Precise Determination of Hong Kong Geoid Using Heterogeneous Data

Dr. LUO Zhicai and Prof. CHEN Yong-qi, Hong Kong, China

Key words: Geoid determination, GPS/leveling, Gravity anomaly, Digital terrain model, Geopotential model.

ABSTRACT

The precise determination of local geoid is of considerable importance not only for surveying and mapping but also for studying some related problems of geophysics, oceanography and geodynamics, and now especially for applying GPS technique to determine orthometric or normal height in geodesy and surveying engineering. In the paper, based on the remove-restore technique, the Hong Kong gravimetric geoid with resolution of one kilometer has been computed using high degree global spherical harmonic geopotential model WDM94, gravity anomalies and digital terrain model which covering the whole territory of Hong Kong and its neighboring region. By comparing with precise GPS/leveling data the standard deviation of the geoid is $\pm 7.6$ cm, and the relative accuracy about 4ppm for baseline length longer than 50 km. The results illustrate that the systematic bias exists obviously between the two types of geoid heights. Employing four parameter transform model the gravimetric geoid can be improved significantly by precise GPS/leveling data. The results show that the standard deviation of the improved geoid is $\pm 1.9$ cm, and the relative accuracy better than 1ppm for baseline length longer than 40 km. Moreover, after the systematic bias removed the combined geoid was obtained by interpolating the residuals at grid points with Shepard surface fitting method, and the final geoid will be evaluated using independent precise GPS/leveling data in near future.

CONTACT

Prof. Y.Q.CHEN and Dr. Z.C. LUO
Department of Land Surveying & Geo-Informatics, The Hong Kong Polytechnic University
Hung Hom, Kowloon, Hong Kong
P.R.China
Tel. + 852 2766 5966
Fax + 852 2330 2994
E-mail: lsyqchen@polyu.edu.hk and lszcluo@polyu.edu.hk
Web site: http://www.lsgi.polyu.edu.hk/staff/YQ.Chen/index.htm
1. INTRODUCTION

The geoid is loosely defined as the equipotential surface of the earth’s gravity field at the mean sea level. It gives a consistent height system of topography on land as well as on sea. The precise determination of local geoid is of considerable importance not only for surveying and mapping but also for studying some related problems of geophysics, oceanography and geodynamics, and now especially for applying GPS technique to determine orthometric or normal height in geodesy and surveying engineering. Using EGM96 global spherical harmonic geopotential model, digital terrain model (DTM), high quality GPS/leveling data and gravity data on land and on sea which obtained at that time, the geoid of Hong Kong has been computed with remove-restore technique by Yang (1999), and he demonstrated that the accuracy of such derived geoid can arrive at about 2.6 centimeters, and that its relative accuracy better than 1ppm for baseline length longer than 40 kilometers. Since then more high precise GPS/leveling stations have been established in Hong Kong. Moreover, more gravity data and better DTM were collected. Therefore it’s necessary to reconstruct the geoid of Hong Kong. In the paper all aspects involved in the practical computation and evaluation of the new geoid are described. The data sources and its preparation is first summarized. It is followed by discussing the selection of the best fitting geo-potential model, the fourth section discusses the method for the computation of gravimetric geoid height using Stokes formula with original spherical kernel function by 1-D fast Fourier transform (FFT) in the remove-restore approach. High quality GPS/leveling data should be used to remove the systematic biases of the gravimetric geoid through 4-parameter transformation model. In the fifth section the combined geoid is constructed with Shepard surface interpolation method.

