# Use of DLT in Photogrammetric Metrology 

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Key words: Metrology, Photogrammetry, DLT.


#### Abstract

SUMMARY

In industrial metrology, the mechanical parts come in a wide range of sizes and shapes. Hence, the measurement process must be adapted. Numerical photogrammetry constitutes a great contribution to industry for certain metrological treatments. In this study, we propose an approach which allows the realization of a photogrammetric system of measurement for the control of the mechanical parts. The method uses DLT transformation (Direct Linear Transformation) between the image space and the object space. The photogrammetry treatment model developed in [1] allows the monoscopic treatment of the photographs of the object under control. The matching by least-squares and the epipolar geometry principle are applied for the calculation of an approximate configuration of the block defined by the perspective beams and the subject. The final compensation uses the rigorous principle of the condition of colinearity while at the same time taking into account all perspectives (multiphoto). The objective of this work is to achieve a photogrammetric system of measurement based on perspective without stereoscopic vision and to see that it is possible to direct this system towards the treatment of blocks (adjustment) relating to a project.


## RESUME

En métrologie industrielle, les pièces mécaniques sont très diversifiées par leurs tailles et par leurs formes. Le processus de mesure doit donc être adapté. La photogrammétrie numérique constitue pour certains de ces traitements métrologiques un grand apport en industrie. Dans cette étude, nous proposons une démarche permettant la réalisation d'un Système de Mesure Photogrammétrique pour le contrôle des pièces mécaniques. La méthode utilise la transformation TLD (Transformation Linéaire Directe) entre l'espace image et l'espace objet. Le modèle de traitement Photogrammétrique développé dans [1] permet le traitement monoscopique des photographies de l'objet à contrôler. L'appariement par les moindres carrés et le principe de la géométrie épipolaire sont appliqués pour le calcul d'une configuration approchée du bloc défini par les faisceaux perspectifs et le sujet. La compensation finale utilise le principe rigoureux de la condition de colinéarité avec une prise en compte de toutes les perspectives (multi-photo). L'objectif de ce travail est de réaliser un système de mesure photogrammétrique basé sur la perspective sans vision stéréoscopique et de voir qu'il est possible d'orienter ce système vers le traitement de blocs (ajustement) relatif à un projet.

# Use of DLT in Photogrammetric Metrology 

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## 1. INTRODUCTION

In industrial metrology, the mechanical parts come in a very wide range of sizes and shapes. Three-dimensional measurement compared with a space reference gives a good definition of the object in space. The extraction and development processes of spatial information require more and more the exploitation of digital imagery. Digital photogrammetry remains ever present for these metrological treatments, making perhaps a great contribution to industry [2]. This study proposes an approach which allows the realization of a photogrammetric system of measurement for the mechanical parts. The method used is a transformation between the image space and the object space. The photogrammetric treatment model developed in [1] allows a monoscopic analysis of the photographs of the object under control. The matching by least-squares and the epipolar geometry principle are applied for the calculation of an approximate configuration of the block defined by the perspective beams and the subject. Photography, as a source of information is intended to provide in convenient form information concerned with several fields and presenting zones which overlap:

- the geometric field in which position, shape and dimension of the object are the principal unknown factors.
- the physical field linked to the recording of information based on meta-data (geometric and semantic).
- the optical field which allows treatments by comparison.

Hence, photogrammetry is an indirect measurement technique of objects recorded in the form of photographic perspectives. Thus the image of the object tends to replace the object itself as a data carrier during the actual measurement.

## 2. MODELS IN DIGITAL PHOTOGRAMMETRY

Digital photogrammetry has existed since 1980, through an adapted instrumentation and various models of restitutors were presented at the congress of Ottawa [3]. The suggestion of interesting solutions in terms of cost / effectiveness, amongst which the use of a of PC type platform, appears to be appropriate for most applications [4]. The aim of this work is to determine by the calculation the external elements of two perspective beams, using points known on the object, and of points of connections common to both beams.
Analytical restitution leads to specific determinations by exploiting the measurements made directly in the plan of the plate. The object point / image point correspondence is done analytically applying mathematical relations.

