASEAN countries, a method of establishing a common and unified height system is proposed.

2. DISADVANTAGES OF THE CLASSICAL METHOD OF ESTABLISHING THE NATIONAL HEIGHT SYSTEM

According to the classical method of establishing the national height system, the mean sea level, determined by long-term measurement of the sea level at tide gauge, is used for starting origin in national height system. Elevation of any point in the country is determined by leveling measurement, whereby height difference at bench mark has to be corrected, using gravity data. Depending on which kind of gravity data being taken in consideration, an orthometric height or a normal height is acquired. In the first case, the mean actual gravity and in the second case the mean normal gravity is used for height difference correction.

It is well-known that the orthometric height was introduced long time ago, while the normal height was only acknowledged since 1945 from the Molodenski theory as a term used to determine the Earth's gravity field and figure (*Molodenski 2001*) The derivation of the orthometric height is not rigorous enough and hence less accurate, moreover complicate in practical computation. On the contrary, the normal height is rigorous, more accurate and simple in computational realization.

As a rule, starting origin used in the national height system must be unique. If more than one starting origin are used, elevation discrepancy may occur at the same ground point. In this case it is nearly impossible to improve the reliability and accuracy of the national height network, even when proper adjustment procedures are applied. Moreover it seems to be very difficult, even impossible to compare elevations collected by means of traditional leveling with those collected with GPS-leveling as the first one relates to the gravimetric ellipsoid with unknown size and orientation parameters.

3. THE PROPOSED METHOD OF BUILDING A UNIFIED HEIGHT SYSTEM

The above mentioned disavanages of the classical method in building height system can be overcome in the proposed method, based on the use of the normal height given by Molodenski, interpreted initially as height of the first approach of the Earth's surface (teluroid) from the surface of the normal ellipsoid. In this sense, the normal height H^{γ} can be understood as the difference between the geodetic height h and the height anomaly ζ :

$$H^{\gamma} = h - \zeta \ . \tag{1}$$

The geodetic height h can be acquired by GPS measurement and the height anomaly bt the use of gravity values given for the whole terrestrial globe. So the new method of establishing the height system consists in GPS-leveling. As such the derived normal height refers everywhere to the normal ellipsoid, just like the geodetic height does. For that reason, height systems of different starting origin (or of many countries) can simply be integrated into a unified system, using the normal ellipsoid as the common reference. In order to merge unifiedly height values, acquired by traditional leveling with those acquired by GPS-leveling, the normal ellipsoid and the WGS84 ellipsoid (GPS) must be the same.

