Refractive Index Formulae for Radio Waves

Jean M. RÜEGER, Australia

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ABSTRACT

The radio refractive index formula adopted in 1963 by the International Union of Geodesy and Geophysics (IUGG) and the International Association of Geodesy (IAG) is being reviewed. Forty years ago, this formula was essential for the reduction of distances measured with microwave EDM instruments. Since then, long-range EDM has been replaced by the Global Positioning System (GPS). Today, the formulae are important for accurate measurements using radio waves, including GPS and VLBI (Very Long Baseline Interferometry). The new radio wave refractive index formulae of the last decades are discussed and the inherent problems of some of them are pointed out. State-of-the art formulae for routine and precise measurements in the radio wave spectrum are discussed as are computer programs that model the influences of resonance lines.

Two formulae for hand calculations of the radio refractive index N_r (in ppm, for infinite wavelengths) have been designed, one based on 'best available' coefficients and one based on 'best average' coefficients. For air with 0.0375% (375 ppm) content of CO_2, the latter is:

$$N_r = 77.6890 \frac{p_d}{T} + 71.2952 \frac{p_w}{T} + 375463 \frac{p_w}{T^2}$$

where p_d (= p_{tot} - p_w) is the partial dry air (including 375 ppm carbon dioxide) pressure (in hPa), p_w is the partial water vapour pressure (in hPa), and T is the temperature (in K). The accuracy of the dry air refractivity component N_d (first term on right hand side) is, conservatively, 0.02% of N_d. A conservative value for the accuracy of the water vapour component N_w (sum of second and third term on the right hand side) is 0.2% of N_w.

CONTACTS

Assoc. Prof. Jean M. Rüeger
School of Surveying and Spatial Information Systems
University of New South Wales
UNSW SYDNEY NSW 2052
AUSTRALIA
Tel. +61-2-9385-4173
Fax +61-2-9313-7493
E-mail: J.Rueger@unsw.edu.au
Web site: www.gmat.unsw.edu.au

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1. INTRODUCTION

The last resolutions of the International Union of Geodesy and Geophysics (IUGG) on refractive indices date back to 1960 and 1963. Because of more recent determinations of the radio wave refractivity of air, the 1960 and 1963 resolutions do no longer satisfy the needs of geodesy and surveying. In consequence, the radio wave formula adopted by IUGG in 1963 is rarely (if ever) used in connection with the GPS (Global Positioning System) and VLBI (Very Long Baseline Interferometry). Even in microwave electronic distance measurement (EDM), the 1963 IUGG resolutions were not universally followed. Although terrestrial microwave distance measurement (λ = 8 – 30 mm, f = 10 – 35 GHz) is no longer used on a routine basis in most parts of the world, very long baseline interferometry (VLBI: λ = 13 – 210 mm, f = 1.4 – 23 GHz) is, and the Global Positioning System (GPS: λ = 190 – 250 mm, f = 1.2 – 1.6 GHz) even more so. A re-evaluation of the radio wave refractive index in geodesy and surveying is, thus, warranted.

In 1991, at the 20th General Assembly of the International Association of Geodesy (IAG) in Vienna, it was suggested that new IUGG resolutions on refractive indices be prepared for adoption at a future General Assembly of IUGG. An ad-hoc working party was formed in due course (Rüeger 1999). Two resolutions on the refractive index of visible and infrared waves were adopted by IAG in 1999 and saw the introduction of a revised default carbon dioxide content of 375 ppm (0.0375 percent). Since then, the review the radio and millimetre wave refractive index has progressed and an interim report has been published recently (Rüeger 2002). Some, but by no means all, key aspects of the interim report (Rüeger 2002) are discussed in this paper.

Unless stated otherwise, the following formula is used for the conversion between the pressure units of mm Hg (or Torr) and hPa (hectopascal): p [hPa] = (1013.25/760) p [mm Hg]. The temperature T (in kelvin, K) is computed from T = 273.15 + t, where t is the temperature in degrees Celsius.