Depending on whether the photogrammetric unit is the beam or the model, one of the two following conditions is applied:

- colinearity
- planarity
the characteristics of a restitutor on PC are close to those required of a video game, and so it is worth looking closely into the programming techniques, since different applications [5] pose different problems regarding the acquisition of three dimensional co-ordinates and their stereoscopic visualization.


### 2.1 Equation of Colinearity

The problem arising from this condition is to find a rotation which makes the image vector Om and the object vector $\mathbf{O M}$ colinear, figure (1). If K indicates the scale factor relating to point M , the following relation will occur:

$$
\left[\begin{array}{c}
\mathrm{x}  \tag{1}\\
\mathrm{y} \\
-\mathrm{c}
\end{array}\right] \text { k.R. }\left[\begin{array}{c}
\mathrm{X}-\mathrm{X}_{0} \\
\mathrm{Y}-\mathrm{Y}_{0} \\
\mathrm{Z}-\mathrm{Z} 0
\end{array}\right]
$$



Figure 1: Co linearity Condition

### 2.2 Equation of Coplanarity

The problem to be treated is no other than that of the formation of the image - analytical model, i.e. the realization of the relative orientation and it results in:

1. Finding the rotation matrices R1 and R2.
2. Or finding the rotation matrix R2 and the translations according to axes which make the three vectors coplanar, as defined in figure (2) below:

The base $\mathrm{b}=\mathrm{O}_{1} \mathrm{O}_{2}$
The homologous rays: $\mathrm{O}_{1} \mathrm{~m}_{1}$ and $\mathrm{O}_{2} \mathrm{~m}_{2}$


Figure 2: coplanarity condition
If the left perspective center $\mathrm{O}_{1}$ is taken as origin, the vectors co-ordinates will then be:

$$
\begin{gathered}
\mathrm{O}_{1} \mathrm{O}_{2}, \mathrm{O}_{1} \mathrm{~m}_{1}, \mathrm{O}_{2} \mathrm{~m}_{2} \\
\mathrm{O}_{1} \mathrm{O} 2=\left[\begin{array}{l}
\mathrm{b}_{2} \\
\mathrm{by}_{1} \\
\mathrm{bz}
\end{array}\right] \quad ; \mathrm{O}_{1} \mathrm{~m}_{1}=\left[\begin{array}{l}
\mathrm{x}_{1} \\
\mathrm{y}_{1} \\
\mathrm{z}_{1}
\end{array}\right] ; \mathrm{O}_{2} \mathrm{~m}_{2}=\left[\begin{array}{c}
\mathrm{x}_{1}+\mathrm{b}_{\mathrm{x}} \\
\mathrm{y}_{1}+\mathrm{b}_{\mathrm{y}} \\
\mathrm{z}_{1}+\mathrm{b}_{\mathrm{z}}
\end{array}\right]
\end{gathered}
$$

In mathematical terms, the condition of coplanarity is expressed in the equation (2):
(2)

$$
\left|\begin{array}{ccc}
\mathrm{b}_{\mathrm{x}} & \mathrm{~b}_{\mathrm{y}} & \mathrm{~b}_{\mathrm{z}} \\
\mathrm{X}_{1} & \mathrm{y}_{1} & \mathrm{Z}_{1} \\
\mathrm{X}_{1}+\mathrm{b}_{\mathrm{x}} & \mathrm{y}_{1}+\mathrm{b}_{\mathrm{y}} & \mathrm{Z}_{1}+\mathrm{b}_{\mathrm{z}}
\end{array}\right|=0
$$

In order to restore the object, either the beam unit or the model unit may be chosen, but it is necessary to make the correction of the plate coordinates which must be used to realise the different orientations.