2. DATA SOURCES

2.1 Digital terrain model

To compute terrain effects DTM of Hong Kong and its neighboring region must be constructed. 16 sheets of 1:20 000 digital topographical maps covering the whole territory of Hong Kong were used to generate DTMs with resolution 100m, 500m, and 1 km using software 3D Analyst of ArcView GIS version 3.1. 100m resolution DTM of Shenzhen was obtained. Figure 1 shows the topography of Hong Kong and its neighboring region of 70km by 70km. In the area the highest point is 823.0m above Hong Kong Principal Datum, the lowest –60.8m and the on average 73.7m.
2.2 GPS/leveling data

There are 55 high quality GPS stations in Hong Kong. Their heights above the Hong Kong Principal Datum were determined with precise geometric leveling or trigonometric leveling. The accuracy specification of geometric leveling is \(4\sqrt{k} \sim 8\sqrt{k}\) (mm) where \(k\) is the length of leveling routine in km. The estimated accuracy of leveled heights is 1~2 cm at 17 stations, 3~5 cm at 26 stations, and no better than 10 cm at 12 stations. The distribution of these GPS/leveling stations is displayed in Figure 2.

Figure 2 Distribution of the 55 GPS/leveling stations in Hong Kong (HK80 Grid Coordinates)
2.3 Gravity data

Hong Kong has 640 discrete gravity observations with station spacing 2 km on land and 4 km on sea. They were collected using Lacoste and Romberg model ‘G’ land gravity meter and model ‘H/U’ seabed gravity meter by Electronic and Geophysical Services Ltd. (EGS, 1988; 1991). The local gravity base is connected to International Gravity Standardization Net 1971 (IGSN 71) with accuracy of 0.03 mGal (Evans, 1990). The gravity measurements in neighboring region Shenzhen are available: 3609 points on land and 1262 points on sea with 1 km resolution. They were measured with Lacoste & Romberg model ‘G’ and ‘D’ land gravimeter and model ‘S’ sea gravimeter in 2001. We selected 2158 gravity measurements near Hong Kong territories together with 640 gravity measurements in Hong Kong for the determination of Hong Kong geoid. Figure 3 shows the distribution of these gravity measurements.

![Figure 3 Distribution of gravity observations over Hong Kong and its neighboring region](image)

3. SELECTION OF THE BEST GEO-POTENTIAL MODEL

In the determination of local or regional geoid, the remove-restore approach is usually employed. The medium and long wavelength components of local gravity field is first modeled by a high degree geo-potential model. For this purpose the geo-potential model with the best fitting to the area of interest is preferred. There are many high degree global geo-potential models, including the OSU series (OSU89a, OSU89b and OSU91a), the GFZ series (GFZ93a, GFZ93b and GFZ95a), the NASA GSFC and NIMA joint model EGM96 (Pavlis, 1996), WDM94 developed by Wuhan Technical University of Surveying and Mapping (Ning et al., 1994), GPM98CR (Wenzel, 1998), and etc. We evaluated three models (i.e., EGM96, WDM94, and GPM98CR) using the following data: 43 GPS/Leveling observations and 640 gravity measurements in Hong Kong, and 62 GPS/Leveling observations and 4623 gravity measurement in Shenzhen.

Let $h$ and $H$ denote the GPS-derived ellipsoidal height and the leveled height of a station, respectively. The geoid height at the station can be calculated by
Value $N_{obs}$ so obtained here called observed geoid height. The geoid height $N^{GM}$ from a geopotential model can be formulated by (Heiskanen and Moritz, 1967)

$$N^{GM} = \frac{GM}{r \cdot \gamma} \sum_{n=2}^{n_{max}} \left( \frac{a}{r} \right)^n \sum_{m=0}^{n} \left( C_{nm} \cos m \lambda + S_{nm} \sin m \lambda \right) P_{nm}(\sin \varphi)$$

where $GM$ is the geocentric gravitational constant, $\gamma$ is the normal gravity of the computation point, $a$ is the long radius of reference ellipsoid, $n_{max}$ is the maximum degree of geopotential model, $\varphi$, $\lambda$ and $r$ are the geocentric latitude, longitude and radial distance of the computation point, $C_{nm}$ and $S_{nm}$ are the fully normalized coefficients of the anomalous potential, and $P_{nm}(\sin \varphi)$ is the fully normalized associated Legendre functions, given degree $n$ and order $m$. The differences between the observed geoid heights and those from geopotential model at the same position can be computed by

$$\Delta N = N_{obs} - N^{GM}$$

The values $\Delta N$ reflect the goodness of fitting. Table 1 lists the statistic values of $\Delta N$ for three geo-potential models in Hong Kong and Shenzhen. To remove the possible systematic biases, four parameter transformation is applied, i.e.,

$$N_{obs}^{(x, y)} = N^{GM}^{(x, y)} + a_0 + a_1 x + a_2 y + a_3 xy + v$$

where $a_i (i = 0, 1, \cdots)$ is unknown parameters, $v$ denotes a residual random noise term, $x$ and $y$ are HK80 grid coordinates or Shenzhen local grid coordinates in kilometer.

