Monitoring Displacements at Large Dams by Means of Precision Traverses

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Key words: Traverse, Geodetic Surveys, Monitoring of Displacements, Theodolite, Motorized Tachymeter.

SUMMARY

The paper presents the results of a test on the angular accuracy of a motorized tachymeter recently used for precision traversing during the first fill of a portuguese large dam. Angular measurements were carried out at two precision traverses, one at the crest of the dam and the other at an inspection gallery. Horizontal angles were measured with the automatic target recognition procedure of the motorized tachymeter and with a conventional electronic theodolite, with prior equivalent accuracy. Statistical inference methods were used to test the accuracy of automatic angular measurement against conventional measurement.

RESUMO

A comunicação apresenta o resultado de um teste à precisão angular de um taqueómetro recentemente utilizado em poligonação de precisão durante o primeiro enchimento de uma grande barragem portuguesa. As medições angulares foram efectuadas em duas poligonais de precisão, uma no coroamento e a outra numa galeria de visita. Os ângulos horizontais foram medidos recorrendo ao sistema de posicionamento automático do taqueómetro motorizado e com um teodolito electrónico convencional, com exactidão à priori equivalente. Foram utilizados métodos de inferência estatística para testar a exactidão da medição angular automática em oposição ao método convencional de medição.
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1. INTRODUCTION

Some Portuguese dams include traverses in their geodetic observation system, in order to control horizontal displacements of points on the crest or in inspection galleries (Casaca et al., 2002). Until 2001, surveying teams of the Applied Geodesy Division of the Dam Department of National Laboratory for Civil Engineering (NLCE) had to measure distances and angles using distinct equipment: distance measuring devices and theodolites, respectively. In 1963 took place the first measuring campaign of a traverse, this one placed in an inspection gallery of a concrete dam. Then and in following years, theodolites Wild T3 or Kern DKM3 were employed to make angular observations while distances were measured with the help of invar wires. In the eighties electronic equipment started to be used: electro-optical distancemeters (a Mekometer ME3000) and electronic theodolites (Kern E2 or Leica T2002). After 2001, when NLCE bought a high precision motorised tachymeter, was possible observe angles and distances simultaneously. Since motorized tachymeters aren’t suitable to carry on manual pointings, and automatic pointings, made possible with the help of an automatic target recognition system, are less accurate than manual pointings, it was necessary to perform some tests on the angular accuracy of the motorized tachymeter. These tests make use of displacements computed from different sets of data: i) angles and distances measured by the tachymeter, ii) angles measured by the theodolite and distances measured by the tachymeter.

This paper presents the computational model, used to compute the displacements, and the results of the tests on the accuracy of automatic angular measurement against conventional measurement.

2. THE COMPUTATIONAL MODEL

2.1 The Functional Model

The functional model relates displacements of the object, ancillary and reference points to the observable variables (horizontal angles and distances) which are often called the observables. The functional model is a set of linear observation equations which result from “Taylorization” of the non-linear relations between displacements and variations of the observables.

Considering a local cartesian reference frame (xOy), with the xx axis oriented from the right bank to the left bank and the yy axis oriented from downstream to upstream, the observation equation correspondent to an oriented horizontal angle with origin at (j), station at (i), and target point at (k) is:

\[
\begin{align*}
\Delta A_{ijk} & = \frac{y_{jk}}{D_{ik}^2} dx_k - \frac{x_{ik}}{D_{ik}^2} dy_k + \frac{y_{ij}}{D_{ij}^2} dx_j + \frac{x_{ik}}{D_{ik}^2} dx_j + \frac{x_{ij}}{D_{ij}^2} dy_j + \frac{y_{ij}}{D_{ij}^2} dy_j + \frac{x_{ij}}{D_{ij}^2} dy_j
\end{align*}
\]
where:

\[
D_{ik} = \sqrt{x_{ik}^2 + y_{ik}^2}, \quad x_{ik} = x_k - x_i, \quad y_{ik} = y_k - y_i, \quad \text{etc.}
\]