### 2.3 DLT Approach

The direct linear transformation method (D L T) of the co-ordinates comparator with the ground co-ordinates, figure (3) was developed in [1]. It is based on the following pair of equations:


Figure 3: Diagram of the DLT Principle.
(3)

$$
\begin{aligned}
& x+\left(x-x_{0}\right) r^{2} k_{1}+\left(r^{2}+2\left(x-x_{0}\right)^{2}\right) p_{1}+2\left(y-y_{0}\right)\left(x-x_{0}\right) p_{2}=\frac{L_{1} X+L_{2} Y+L_{3} Z+L_{4}}{L_{9} X+L_{10} Y+L_{11} Z+1} \\
& y+\left(y-y_{0}\right) r^{2} k_{1}+\left(r^{2}+2\left(y_{1}-y_{0}\right)^{2}\right) p_{2}+2\left(y-y_{0}\right)\left(x-x_{0}\right) p_{1}=\frac{L_{5} X+L_{6} Y+L_{7} Z+L_{8}}{L_{9} X+L_{10} Y+L_{11} Z+1}
\end{aligned}
$$

with $r=\left(x^{2}+y^{2}\right)^{1 / 2}$
where:

- $\quad \mathrm{X}$ and Y are the co-ordinates comparator of the image points.
- $x_{0}$ and $y_{0}$ define the position of the center of the plate in the comparator system.
- $\quad \mathrm{k}_{1}, \mathrm{p}_{1}$ and $\mathrm{p}_{2}$ are the parameters of distortion of the objective;
- $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ are the ground co-ordinates of the points photographed
- $\mathrm{L}_{1}$ through $\mathrm{L}_{14}$ are the unknown coefficients of the beam.

The following expressions can be deduced directly from the system (3):
$\mathrm{L} 1 \mathrm{X}+\mathrm{L}_{2} \mathrm{Y}+\mathrm{L} 3 \mathrm{Z}+\mathrm{L} 4-\mathrm{xXL} 9-\mathrm{xYL} 10-\mathrm{xZL}_{11}-\mathrm{xr}^{2} \mathrm{Ak}_{1}-\left(\mathrm{r}^{2}+2 \mathrm{x}^{2}\right) \mathrm{Ap}_{1}-2 \mathrm{yxAp}_{2}-\mathrm{x}=\mathrm{r}_{\mathrm{x}}$ $\mathrm{L} 5 \mathrm{X}+\mathrm{L} 6 \mathrm{Y}+\mathrm{L} 7 \mathrm{Z}+\mathrm{L} 8-\mathrm{yXL} 9-\mathrm{yYL} 10-\mathrm{yZL} 11-\mathrm{yr}^{2} \mathrm{Ak}_{1}-\left(\mathrm{r}^{2}+2 \mathrm{x}^{2}\right) \mathrm{Ap} 2-2 \mathrm{yxAp}_{1}-\mathrm{y}=\mathrm{r}_{\mathrm{y}}$
(4)
where :

- $\quad \mathrm{A}=\mathrm{L}_{9} \mathrm{X}+\mathrm{L}_{10} \mathrm{Y}+\mathrm{L}_{11} \mathrm{Z}+1$
- $\quad r_{x}$ and $r_{y} y$ are residue errors in condition equations.
- $\quad \mathrm{L}_{1}, \ldots ., \mathrm{L}_{11}$ are parameters drawn from spatial transformation (scale factor, rotaations and translations). They are considered independent as far as DLT is concerned.
- $\quad L_{12}, L_{13}, L_{14}$ are the distortion coefficients of the objective.
with $\mathrm{Ak}_{1}=\mathrm{K}_{1}^{\prime}=\mathrm{L}_{12}, \mathrm{Ap}_{1}=\mathrm{P}_{1}{ }_{1}=\mathrm{L}_{13}, \mathrm{Ap}_{2}=\mathrm{P}_{2}{ }_{2}=\mathrm{L}_{14}$; the system of equations (4) can be reduced to the following matrix form :

$$
\left[\begin{array}{cccccccccccccc}
-X & -Y & -Z & -1 & 0 & 0 & 0 & 0 & x X & x Y & x Z & \mathrm{xr}^{2} & \left(r^{2}+2 x^{2}\right) & 2 y x \\
0 & 0 & 0 & 0 & -X & -Y & -Z & -1 & y X & y Y & y Z & \mathrm{yr}^{2} & 2 y x & \left(r^{2}+2 y^{2}\right)
\end{array}\right] . \operatorname{L+}\left[\begin{array}{l}
\mathrm{x} \\
y
\end{array}\right]=\left[\begin{array}{l}
r_{x} \\
\mathrm{ry}_{\mathrm{y}}
\end{array}\right]
$$

Each object point known in ground co-ordinates lends itself to two similar equations. A minimum of seven points is necessary to solve the problem correctly, but to have a good determination of the unknown parameters of the transformation, the number of fulcrums must be higher than that which would be necessary, which leads us to a resolution by the method of the least-squares. It is worth pointing out that it is possible to use this method together with the two conditions (colinearity and coplanarity) for a more refined treatment [6].