It is shown that the STD from geo-potential models EGM96, WDM94 and GPM98CR are $\pm 0.039m$, $\pm 0.037m$ and $\pm 0.030m$ in Hong Kong, and $\pm 0.052m$, $\pm 0.032m$ and $\pm 0.095m$ in Shenzhen, respectively. In general the accuracy of geoid heights from WDM94 is better. This is because more observed gravity data covering the territory of China were used to develop WDM94 geo-potential model.

Gravity measurements can also be used to evaluate these geo-potential models. The gravity observations are reduced to the free-air gravity anomalies $\Delta g_{obs}$ on the geoid (approximately the mean sea level). Furthermore the gravity anomalies $\Delta g^{GM}$ on the gravity stations from geo-potential model up to degree 360 for EGM96 and WDM94, and 720 for GPM98CR can be calculated by (Heiskanen and Moritz, 1967)

$$\Delta g^{GM} = \frac{GM}{r^2} \sum_{n=2}^{n_{max}} \left( \frac{a}{r} \right)^n (n-1) \sum_{m=0}^{n} (C_{nm} \cos m \lambda + S_{nm} \sin m \lambda) P_{nm}(\sin \varphi)$$

The difference between the observed gravity anomalies and those computed from geo-potential model can be obtained by

$$\Delta g = \Delta g_{obs} - \Delta g^{GM}$$

As $\Delta N$, values $\Delta g$ can also reflect the goodness of fitting. Table 2 lists the statistic values of $\Delta g$. From Table 2 we can see the STD values for EGM96, WDM94 and GPM98CR are $\pm 14.136mGal$, $\pm 13.559mGal$ and $\pm 15.130mGal$ in Hong Kong, respectively, and $\pm 8.913mGal$, $\pm 8.613mGal$ and $\pm 11.750mGal$ in Shenzhen. These results also indicate that WDM94 is better than others. Therefore WDM94 is a preferred reference geo-potential
model for the determination of new Hong Kong geoid.

Table 1 Statistics of values $\Delta N$ in Hong Kong (HK) and Shenzhen (SZ) (Unit: meter)

<table>
<thead>
<tr>
<th>Geopotential model</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HK</td>
<td>SZ</td>
<td>HK</td>
<td>SZ</td>
</tr>
<tr>
<td>EGM96</td>
<td>before</td>
<td>0.356</td>
<td>0.033</td>
<td>-0.119</td>
</tr>
<tr>
<td></td>
<td>after</td>
<td>0.090</td>
<td>0.127</td>
<td>-0.105</td>
</tr>
<tr>
<td>WDM94</td>
<td>before</td>
<td>0.580</td>
<td>0.895</td>
<td>0.239</td>
</tr>
<tr>
<td></td>
<td>after</td>
<td>0.080</td>
<td>0.076</td>
<td>-0.118</td>
</tr>
<tr>
<td>GPM98CR</td>
<td>before</td>
<td>0.284</td>
<td>0.308</td>
<td>-0.280</td>
</tr>
<tr>
<td></td>
<td>after</td>
<td>0.082</td>
<td>0.227</td>
<td>-0.064</td>
</tr>
</tbody>
</table>

Note: ‘before’ denotes the values before systematic bias removed, and ‘after’ denotes the values after systematic bias removed.