The observation equation correspondent to an horizontal distance (distance reduced to the local horizon) between the station (i) and the target point (k) is:

\[
dD_{ik} \approx \frac{x_{ik}}{D_{ik}} dx_k + \frac{y_{ik}}{D_{ik}} dy_k = \frac{x_{ik}}{D_{ik}} dx_i - \frac{y_{ik}}{D_{ik}} dy_i
\]

The functional model requires reference equations, where the displacements of two points, at least, is supposed to be measured by an independent method (pendulum) or is supposed to be null. Those reference equations are of the type:

\[
dx_i = d\bar{x}_i, \quad dy_i = d\bar{y}_i
\]

The functional model correspondent to a plane traverse with p points is composed of (2p+1) equations: (p–2) horizontal angle equation observations, (p–1) horizontal distance equations and four reference equations. The functional model of a traverse has the minimum level of redundancy: (n= 2p) unknowns and (m= 2p+1) equations.

The functional model may be represented as a vector equation:

\[
A(m,n)\Delta X(n,1) = \Delta Y(m,1)
\]

where A is the matrix of the coefficients of the observation and reference equations, \( \Delta X \) is the vector of displacements and \( \Delta Y \) is the vector of the variation of the observables (horizontal angles and distances). The matrix A, which elements may be computed with approximate coordinates, even before any field work, is called the first order design (FOD) matrix.

### 2.2 The Stochastic Model

The operational mathematical model results from the connection of a stochastic model to the functional model, to enable the design and the quality control of the networks that compose the geodetic observation systems.

The vector of the variation of the observables (\( \Delta Y \)) may be regarded as an one-sized sample of a multinormal distribution

\[
\Delta Y \in N(\mu, \Sigma), \quad E(\Delta Y) = \mu, \quad V(\Delta Y) = \Sigma
\]

where: E and V stand for the mathematical expectation and variance operators; \( N(\mu, \Sigma) \) stands for the family of the m-dimensional normal random vectors with mean vector \( \mu(m,1) \) and variance matrix \( \Sigma(m,m) \), which is called the second order design (SOD) matrix.
To estimate the unknown vector of the displacements ($\Delta X$), the statistical inference theory recommends the use of the best (minimum variance) linear unbiased estimator (BLUE):

$$\Delta \bar{X} = C^{-1} A^T \Sigma^{-1} \Delta Y, \quad C = A^T \Sigma^{-1} A$$

The BLUE is a normal random vector, which mean vector and variance matrix are:

$$E(\Delta \bar{X}) = \Delta X, \quad V(\Delta \bar{X}) = C^{-1}$$

The SOD matrix of a traverse, with angles and distances, is a diagonal matrix. The diagonal elements of the SOD matrix are the variances of the components of the vector of the observables ($\Delta Y$), which are variations of angles and distances, measured at different observation epochs.

If the horizontal angles and distances are observed independently, at two observation epochs, with instruments of the same class of accuracy and precision, the prior variances of their variations are:

$$V(dA) = 2\sigma_A^2, \quad V(dD) = 2\sigma_D^2$$

where $\sigma_A$ and $\sigma_D$ are the prior standard deviations of the instruments.

### 3. APPLICATION ON PRECISE TRAVERSING

To test the accuracy of automatic angular measurement against conventional measurement displacements of points of a traverse were used. These were computed from different sets of data measured by a tachymeter and a theodolite. The traverse chosen for this study is included in the geodetic observation system of a Portuguese dam (Fig. 1) and is used to monitoring horizontal displacements of points on the crest.