### 2.4 Adjustment of Blocks

In this type of work, one cannot genuinely speak of block compensation, since the nature of the object is different from its geographic space. However, there is a great similarity with the aero triangulation considering the principle of the functional model used, namely the beams method. In this situation where one is concerned with in an exact definition of the object, the total compensation principle (block adjustment) is based on an installation through DLT calculation and a final compensation using the external parameters. The observation relations used must take into account all the perspectives representing the subject.

Each beam gives place to fourteen parameters and each object-point known gives two equations. It is thus possible to pose an observation relations system between co-ordinates, parameters and distances.

The compensation principle is based on the rigorous method [7] which deals with the adjustment of the plate observations.

## 3. TREATMENT PROCESSES

### 3.1 Preliminary Treatments

These treatments include:

- The orientation of the photographies: $(x, y)=R\left(x^{\prime}, y^{\prime}\right)$ with $\square$ rotation matrix of the photography which is written as :

$$
\left.\left\lvert\, \begin{array}{l}
x^{\prime}  \tag{5}\\
y^{\prime}
\end{array}\right.\right\}\left[\begin{array}{cc}
\cos \alpha & -\sin \alpha \| \\
\sin \alpha & \cos \alpha
\end{array}\right]
$$

- The observation treatment :

The observations in space image of the plate co-ordinates are sullied with errors in dots since measurements are done separately (non stereoscopic dots). In this case, it is necessary to check the reliability of the observations by taking into account the parallax equations (6).
(6) $\mathrm{PL}_{\mathrm{i}}=\mathrm{xi}^{\prime}-\mathrm{xi}^{\prime \prime} \quad \mathrm{i}=1 \div \mathrm{n}$

The calculation reference is based on the following parameters:

- $\quad \mathrm{H}_{\mathrm{i}}, \mathrm{H}_{0}$ distance between i and the reference (0), figure (4).
- $\mathrm{PL}_{0}$ longitudinal linear parallax of the point of reference.
- $\Delta \mathrm{H}_{\mathrm{i}}=\mathrm{H}_{\mathrm{i}}-\mathrm{H}_{0}$ difference in distance.
- $\Delta \mathrm{Pl}_{\mathrm{i}}$ difference in parallaxes between the reference and point I.

(7)

Figure 4: General diagram of the parallax

$$
\Delta \mathrm{PLi}=\mathrm{PLi}-\mathrm{PL} 0=\text { B.c. }\left(\frac{1}{\mathrm{H}_{\mathrm{i}}}-\frac{1}{\mathrm{H}_{0}}\right)=\frac{\text { B.c. } \Delta \mathrm{H}_{\mathrm{i}}}{\mathrm{H}_{\mathrm{i}} \mathrm{H}_{0}}
$$

By developing (7) above, we could get:

$$
\mathrm{H}_{0}^{2} \cdot \Delta \mathrm{PLi}_{i} \Delta \mathrm{H}_{\mathrm{i}}=\mathrm{B} . \mathrm{c} \cdot \Delta \mathrm{H}_{\mathrm{i}}
$$

which gives:

$$
\Delta \mathrm{H}_{\mathrm{i}}\left(\mathrm{~B} . \mathrm{c}+\mathrm{H}_{0} . \Delta \mathrm{PL}_{\mathrm{i}}\right)-\mathrm{H}_{0}^{2} . \Delta \mathrm{PL}_{\mathrm{i}}=0
$$

that is to say a function of the type $\mathrm{F}(\Delta \mathrm{Hi}, \Delta \mathrm{PLi})=0$.
The total differential $(\mathrm{dF}=0)$ leads to the matrix equation $\mathrm{AX}=0$ where :