Table 2 Statistics of values $d_g$ in Hong Kong and Shenzhen (Unit: mGal)

<table>
<thead>
<tr>
<th>Geopotential model</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>RMS</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HK</td>
<td>SZ</td>
<td>HK</td>
<td>SZ</td>
<td>HK</td>
</tr>
</tbody>
</table>

4. GRAVIMETRIC GEOID DETERMINATION

In theory, local gravimetric geoid can be determined by solving the geodetic boundary value problem (such as Stokes’ problem or Molodensky’ problem) using gravity anomaly covering the area of interest as boundary value. But in practical computation the remove-restore technique has been widely used to improve the Stokes integral solution, especially in mountain area (see, e.g. Moritz, 1983; Sideris and Forsberg, 1990; Tziavos et al., 1992; Forsberg and Tscherning, 1997; Zhang and Featherstone, 1997). With the technique the long-medium wavelength component derived from a geopotential model and the short wavelength component due to the topographic effect are removed mathematically from the observed gravity anomalies. And then the Stokes’ integration of the remaining parts of the gravity anomalies (called residual gravity anomaly) provides the residual geoid height. The final geoid heights of points are obtained by restoring the previous removed components, i.e., the gravimetric geoid at point $P(\phi_p, \lambda_p)$, denoted as $N^G(\phi_p, \lambda_p)$, can be expressed as

$$N^G(\phi_p, \lambda_p) = N(\phi_p, \lambda_p) + N^{GM}(\phi_p, \lambda_p) + N^T(\phi_p, \lambda_p)$$

(7)

where, $N^T(\phi_p, \lambda_p)$ is the topographic indirect effect on the geoid due to the second method of Helmert’s condensation of the topography (Wicheincharoen, 1982), $N^{GM}(\phi_p, \lambda_p)$ is computed from a geopotential model by equation (2) and $N(\phi_p, \lambda_p)$ is the contribution of...
residual gravity anomaly $\delta g(\varphi, \lambda)$, computed by the Stokes' integration. For the detail readers are referred to (Yang and Chen, 2001).

The grid terrain correction with spacing 100m was first computed using the mass prism topographic model and 100% zero padding technique (Sideris, 1990; Sideris and Li, 1992; Li, 1993). The grid terrain corrections with grid interval 500 meters and one kilometer were interpolated respectively from the 100 meters grid. These terrain corrections, the gravity anomalies from WDM94 by equation (5) and the observed gravity anomalies were employed to construct the residual gravity anomalies of 1km grid. Figure 4 provides the residual gravity anomalies of 1km grid. Secondly, using original spherical Stokes kernel function and 1-D FFT technique (Haagmans et al., 1993; Li, 1993; Sideris and She, 1995) the residual geoid was computed. Figure 5 show the residual geoid with 1km grid. Finally, the gravimetric geoid is obtained by restoring the previous removed components. Figure 6 displays the gravimetric geoid with resolution of 1km, and the statistic for different geoid components was listed in Table 3.

Table 3 Statistic of different geoid components for 1 km grid (unit : meter)

<table>
<thead>
<tr>
<th>Item</th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual geoid</td>
<td>-0.065</td>
<td>-0.275</td>
<td>-0.162</td>
<td>±0.047</td>
</tr>
<tr>
<td>WDM94 to degree and order 360</td>
<td>-0.236</td>
<td>-3.893</td>
<td>-2.064</td>
<td>±0.835</td>
</tr>
<tr>
<td>Terrain indirect effects on geoid</td>
<td>0.000</td>
<td>-0.024</td>
<td>-0.001</td>
<td>±0.002</td>
</tr>
<tr>
<td>Terrain direct effects on geoid</td>
<td>0.070</td>
<td>0.015</td>
<td>0.034</td>
<td>±0.010</td>
</tr>
<tr>
<td>Gravimetric geoid</td>
<td>-0.325</td>
<td>-4.058</td>
<td>-2.260</td>
<td>±0.868</td>
</tr>
</tbody>
</table>