![Crest of the dam](image)

**Figure 1** – Crest of the dam
3.1 The Traverse

The traverse has ten points, as seen in Figure 2: eight points (P1 to P8) are placed on the crest of the dam and are object points; two points (PD and PE) are placed in the banks of the river and are the reference points of the traverse. All points are materialized by forced centring pieces (Wild type) in the top of concrete pillars (Figure 6). The reference pillars have large concrete foundations on rock. All pillars have thermic isolation. The eight pillars on the crest of the dam are placed in the same profile that the inverted pendulums of the dam. Distances between pillars vary from \( \approx 55 \text{ m} \) until \( \approx 340 \text{ m} \).

![Figure 2 – Traverse points position](image)

3.2 The Equipment

The equipment used for the measurements were: i) a motorized tachymeter Leica TCA2003 (Figure 3), equipped with an automatic target recognition (ATR) system; ii) a theodolite Leica T2002 (Figure 5). The accessories were: i) for the TCA2003, two precision circular retro-reflectors (Figure 4), each mounted on a carrier with a built in tubular bubble; ii) for the T2002, two circular targets of the Wild type (Figure 6).

![Figure 3 – Tachymeter](image)  ![Figure 4 – Retroreflector](image)
The prior accuracies of the instruments (Leica 1991 and 2000) are displayed at the Table 1.

<table>
<thead>
<tr>
<th>Instrument</th>
<th>Observable</th>
<th>Measurement Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>T2002</td>
<td>Angle</td>
<td>0.15 mgon</td>
</tr>
<tr>
<td></td>
<td>Angle</td>
<td>0.15 mgon</td>
</tr>
<tr>
<td>TCA2003</td>
<td>Distance</td>
<td>1 mm + 1 ppm</td>
</tr>
<tr>
<td></td>
<td>Searching</td>
<td>1 mm until 400 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.15 mgon after 400 m</td>
</tr>
</tbody>
</table>

### 3.3 The Observation Campaigns

The displacements were computed from measurements made during two campaigns that took place during the first fill of the dam. In each campaign, measurements were made during the same period of the day: end of the afternoon, always starting by theodolite observations, immediately followed by tachymeter observations, with the ATR system.

Horizontal angles were measured with the theodolite and with the tachymeter. Slope distances were measured with the tachymeter. Atmospheric corrections for the distances were computed based on temperature (measurements made near each pillar), air pressure (measured at the beginning, middle and end of distances measurements) and relative air humidity (measured at the end of distances measurements). After atmospheric corrections of the slope distances, horizontal distances were computed with the help of the vertical angles.

### 3.4 Data Processing

Several adjustments were made. All used the same functional model, composed of 28 equations: eight horizontal angle equation observations, 16 horizontal distance equations (two in each station, one "backwards", other "forwards") and four reference equations. The
observables were the variation of horizontal angles (dAh) and the variation of horizontal distances (dD). Two different vectors of observables were used: i) variations of tachymeter horizontal angles and horizontal distances; ii) variations of theodolite horizontal angles and tachymeter horizontal distances.

Concerning the stochastic model, three variance matrices, Σ, were used, as displayed in Table 2.

### Table 2 – Variance matrix

<table>
<thead>
<tr>
<th>Observables</th>
<th>Version</th>
<th>Variance Matrix</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>σ(dAh): function of distance</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>σ(dD): short distances1.4 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>long distance 2 mm</td>
</tr>
<tr>
<td>tachymeter</td>
<td>1</td>
<td></td>
<td>σ(dAh) = 0.3 mgon</td>
</tr>
<tr>
<td>angles and</td>
<td></td>
<td></td>
<td>σ(dD): short distances1.4 mm</td>
</tr>
<tr>
<td>distances</td>
<td></td>
<td></td>
<td>long distance 2 mm</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>theodolite</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>angles and</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tachymeter</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>distances</td>
<td>I</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