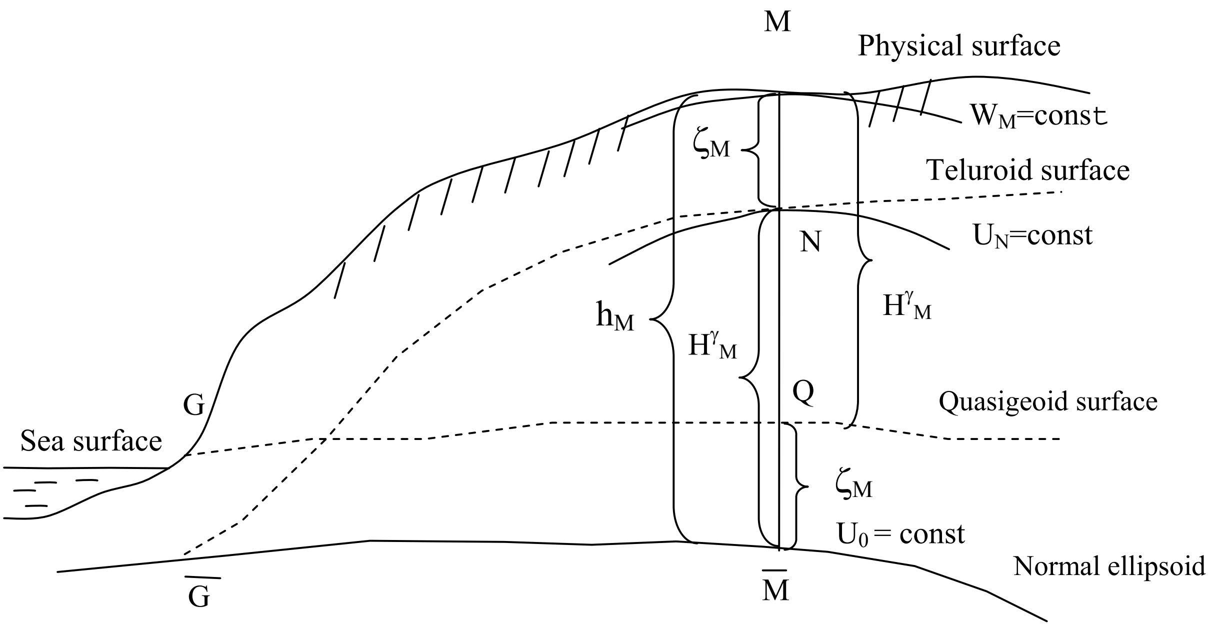

Fig.1. Main surfaces of the Earth

4. PROBLEMS TO BE SOLVED FOR BUILDING A UNIFIED HEIGHT SYSTEM FOR THE ASEAN COUTRIES

In order to proceed successfully the building of a unified height system, we would propose the FLAG Council to set up a project and appoint a working group, responsible:

- To analyse and evaluate height system, existing in each member country and to decide the actual gravity system and normal gravity formulas to be used in establishing the unified height system.

- To choose the unique ellipsoid used in the region. For that the world newest ellipsoid is recommended.

- To collect and evaluate the existing actual gravity data in each member country and convert these data to the choosen unified gravity system

- To compute the gravity anomaly values in the unified actual and normal gravity system.

- To collect and choose the best-fited to the local gravity data geopotential models.

- To calculate the height anomaly values at the accepted starting points of the new height system, using local and worldwide gravity data.

- To determine geodetic height of these starting points by use of precise GPS measurements.

- To derive and adjust the normal heights at the starting points in the common network.

- To process each national height network, based on the adjusted starting points in the unified network of the region.

5. AN EXPECTED ACCURACY

A rms error of the normal height difference derived at the origins by using GPS – levelling can be expressed according to (1) as follow :

$$m_{\Delta H^{\gamma}} = \sqrt{m_{\Delta h}^2 + m_{\Delta\zeta}^2} \quad . \tag{2}$$

The quality $m_{\Delta h}$, as it is known, does not exceed 1 *cm*, so the magnitude of $m_{\Delta H}{}^{\gamma}$ depends mainly on the magnitude of $m_{\Delta\varsigma}$ **.**

The height anomaly ζ is obtained by using gravity data given on the whole Earth's surface in two forms: gravity anomalies inside a zone Σ of some size around the considered point and harmonic coefficients of any model of the Earth gravity field.

Let Σ be a plane square subdeviebed into N standard compartments of size S with gravimetric point in the center. Then a height anomaly ζ in the considered point can be computed as

$$\zeta = \frac{1}{2\pi\gamma}\sum_{j=1}^{n}\Delta g_j F_j \, , \tag{3}$$

where Δg_j is a value of gravity anomaly in compartment j ; F_j is its influence coefficient;

$$F_j = \int_{x_1}^{x_2}\int_{y_1}^{y_2}\frac{dxdy}{\sqrt{x^2+y^2}} \quad , \tag{4}$$

x_1,x_2,y_1,y_2 are coordinates of limiting grids of the compartment with gravity value Δg_j .
The error variance of ζ is given in the following form (*Lan 2002*):

$$M\left\{\delta\zeta^{2}\right\}=\frac{1}{(2\pi\gamma)^{2}}\left[\sum_{i=1}^{N}\sum_{i'=1}^{N}F_{i}F_{i'}C_{ii'}-\sum_{i=1}^{N}\sum_{i'=1}^{N}\sum_{j'=1}^{n}F_{i}F_{i'j'}C_{ii'j'}\;-\right.$$