$$
\left.\mathrm{A}=\left[\begin{array}{ll}
\mathrm{A}_{1} & \mathrm{~B}_{1} \\
\mathrm{~A}_{\mathrm{i}} & \mathrm{~B}_{\mathrm{i}} \\
\mathrm{~A}_{\mathrm{n}} & \mathrm{~B}_{\mathrm{n}}
\end{array}\right] ; \quad \mathrm{X}=\left\lvert\, \begin{array}{l}
\varepsilon_{x} \\
\varepsilon_{y}
\end{array}\right.\right] ; \text { consequently : } \quad \mathrm{X}=\left[{ }^{\mathrm{t}} \mathrm{AA}\right]^{-1}
$$

Vector X represents the oblique of measurements in distance and parallaxes.

### 3.2 DLT treatment

DLT treatment must be made beam by beam. Stages of this process include:

- Calculation of the beam parameters.
- Calculation of the distortion factors.
- Following beam.
- Calculation of the new points.

The distortion of the objective concerns the radius and the revolution and the parameters vary in a linear way. A linear interpolation can be carried out to assess the distortion parameters relating to the control points and to the new points.

Thus for each $\mathrm{dr}_{\mathrm{j}}=\left(\mathrm{r}_{\mathrm{j}+1}-\mathrm{r}_{\mathrm{j}}\right)$ there is a corresponding increase $\mathrm{dk}_{1}(\mathrm{j})=\mathrm{k}_{1}(\mathrm{j}+1)-\mathrm{k}_{1}(\mathrm{j})$ which involves for a $\mathrm{dr}=\left(\mathrm{r}-\mathrm{r}_{\mathrm{j}}\right)$ increase

$$
\mathrm{dk}_{1}=\frac{\left(\mathrm{r}-\mathrm{r}_{\mathrm{j}}\right)\left(\mathrm{k}_{1}(\mathrm{j}+1)-\mathrm{k}_{1}(\mathrm{j})\right.}{\mathrm{r}_{\mathrm{j}+1}-\mathrm{r}_{\mathrm{j}}}
$$

and hence, the value of $\mathrm{k}_{1}$ corresponding to the radial distance R is:

$$
\begin{aligned}
& \mathrm{p}_{1}=\mathrm{p}_{1}(\mathrm{j})+\mathrm{dp}_{1} \\
& \mathrm{p}_{2}=\mathrm{p}_{2}(\mathrm{j})+\mathrm{dp}_{2}
\end{aligned}
$$

the verification constraints being expressed as [8] :

$$
\begin{align*}
& \quad\left(\mathrm{L}_{1}^{2}+\mathrm{L}_{2}^{2}+\mathrm{L}_{3}^{2}\right)-\left(\mathrm{L}_{5}^{2}+\mathrm{L}_{6}^{2}+\mathrm{L}_{7}^{2}\right)+\left(\left(\mathrm{C}^{2}-\mathrm{B}^{2}\right) / \mathrm{D}^{2}\right)=0  \tag{8}\\
& \mathrm{~A}-(\mathrm{BC} / \mathrm{D})=0
\end{align*}
$$

However, if the twenty eight coefficients of the two homologous perspective beams are determined, it is possible then to determine the objects co-ordinates of any point common to these two beams, considering that for each point, there will be the following system:

$$
\begin{align*}
& P \cdot W-Q=0  \tag{9}\\
& P=\left[\begin{array}{lll}
a_{11} & a_{21} & a_{31} \\
a_{41} & a_{51} & a_{61} \\
a_{12} & a_{22} & a_{32} \\
a_{42} & a_{52} & a_{62}
\end{array}\right] ; W=\left[\begin{array}{l}
x \\
y \\
z
\end{array}\right] ; Q=\left[\begin{array}{l}
b_{11} \\
b_{12} \\
b_{21} \\
b_{22}
\end{array}\right]
\end{align*}
$$

We have here a problem with three unknown (3) factors which can be solved via the leastsquares method (Cholevski method) wherein W indicates the vector of object co-ordinates ( $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ ). Furthermore, this equation can be adapted through the MGPC (Multi Geometrical Photo Constraint) (9) for a better analysis (camera / object).