2. HISTORY OF FORMULAE FOR THE RADIO REFRACTIVE INDEX

The 12th General Assembly of IUGG (Helsinki, 26 July - 6 August 1960) passed a resolution that adopted the following equation (after Essen and Froome 1951) for the reduction of microwave electronic distance measurements (IUGG 1960, Edge 1962):

$$N_r = (n_r - 1)\,10^6 = \frac{103.49}{T}\,(p_{tot} - p_w) + \frac{86.26}{T}\,\left(1 + \frac{5748}{T}\right)\,p_w \qquad (1)$$

where T = temperature (in K), p_{tot} = total atmospheric pressure (in mm Hg), p_w = partial water vapour pressure (in mm Hg), n_r = refractive index of radio waves in air at ambient conditions, N_r = refractivity (in ppm) of radio waves in air at ambient conditions. The same formula was confirmed in another resolution (IUGG 1963) at the 13th General Assembly of IUGG (Berkley, 19 - 31 August 1963). After conversion to hPa (hectopascal) as the pressure unit and into the more common form, the above equation becomes:

$$N_r = (n_r - 1) 10^6 = 77.624 \frac{p_d}{T} + 64.700 \frac{p_w}{T} + 371897 \frac{p_w}{T^2} \qquad (2)$$

where p_d (= p_{tot} - p_w) is the partial pressure of the dry air, the temperature is taken in K and both partial pressures in hPa. The accuracy of Eqs. (1) and (2) was perceived to be ±0.1 ppm under 'normal' conditions and better than ±1.0 ppm under 'extreme' conditions (Edge 1962). According to Deichl (1984), the simplifications made in the above equation (with respect to the original Essen and Froome equations of 1951) cause systematic errors of about 0.35 ppm (refractive index too small).

As the 1963 IUGG Resolution is based on Essen & Froome (1951), a brief summary of that work is warranted. At 24 GHz, these authors measured the refractive index of dry carbon-dioxide-free air, nitrogen, oxygen, argon and carbon dioxide at mean temperature of 20°C and a mean pressure of 760 mm Hg, and reduced the measurements to 0°C. They measured the refractive index of water vapour in a very limited range of temperatures (15°C and 25°C) and water vapour pressures (8.0 hPa to 18.7 hPa). They give a precision of ±0.10 ppm (0.035% of the refractivity value) for their dry air value and of ±0.1 ppm (0.165% of the refractivity value) for their water vapour value (at 20°C and 13.3 hPa). They stated that the weak water vapour absorption at 22.23 GHz (13 mm) 'should have an entirely negligible effect' and noted that the water vapour 'extrapolation formula (to other conditions) cannot be given the same confidence as that of the other gases'. Because of their limited temperature range, Essen & Froome (1951) had to predict the K_2 term of the water vapour refractivity from visible values. Hill et al. (1982, p. 1256) noted that Essen & Froome's use of the optical frequency water vapour coefficient K_2 was 'an inaccurate assumption'. Essen & Froome (1951) stated that their simplified equation (Eq. (12), p. 873 of their paper, adopted by IUGG in 1963), introduces errors in refractivity of 0.5 ppm at the extremes of the specified range of temperatures (-20°C, +60°C) and with 'normal' water vapour pressures. (Table 2 shows that, at high temperature and humidity, the simplified Essen & Froome formula differs much more than 0.5 ppm from state-of-the-art formulae.) Their definition of T = 273 + t introduces further errors. Because of all these deficiencies (of the water vapour terms, mainly), the IUGG formula of 1960 and 1963 should no longer be used.

Since the adoption of the Essen & Froome equation by IUGG in 1960/1963, a great number of investigations into the refractivity of radio and millimetre waves were carried out. The discussion here is restricted to a few contributions that are particularly relevant in the context of geodesy and surveying. For a more complete discussion of the developments in the radio refractive index, readers are referred to Rüeger (2002).

In 1974, Thayer proposed an 'improved' three-term equation for the radio refractive index for precise geodetic and laboratory use that included compressibility factors. (Although Thayer might have been the first author to use compressibility factors as such for radio refractivity, Essen & Froome (1951, Eq. (11), p. 872) already corrected for the non-ideal gas behaviour in their full equation.) Thayer noted that the omission of compressibility factors leads to errors in the radio wave refractivity of 0.04 ppm in the dry term and 0.1 ppm in the wet term at high humidities. Thayer (1974) derived his K_1 from the refractivity value published by Smith & Weintraub (1953). The latter obtained their value from a conversion of three published values of the dielectric constant of dry air, one being measured at optical wavelengths and one at 24 GHz. Like Essen & Froome (1951) and, in fact, Thayer himself (1974, before Eq. 6), Hill (1996) points out that the measured radio refractive index is larger than the extrapolated visible one. The accuracy claimed by Thayer (1974) and Smith & Weintraub (1953) is, essentially, that of the optical measurements. (The better of the two microwave measurements is worse by a factor two.) Considering the comments above, Thayer's accuracy claim and the adoption of a partly optical K_1 was inappropriate. Thayer (1974) also 'extrapolated' the 'visible' water vapour refractivity to radio wavelengths and computed the K_2 coefficient from it. He then used Boudouris' (1963) water vapour measurements to derive the K_3 value. Hill (1996) stated that 'the contributions to refraction by water vapour cannot be extrapolated to the infrared and radio regions because of the strong contribution by the infrared resonances of water vapour'. In consequence, the coefficients K_1, K_2 and K_3 proposed by Thayer (1974) should **not** be used. Unfortunately, Thayer's coefficients were used later by many authors, particularly in the field of geodesy (e.g. Herring 1992, Jarlemark 1994, Mendes 1999).