Figure 4 Residual gravity anomalies of Hong Kong at 1km grid (HK80 Grid Coordinates, Contour interval 10mGal)
The accuracy of such derived gravimetric geoid is always estimated by comparing their geoid
heights with that derived from GPS/Leveling in relative or absolute sense. The comparison
results show that the standard deviation of the gravimetric geoid is ±7.6 cm, and the relative
accuracy can arrive at about 4ppm for baseline length longer than 50 km. After the systematic
bias removed by equation (4) the standard deviation of transformed geoid is ±1.9cm, and its
relative accuracy is better than 1ppm for baseline length longer than 40km (see Figure 7).
These results also illustrate that the gravimetric geoid can be improved significantly by
precise GPS/leveling data.
In this study the effects of gravity data outside the computed area were investigated. The Hong Kong geoid was determined both with gravity measurements in Shenzhen and without the data there. The differences are given in Table 4, where the transformed geoid means the systematic biases were removed using the GPS/Leveling data. It can be seen that the differences are significant for some parts of the area. Figures 8 and 9 show these differences. It is clear that the significant differences are in the northern part of the territory.

Table 4  Differences  between the gravimetric geoid heights at 1 km grid points with and without Shenzhen gravity data included (unit: meter)

<table>
<thead>
<tr>
<th></th>
<th>Max</th>
<th>Min</th>
<th>Mean</th>
<th>STD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravimetric geoid</td>
<td>0.070</td>
<td>-0.020</td>
<td>0.007</td>
<td>±0.014</td>
</tr>
<tr>
<td>Transformed geoid</td>
<td>0.062</td>
<td>-0.015</td>
<td>0.003</td>
<td>±0.013</td>
</tr>
</tbody>
</table>

Figure 8 Differences between the gravimetric geoid with and without Shenzhen gravity data included before systematic biases removed (HK80 Grid Coordinates, contour interval 0.01m)
5. THE GEOID DETERMINED WITH HETEROGENEOUS DATA

To precisely determine local geoid a strategy to take advantage of all the data/information is discussed in detail in (Chen and Yang, 2001). The gravimetric geoid was first constructed using the remove-restore technique with WDM94 as reference geo-potential model, terrain correction applied, and gravity data both in Hong Kong and Shenzhen. The possible systematic biases were then removed with GPS/Leveling data. To improve the accuracy of the geoid, the GPS/Leveling data should be fully utilized. Therefore local Shepard surface interpolation method is used to interpolate the residuals at one kilometer grid, i.e.,

$$\delta N(\phi_p, \lambda_p) = \sum_{i=1}^{n} \delta N(\phi_i, \lambda_i) \cdot K(r_i)^m / \sum_{i=1}^{n} K(r_i)^m, \quad r_i \neq 0$$

$$\delta N(\phi_i, \lambda_i), \quad r_i = 0$$

(8)

where $\delta N$ is the difference between the GPS/leveling derived geoid height and gravimetric geoid height, $r_i$ is the distance between interpolating point $(\phi_p, \lambda_p)$ and known point $(\phi_i, \lambda_i)$, $m$ the fitting power should be integer ($m = 1, 2, \cdots$), and $K(r_i)$ the kernel function or so-called weighting function, i.e.,

$$K(r) = \begin{cases} 
1/r, & 0 < r \leq D/3 \\
27 \left( r/D \right)^2 - 1, & D/3 < r \leq D \\
0, & r > D 
\end{cases}$$

(9)

with $D$ interpolating or searching radius. Such derived geoid is called hybrid or combined geoid. Figure 10 is the hybrid geoid with 1km grid. This geoid will be evaluated using independent precise GPS/leveling data in near future.

Figure 9 Differences between the gravimetric geoid with and without Shenzhen gravity data included after systematic biases removed (HK80 Grid Coordinates, contour interval 0.01m)
6. CONCLUSIONS

This paper summarizes the practical computation of the new Hong Kong geoid. On the basis of remove-compute-restore approach, the gravimetric geoid with resolution of 1km covering the 50km by 70km territory of Hong Kong, has been computed using global spherical harmonic geopotential model WDM94 to degree and order 360, gravity anomalies and digital terrain model covering the whole territory of Hong Kong and its neighboring region. Original Stokes spherical kernel function and 1-D FFT technique were employed to determine the residual geoid. Comparison with the geoid heights derived from GPS/leveling data shows systematic biases exist between the two types of geoid heights. However, removal of the systematic biases using high precise GPS/leveling data can significantly improve the gravimetric geoid. Furthermore, the hybrid geoid was computed by interpolating the residuals at grid points with Shepard surface fitting method. The final geoid will be evaluated using independent precise GPS/leveling data in near future.