### 3.3 Total Compensation

The objective is to establish an observation relation bringing together the point, the beam and external data such as the distances measured.

$$
\begin{equation*}
\mathrm{PjL}_{\mathrm{i}}^{\mathrm{j}}-\mathrm{W}_{\mathrm{i}}^{\mathrm{j}}=0 \tag{10}
\end{equation*}
$$

where :

$\mathrm{L}_{\mathrm{i}}^{\mathrm{j}}$ : point i in beam j ;
$W_{i}^{j}$ : plate co-ordinates of point $i$ in beam $j$

- for each beam there are $\mathrm{N} \times 2$ equations;
- for $m$ beam one obtains $m \times N \times 2$ equations;
- the unknown factors will be Li parameters of each beam, i.e. $14 \times \mathrm{m}$;

The compensation principle is to reduce the gap of the plate observations, $\left[\begin{array}{rl}r_{x} & r_{y}\end{array}\right]=0$. The matrix is diagonal in shape and its resolution goes through the RNM (Reduced Normal Matrix) [10]. Concerning the external data, a condition equation on the distances reduced to the scale of the plate. This fusion of photogrammetric precision and the potential of interpretation of the object characteristics is a new approach associating geometry and radiometry [11].

With the evolution of the photogrammetric systems with video images, combination with external data in the total treatment will allow a better mastery of space reference and a more precise assessment on the object [12].

## Algorithm figure (5):

$\Rightarrow$ To acquire perspective i and $\mathrm{i}+1$;
$\Rightarrow$ Pre-treatments ( $\mathrm{x}, \mathrm{y}$ ), ( $\mathrm{x}^{\prime}, \mathrm{y}$ '), oblique;
$\Rightarrow$ DLT treatment - calculation of distortion - interpolation;
$\Rightarrow$ Storage of parameters (14) ;
$\Rightarrow$ Repeat $1,2,3,4$ until $\mathrm{i}+1=$ NR beams
$\Rightarrow$ Integration of other measurements (distances, co-ordinates, etc..)
$\Rightarrow$ Adjustment of parameters (beams) - Rigorous method;
$\Rightarrow$ dimensional control.


Figure 5: Software architecture

## 4. APPLICATION

### 4.1 Test Object

Our example concerns an experiment carried out in the National Center of Space Techniques (CNTS) on an industrial object (a Mazda pick up truck), figure (6). The three-dimensional reference observations are carried out using an electronic tachymeter (ZEISS ELTA 46), and
the snapshots using a simple camera (Minolta) with a 45 mm lens. The overlap of the two beams was $80 \%$.
The calculation of the ground co-ordinates (object) and the three dimensional extraction includes all of the image observations of the points. The latter were carried out on a C100 ZEISS Planicomp comparator on the one hand, coupled with an on screen observation on the other hand. This double task was necessary to allow a comparison between the comparator system and the screen system. Indeed, the mastery of the determination of uncertainty is less obvious, especially when calculations of characteristics are concerned (evaluation of the propagation of errors) [13].


Figure 6: Mazda vehicle involved in an accident
The form of the data:
The acquisition of the data is made on comparator, the distinction fulcrum (PA) and control point (PC) is used for the resolution of the system. For each beam, an observation table gathers the objects co-ordinates and the plate co-ordinates. Initial checks of the beam treatment appear in [6] according to the parameters obtained.