$$\left.-\sum_{i=1}^{N}\sum_{i'=1}^{N}\sum_{j=1}^{n}F_{i'}F_{ij}C_{i',ij}+\sum_{i=1}^{N}\sum_{i'=1}^{N}\sum_{j=1}^{n}\sum_{j'=1}^{n}F_{ij}F_{i'j'}C_{ij,i'j'}\right] \quad , \qquad (5)$$

where C denotes the covariance and M- the mathematical expectation:

$$\left.\begin{array}{ll} C_{ij'} = M\{\Delta g_{i}\Delta g_{i'}\} & ; \\ C_{i,i'j'} = M\{\Delta g_{i}\Delta g_{i'j'}\} & ; \\ C_{i',ij} = M\{\Delta g_{i'}\Delta g_{ij}\} & ; \\ C_{ij,i'j'} = M\{\Delta g_{ij}\Delta g_{i'j'}\} & . \end{array}\right\} \qquad (6)$$

In (5) n is a number of the elementary compartments in $\sum$; n≥N.
For numerical evaluating we need a covariance model of height anomaly. We accepted the modified Third-order Markov function (*Lan 1980a*):

$$C = D_{\Delta g}\, e^{-\frac{B}{L}}\left(1+\frac{R}{L}-\frac{R^{2}}{2L^{2}}\right)-d \quad . \qquad (7)$$

The parameters $D_{\Delta g}$, d, L were determined by use of the realistic gravity data in middle Vietnam: $D_{\Delta g}$ = 436 mgal^2 ; d = 65mgal^2 ; L=18km .
We considered the size of standard compartments equal 10km and the size of zone $\sum$ equal 1300km. Then we received the standard error of ζ

$$\sigma_{\zeta}=\sqrt{M\left\{\delta\varsigma^{2}\right\}}=0.035\ m \quad . \qquad (8)$$

Accordingly the difference of height anomalies in two point A and B has the standard error

$$\sigma_{\Delta\varsigma}^{AB}=\sigma_{\zeta}.\sqrt{2}=0.035m.\sqrt{2}=0.050\ m \, . \qquad (9)$$

Let us now consider influence of the gravity anomalies in the remaining part of the Earth globe determined by use of the spherical harmonic coefficient. In this case for the standard error of the difference of plumb line deflections at points A and B we have derived following expressions (*Lan 1980b*):

For $\lambda_A = \lambda_B$:

$$\left(\overline{\sigma}_{\Delta\xi}^{AB}\right)^{2}=\left(\frac{\rho''}{2\gamma}\right)^{2}\sum_{n=2}^{n_{\max}}Q_{n}^{2}\sum_{m=0}^{n}\sigma_{nm}^{2}\left\{(n+1)\left[tg\varphi_{B}P_{nm}(\sin\varphi_{B})-tg\varphi_{A}P_{nm}(\sin\varphi_{A})\right]-\right.$$

$$-(n-m+1)\left[\sec\varphi_B P_{n+1,m}(\sin\varphi_B)-\sec\varphi_A P_{n+1,m}(\sin\varphi_A)\right]^2 \;;$$

$$\left(\overline{\sigma}^{AB}_{\Delta\eta}\right)^2=\left(\frac{\rho''}{2\gamma}\right)^2\sum_{n=2}^{n_{\max}}Q_n^2\sum_{m=0}^{n}\sigma_{nm}^2 m^2\left[\sec\varphi_B P_{n,m}(\sin\varphi_B)-\sec\varphi_A P_{n,m}(\sin\varphi_A)\right]^2 \;. \qquad (10)$$

For $\varphi_A=\varphi_B$:

$$\left(\overline{\sigma}^{AB}_{\Delta\xi}\right)^2=\left(\frac{\rho''}{2\gamma}\right)^2\sum_{n=2}^{n_{\max}}Q_n^2\sum_{m=0}^{n}\sigma_{nm}^2\left[(n+1)tg_A^{\varphi}P_{nm}(\sin\varphi_A)-(n-m+1)\sec\varphi_A P_{n+1,m}(\sin\varphi_A)\right]\sin^2\left[\frac{1}{2}m(\lambda_B-\lambda_A)\right];$$

$$\left(\overline{\sigma}^{AB}_{\Delta\eta}\right)^2=\left(\frac{\rho''}{2\gamma}\right)^2\sum_{n=2}^{n_{\max}}Q_n^2\sum_{m=0}^{n}\sigma_{nm}^2\left\{m\sec\varphi_A P_{nm}(\sin\varphi_A)\sin\left[\frac{1}{2}m(\lambda_B-\lambda_A)\right]\right\}^2 . \qquad (11)$$