All variance matrices are diagonal. Variance matrix “version 2”, applied to tachymeter observations, expresses the use of identical standard deviations for all the observables. On the contrary, in “version 1”, each angle has a different standard deviation due to the fact that, although the searching accuracy is constant and equal to 1 mm, the distances between the tachymeter and the targets are different, since pillars aren’t equally distributed (distances range, see Figure 1: 55 m, between pillars P1 and P2, and 338 m, between pillars P8 and PE). Finally, angles measured by the theodolite have constant standard deviations. The value of 0.3 mgon, applied to variations of horizontal angles measured by theodolites, is adopted by NLCE for precision planimetric networks for the determination of displacements in large dams (Casaca, 2001).
The standard deviation $\sigma(dA_h)$ of a variation of a horizontal angle measured by the tachymeter, (version 1 of Table 2) was computed by:

$$
\sigma (dA_h) = \sqrt{2\sigma_{Or}^2 + 2\sigma_{TP}^2}
$$

where $\sigma_{Or}$ is the standard deviation of the horizontal azimuth measured to the origin and $\sigma_{TP}$ is the standard deviation of the horizontal azimuth measured to the target point. Due to the fact that searching accuracy is 1 mm (Leica, 2000) to targets placed at distances shorter than 400 m, it is possible to estimate the standard deviation of a horizontal azimuth.

The displacements, computed with a network adjustment software developed at NLEC, are shown in Figures 7 and 8. In both figures the displacements computed from readings made by coordinometers to pendulums plumb lines were also included.

![Figure 7](image7.png)

**Figure 7** – Displacements from: i) pendulums; tachymeter horizontal angles and distances with ii) variance matrix type 1 (according to table 2); iii) variance matrix type 2 (according to table 2)

![Figure 8](image8.png)

**Figure 8** – Displacements from i) pendulums; ii) tachymeter horizontal angles and distances, variance matrix type 1 (according to table 2); iii) theodolite horizontal angles and tachymeter horizontal distances

Based on the vectors of the displacements ($\Delta X$) is possible to compute the difference, ($\Delta X_2 - \Delta X_1$), of the vectors of the displacements. In Table 3 is presented the norm, $\| \Delta X_2 - \Delta X_1 \|$, of these vectors, concerning the six possible comparisons.
Table 3 – Norm of the vector differences

<table>
<thead>
<tr>
<th></th>
<th>tachymeter (2)</th>
<th>theodolite</th>
<th>pendulums</th>
</tr>
</thead>
<tbody>
<tr>
<td>tachymeter (1)</td>
<td>3.2</td>
<td>2.5</td>
<td>3.8</td>
</tr>
<tr>
<td>tachymeter (2)</td>
<td>3.8</td>
<td>5.8</td>
<td></td>
</tr>
<tr>
<td>Theodolite</td>
<td>2.8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The analysis of Table 3 shows that:

1) to use a matrix of variance with prior variances, established as a function of equipment accuracy, allows to get better results than with the identity matrix;
2) displacements resulting from the theodolite and from the tachymeter observations do agree, not being substantially different of the displacements resulting from pendulum.

The pendulum displacements were measured in the first inspection gallery, which is 6m beneath the crest (Figure 9). This fact makes them not directly comparable with traverse displacements. If traverse points were placed near pendulums reading points it would have been possible to compare displacements and to apply statistical test of hypothesis (Henriques et al., 2002).

Figure 9 – Cross section of the inverted pendulum in the central block
4. CONCLUSIONS

The results lead to the conclusion that, in the measurement of angles of a precision traverse, is possible to substitute the theodolite for a motorized tachymeter, with prior equivalent accuracy, and to apply automatic target recognition procedures. The necessity of only one instrument has obvious advantages: a shorter observation period, more simple data pre-processing and, possibly, a smaller surveying team.

REFERENCES


BIOGRAPHICAL NOTES

Maria João Henriques is a research officer of the Applied Geodesy Division of the Dam Department of National Laboratory for Civil Engineering. João Casaca is a senior research officer, head of the Applied Geodesy Division of the Dam Department of National Laboratory for Civil Engineering. Their research activities are in the domains of network design, quality control, influence of the atmosphere on the observations, calibration of equipment and automation of field observations and office work.

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