Photo $N^{\circ}$ 14; Center: $X=116.254 \mathrm{~mm} ; ~ y=91.048 \mathrm{~mm}$

| NUMERO | NATURE | X object | Y object | Z object | X comp | Y:comp |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 01 | PA | -.900 | 4.940 | -.2100 | 94.6010 | 88.0290 |
| 02 | PA | -.760 | 4.960 | -.4700 | 97.7040 | 81.4760 |
| 03 | PA | -.540 | 4.940 | -.3000 | 102.9880 | 85.6060 |
| 07 | PA | .120 | 5.000 | -.6100 | 118.7260 | 78.0000 |
| 08 | PA | .420 | 4.990 | -.0100 | 126.2510 | 92.1720 |
| 09 | PA | .430 | 5.000 | -.5700 | 126.1870 | 78.1930 |
| 11 | PA | .820 | 4.940 | -.1800 | 135.8920 | 87.6680 |
| 12 | PA | .930 | 4.980 | -.5300 | 137.9780 | 78.8030 |
| 13 | PA | 1.000 | 5.040 | .4900 | 140.1670 | 103.7260 |
| 17 | PA | 1.320 | 4.990 | -.5000 | 147.1700 | 79.3420 |
| 18 | PA | 1.580 | 4.950 | -.0900 | 153.9460 | 89.4490 |
| 19 | PA | 1.630 | 4.950 | -.3400 | 154.7780 | 83.1590 |
| 05 | PC | -.220 | 4.940 | -.2800 | 110.9460 | 85.7900 |
| 06 | PC | 0.000 | 5.010 | -.1300 | 116.2090 | 89.5940 |
| 10 | PC | .660 | 5.040 | .4500 | 132.2680 | 102.9750 |
| 14 | PC | 1.080 | 4.940 | 0.0000 | 142.3120 | 92.0120 |
| 15 | PC | 1.250 | 5.010 | .4000 | 146.0680 | 101.5290 |
| 16 | PC | 1.320 | 4.980 | .0200 | 147.6470 | 92.3400 |
| 04 | PC | -.440 | 4.970 | -.1400 | 105.6380 | 89.2410 |

### 4.2 Results

The system resolution ( $\mathrm{AX}=\mathrm{B}$ ) by a direct approach gives the solution to the 14 unknown parameters. A deduction of the distortion parameters of elements L12, L13 and L14 makes it possible to define K 1 p 1 p 2 for each plate point observed. In practice, the sources of measurement errors affect the calculation of the parametres in a negative way

PHOTOGRAPH N ${ }^{\circ} 14$ TRANSFORMATION PARAMETERS


DISTORTION PARAMETERS OF FULCRUMS

| NUMERO | K1 | P1 | P2 |
| :---: | :---: | :---: | :---: |
| 01 | 74.555211152234 | .698774755861 | -.199788782076 |
| 02 | 72.888151627576 | .683150105426 | -.195321491501 |
| 03 | 75.990364253676 | -.71222584456 | -.203634623108 |
| 07 | 68.451934305574 | -.64157129922 | -.183433570562 |
| 08 | 66.953990793786 | .627531702315 | -.179419467386 |
| 09 | 68.750463636988 | -.64436928956 | -.184233552355 |
| 11 | 78.074856334814 | .731762915443 | -.209220525525 |
| 12 | 72.943446928494 | .683668365113 | -.195469668680 |
| 13 | 58.375442444611 | .547128563008 | -.156431165158 |
| 17 | 71.666879507795 | .671703633554 | -.192048796466 |
| 18 | 76.965445675007 | .721364874166 | -.206247590419 |
| 19 | 79.082810320863 | .741210045827 | -.211921582851 |

$$
\text { Verification: } \mathbf{P}=0.359768 \mathbf{E - 0 6} ; \quad \mathbf{Q}=-0.337842 \mathbf{E - 0 7}
$$

THE ALTERED FULCRUM COORDINATES

| NUMERO | X | Y | Z | DX | DY | DZ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 01 | -.8968 | 4.9384 | -.2147 | .0032 | -.0016 | -.0047 |
| 02 | -.7742 | 4.9575 | -.4705 | -.0142 | -.0025 | -.0005 |
| 03 | -.5291 | 4.9516 | -.2973 | .0109 | .0116 | .0027 |
| 07 | .1229 | 4.9884 | -.5849 | .0029 | -.0116 | .0251 |
| 08 | .4094 | 4.9639 | -.0234 | -.0106 | -.0261 | -.0134 |
| 09 | .4365 | 4.9960 | -.5745 | .0065 | -.0040 | -.0045 |
| 11 | .8230 | 4.9595 | -.1826 | .0030 | .0195 | -.0026 |
| 12 | .9367 | 4.9961 | -.5416 | .0067 | .0161 | -.0116 |
| 13 | 1.0039 | 5.0226 | .4873 | .0039 | -.0174 | -.0027 |
| 17 | 1.3161 | 4.9742 | -.4955 | -.0039 | -.0158 | .0045 |
| 18 | 1.5852 | 4.9557 | -.0952 | .0052 | .0057 | -.0052 |
| 19 | 1.6353 | 4.9604 | -.3406 | .0053 | .0104 | -.0006 |