ACKNOWLEDGEMENTS

This research was supported by Research Grant Council of Hong Kong SAR government (B.34.37.Q328). The GPS/Leveling data of Hong Kong was provided by Geodetic Survey Section, Survey & Mapping Office, Lands Department, Hong Kong. And the gravity data of Shenzhen were supplied by Land Planning Department of Shenzhen, China.

REFERENCES

Chen, Y.Q. and Yang, Z.J., 2001, A hybrid method to determine the Hong Kong geoid, Presented to FIG Working Week, May, 2001, South Korea. Electronic and Geophysical Services Ltd, 1988, High density gravity survey--Sheet 6 NW 8,
Final report, Job Number HK 36088, Hong Kong.
Electronic and Geophysical Services Ltd., 1991, Regional gravity survey of Hong Kong,
Final report, Job Number HK 50190, Hong Kong.
Evans, R.B., 1990, Hong Kong gravity observations in July 1990 with BGS Lacoste and
Romberg meter No. 97 and international connections to IGSN 71, British and Geology
Survey Technical report WK/90/24R.
Forsberg, R. and Tscherning, C.C., 1997, Topographic effects in gravity field modeling for
BVP, F.Sanso and R.Rummel (eds.), Geodetic Boundary Value Problems in View of the
One Centimeter Geoid, Springer-Verlag, Berlin, Heidelberg, New York, Lecture Notes
integrals on the sphere using 1D-FFT, and a comparison with existing methods for
Stokes’s integral, Manuscr Geod, 18, 227-241.
Heiskanen, W.A. and Moritz, H., 1967, Physical Geodesy, Institute of Physical Geodesy,
Technical University, Graz.
Li, Y. C., 1993, Optimized spectral geoid determination, Report No. 20050, Department of
Geomatic Science and Surveying, The University of Calgary, Canada.
Moritz, H., 1983, Local geoid determination in mountainous areas, Report No.353,
Department of Geodetic Science and Surveying, The Ohio State University.
Ning, J.S., Li, J.C., Chao D.B. and Guan Z.L., 1994, The Research of the Earth's Gravity
Field Model WDM94 Complete to Degree 360, Journal of Wuhan Technical University
cddis.gsfc.nasa.gov/926/egm926/egm96.html.
Sideris, M. G., 1990 Rigorous gravimetric terrain modeling using Molodensky’s operator,
Manuscripta Geodaetica, Vol. 15, 97-106.
Sideris, M. G. and Forsberg, R., 1990, Review of geoid prediction methods in mountainous
regions, Determination of the Geoid, Present and Future, IAG Symposia No. 106, Eds.
Sideris, M. G. and Li, Y. C., 1992, Improved geoid determination for leveling by GPS,
Proceedings of the Sixth International Symposium on Satellite Positioning, Columbus,
Sideris, M. G. and She, B. B., 1995, A new high-resolution geoid for Canada and part of the
Tziavos, I. N., Sideris, R. and Schwartz, C. C., 1992, A study of the contributions of various
gravimetric data types on the estimation of gravity field parameters in the mountains,
Wenzel, G. 1998, Ultra high degree geopotential models GPM98A, GPM98B and GPM98C
to degree 1800, http://www.gik.uni-karlsruhe.de/~wenzel/gpm98abc/gpm98abc.htm
Yang, Z.J., 1999, PRECISE DETERMINATION OF LOCAL GEOID AND ITS
GEOPHYSICAL INTERPRETATION, PhD’s thesis, Department of Land Surveying
and Geo-Informatics, The Hong Kong Polytechnic University.
Yang, Z.J. and Chen, Y.Q., 2001, Determination of the Hong Kong geoid, SURVEY
REVIEW, Vol.36, No.279, 23-34.