In (10) and (11) we have φ_A , λ_A , φ_B ,λ_B – latitude and longtitude of the considered points;

Q_n – truncation function;
P_{nm} – associated Legendre's function of degree n and order m;
σ_{nm} – rms error of the harmonic coefficients;
γ - mean gravity of the Earth.
Numerically we have obtained

for $\varphi_A = 0^o$; $\varphi_B = 5^o$; $\lambda_A = \lambda_B$:

$$\overline{\sigma}^{AB}_{\Delta\xi}=0''05,\quad \overline{\sigma}^{AB}_{\Delta\eta}=0''03 \qquad (12)$$

and for $\varphi_A = \varphi_B = 0^o$, $\lambda_B - \lambda_A = 5^o$:

$$\overline{\sigma}^{AB}_{\Delta\xi}=0''03,\quad \overline{\sigma}^{AB}_{\Delta\eta}=0''05 . \qquad (13)$$

Taking for the two considered points in the same meridian or in the same first vertical near the equator the value $\overline{\sigma}^{AB}_{\Delta\theta}=0''05$, we can accept in the case of deriving height anomaly difference at the separation of 500km the standard error equal

$$\overline{\sigma}^{AB}_{\Delta\zeta}=\overline{\sigma}^{AB}_{\Delta\theta}\cdot\frac{500km}{\rho''}=\frac{0''05\times 500000\ m}{206265''}\approx 0.12\ m . \qquad (14)$$

Taking into consideration the value (9) we finally can expect that the standard error of the difference of height anomaly between two points separated at 500 km from each other is

$$m_{\Delta\varsigma}=\sqrt{\left(\sigma^{AB}_{\Delta\zeta}\right)^2+\left(\overline{\sigma}^{AB}_{\Delta\zeta}\right)^2}=\sqrt{(0.05m)^2+(0.12m)^2}=0.13\ m . \qquad (15)$$

Accordingly the rms error of the normal height difference Δh at separation of 500 km is equal

$$m_{\Delta H^{\gamma}} = \sqrt{m_{\Delta h}^2 + m_{\Delta\zeta}^2} = \sqrt{(0.01m)^2 + (0.013)^2} \approx 0.13m \quad . \qquad (16)$$

6. CONCLUSION

The common and unified height system should be established now for the ASEAN countries. For this purpose the normal height in the original Molodenski's recommendation and its realization by use of GPS – levelling is advisable. Accordingly there may be more than one starting point in the region height system and we need no more in the mean sea level which is various for different countries and changing with a time. We can expect such a height network with the rms error of the height difference less than 0.13 m between the separated at 500 km from each other starting points.

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BIOGRAPHICAL NOTES

- Studied in the Moscow Institute of Aerial survey, Geodesy and Cartography (1963 – 1968)
  Had been awarded the PhD degree in 1980 and the Doctor of Science degree in 1988
- Since 12/1968 lecturer at the Faculty of geodesy , Hanoi University of Mining and Geology (HUMG)
  Had been appointed as Head of the Department of Higher geodesy (1983 – 2006), Dean of the Faculty of Geodesy (1991 – 2004), awarded the title of Professor in 1991
- Since 1992 member of the Executive committee of the Vietnam's Society of Geodesy, Cartography and Remote sensing
- Main scientific activity on speciality:
  Using geodetic, astronomical, gravimetric and satellite data to study local gravity field and figure of the Earth

PUBLICATIONS

Has about 20 publications in the scientific - technical journals abroad (Tchechoslovakia, Russia, Germany, Australia) and more than 50 publications in the domestic journals.

AWARD

Has been awarded the Ho Chi Minh Price on science and technology of Vietnam of the year 2005 for the contribution to establishing the National Reference and Coordinate System VN – 2000.

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