| Number | X | Y | Z | DX | DY | DZ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 05 | .2111 | 4.9655 | .2851 | .0089 | .0255 | -.0051 |
| 06 | .0295 | 4.9487 | .1284 | .0295 | -.0613 | .0016 |
| 10 | .6226 | 5.0141 | .4378 | -.0374 | -.0259 | -.0122 |
| 14 | 1.0810 | 4.9594 | 0.0091 | .0010 | .0194 | -.0091 |
| 15 | 1.2431 | 4.9898 | 0.3900 | -.0069 | -.0202 | -.0100 |
| 16 | 1.2941 | 4.9502 | 0.0115 | -.0259 | -.0298 | -.0085 |
| 04 | 0.4309 | 4.9518 | .1570 | .0091 | -.0182 | -.0170 |

Traditional results of photogrammetry (comparator ) :

| Model | $\mathbf{1 4 / \mathbf { 1 5 }}$ |
| :---: | :---: |
| Number of Control Points | 7 points |
| EMQ Projection | 0.0452 m |
| EMQ Elevation | 0.0110 m |

Results of the on-screen measurements:

| Model | $\mathbf{1 4} / \mathbf{1 5}$ |
| :---: | :---: |
| Number of Control Points | 7 points |
| EMQ Projection | 0.0248 m |
| EMQ Elevation | 0.0049 m |

## 5. CONCLUSION

The photogrammetric methods bring without a shadow of doubt a considerable new dimension in industrial metrology. The use of the various types of processes, and particularly the aspects of integration for a better definition of the adjustments, is most helpful for several reasons in mechanics, particularly in CADM (Computer-assisted Design and Manufacture). For the sample test described, one is interested in the precision of acquisition and in the approach of observation of the images of objects in a previously defined space to allow a construction of a prototype (scale model) and to envisage simulations (design). The degree of precision reached will be modest for applications requiring a great exactitude, but this does not pose a problem as to the handling of equations of observations.
The complementarity with other methods of measurements such as MMT is a good approach which can find applications in the comparison and inverse engineering.

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Nomenclature of the variables used :

| Variable | Signification |
| :---: | :--- |
| $\mathrm{x}, \mathrm{y}$ | comparator co-ordinate (plate co-ordinate) |
| XYZ | object co-ordinates |
| $\mathrm{X}_{0} \mathrm{Y}_{0} \mathrm{Z}_{0}$ | Co-ordinate of the perspectif centre. |
| $\mathrm{R}, \mathrm{k}$ | Rotation and scale factor |
| $\mathrm{b}_{\mathrm{x}} \mathrm{b}_{\mathrm{y}} \mathrm{b}_{\mathrm{z}}$ | Component of the base |
| $\mathrm{x}^{\prime}, \mathrm{y}^{\prime}$ | Plate co-ordinate after correction of rotation |
| $\mathrm{Pl}_{0}$ | longitudinale linear parallax of the point ( référence) |
| $\mathrm{H}_{0}$ | Distance between the perspectif center and the reference surface |
| $\mathrm{Pl}_{\mathrm{i}} \Delta \mathrm{Pl}_{\mathrm{i}}$ | longitudinale liear parallax difference between point $\mathrm{i} /$ reference |
| $\mathrm{H}_{\mathrm{i}}, \Delta \mathrm{H}_{\mathrm{i}}$ | Remoteness of the point i - difference of distance between i/ référence |
| B | Ground Base |
| $\mathrm{c}, \mathrm{r}$ | Focal distance and radial distance |
| $\mathrm{L}_{\mathrm{i}}$ | transformation parameters of DLT |
| $\mathrm{k}_{1}, \mathrm{p}_{1}, \mathrm{p}_{2}$ | Parameters of distortion |
| $\mathrm{dk}_{1}, \mathrm{dp}, \mathrm{dp}_{2}$ | Value of the linear interpolation |
| $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ | Coefficients of Bopp and krauss |

## BIOGRAPHICAL NOTES

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