Hasegawa & Stokesberry (1975) gave a good summary of 'measured' K_1, K_2, and K_3 values. They computed weighted means for each of the three coefficients, weighting according to the author's standard deviations. Hill et al. (1982) criticised these authors for including the Essen & Froome (1951) and Essen (1953) data because the latter were established over a very limited temperature range. In addition, these authors did not list the frequency at which the experiments were carried out and, thus, ignored the aspect of anomalous refractivity. The authors also ignored the correlation between the K_2 and K_3 terms that degraded some data, including those of Boudouris.

Bevis et al. (1994) revisited the data used by Hasegawa & Stokesberry (1975). They plotted the data, eliminated outliers and computed mean values for K_1, K_2 and K_3. Again, anomalous refractivity was not considered nor the appropriateness of averaging the K_2 and K_3 values separately, considering their high correlation. Like Hasegawa & Stokesberry (1975) before, the authors included the Essen and Froome (1951) and the Essen (1953) data despite the cautionary remarks of Essen & Froome (1951). They also included Barrell's 1951 value that was extrapolated from the visible. In consequence, their standard errors (of the unweighted mean values) of ±0.05 k/hPa for K_1, ±2.2 K/hPa for K_2 and ±1200 K^2/hPa for K_3 should be treated with care.

Mendes (1999) reviewed a number of radio refractivity formula and used them to predict the total delay in distance measurements to GPS satellites. The dry air delays (in zenith direction), computed with the Boudouris (1963, Eq. (3.10) p. 660) and the Smith and Weintraub (1953) formulae (see Mendes, 1999, Table 3.1, p. 59, for coefficients), differ by only 0.6 mm (being 0.026 percent of the dry zenith delay of 2.3 m). Mendes, comparing different water vapour refractivity formulae, found differences smaller than 0.1 mm in the wet zenith delay (Mendes 1999, personal communication). The omission of the compressibility factor for dry air did not change the dry (zenith) delay and that of the compressibility factor for water vapour did change the wet zenith delay by only 0.1 to 0.2 mm. On the other hand, the omission of the enhancement factor (when computing the water vapour pressure in moist air) changed the wet zenith delay (of 0.258 m) by a maximum of 1.3 mm (at 30° latitude and in summer). Mendes also noted that the wet zenith delay can change by as much as 3 mm, depending on the formula used for the computation of the saturation water vapour pressure. To relate this information to terrestrial measurements, we note that similar dry and wet delays are experienced for a (one-way) horizontal path of 8.58 km length at sea level, 20°C and 28% relative humidity. For low lying satellites (elevation angle of 15°, zenith angle of 75°) the delays are 3.86 times larger (8.9 m for the dry delay and 1.00 m for the wet delay, at 30° latitude and in summer) and the corresponding (one-way) horizontal sea level path length 33.1 km.

3. FORMULAE FOR HAND CALCULATIONS (1 HZ TO ABOUT 1 GHZ, M TO 0.3 M)

It is valuable to have a relative simple closed solution for the refractive index of radio waves for easy calculation with pocket calculators and personal computers. The equations given here are empirical, based on experiment and ignore the non-ideal gas behaviour (compressibility) of air. However, some constants have been derived after taking the real gas behaviour into account (e.g. Birnbaum & Chatterjee 1952, and Boudouris 1963). Thayer (1974) stated that omission of the dry air compressibility factor leads to errors of about 0.04 ppm at 500 hPa and that of the water vapour compressibility factor to 0.1 ppm at high humidities. Owens' (1967, p. 55) noted that 'the partial density of CO_2 is always so small that ideal gas behaviour my be assumed' for carbon dioxide. The refractivity N_r of radio waves can be expressed as:

$$N_r = K_1' \frac{p_{d-c}}{T} + K_2 \frac{p_w}{T} + K_3 \frac{p_w}{T^2} + K_4 \frac{p_c}{T} \tag{3}$$

where p_{d-c} (= $p_d - p_c = p_{tot} - p_w - p_c$) is the (partial) pressure of the dry carbon-dioxide-free air, p_d is the (partial) pressure of the dry air (= $p_{tot} - p_w$), p_w is the partial water vapour pressure and p_c is the partial carbon dioxide pressure and where the K_i are constants and T the temperature. K_1' is the constant K_1 without the CO_2 component. Because of its polar nature, water vapour has a density (K_2) and a density-temperature (K_3) term.

3.1 Equation with 'Best Available' Coefficients

After pointing out some erroneous assumption in Thayer's derivations, Hill et al. (1982) supported the use of the coefficients by Boudouris (1963) and Birnbaum & Chatterjee (1952). Following a later suggestion by Hill (1995), the K_2 and K_3 terms are taken from Boudouris (1963) and the K_1' term (CO_2 free dry air) as well as the carbon dioxide term (K_4) from Newell & Baird (1965).

$$K_1' = 77.674 \pm 0.013 \text{ [K/hPa]} \tag{4a}$$

$$K_2 = 71.97 \pm 10.5 \text{ [K/hPa]} \tag{4b}$$

$$K_3 = 375406 \pm 3000 \ [K^2/hPa] \tag{4c}$$

$$K_4 = 133.484 \ \pm 0.022 \ [K/hPa] \tag{4d}$$

The precisions listed against K_1' and K_4 are one half of the 2σ values quoted by Newell & Baird (1965). Boudouris (1963) did derive the coefficients K_2 and K_3 from a linear regression solution of his measurements at temperatures from 0°C to +63°C and water vapour pressures from 0 to 127 mm hPa. Measured values for K_4 were also given by Liebe et al. (1977), as 133.5 ±0.15 K/hPa, and by Zhevakin & Naumov (1967), as 129.30±0.02 K/hPa. Considering that the omission of the CO_2-term leads to an error of 0.02% only (Hartmann 1993, Hartmann & Leitinger 1984) and that the carbon dioxide content of air is rarely measured by geodesists and surveyors, it is often appropriate to adopt a current value for the CO_2 content and to merge the terms K_1' and K_4 to give the 'dry-air' K_1 term:

$$K_1 \frac{p_d}{T} = K_1' \frac{p_{d-c}}{T} + K_4 \frac{p_c}{T} = K_1' \frac{p_d - p_c}{T} + K_4 \frac{p_c}{T} = K_1' \frac{p_d}{T} + (K_4 - K_1') \frac{p_c}{T} \tag{5a}$$

Assuming, initially, the traditional 300 ppm (0.03%) content of carbon dioxide, p_c can be taken as 0.0003 p_d.

$$K_1 \frac{p_d}{T} = \{K_1' + 0.0003\,(K_4 - K_1')\} \frac{p_d}{T} = \{77.674 + 0.017\} \frac{p_d}{T} = 77.691 \frac{p_d}{T} \tag{5b}$$

It can be shown that the precision of K_1 is the same as that of K_1'. The final three-term equation (after Boudouris, Newell and Baird) for air with 0.03% (300 ppm) CO_2 content is:

$$N_r = 77.691 \frac{p_d}{T} + 71.97 \frac{p_w}{T} + 375406 \frac{p_w}{T^2} \tag{6}$$

where the radio refractivity N_r is in ppm, the dry air (including carbon dioxide) pressure p_d (= p_{tot} - p_w) and the partial water vapour pressure p_w are taken in hPa and the temperature T in K. A comparison of this with other formulae can be found in Tables 1 and 2. Recomputing the K_1 term for a carbon dioxide content of 375 ppm (0.0375%), expected around the year 2004, gives the final form of the formula for the non-dispersive radio wave refractivity N_r (in ppm, temperature in K and pressures in hPa)

$$N_r = 77.695 \frac{p_d}{T} + 71.97 \frac{p_w}{T} + 375406 \frac{p_w}{T^2} \tag{7}$$

3.2 **Equation with 'Best Average' Coefficients**

Some authors that compiled experimental values of refractivity coefficients did compute mean values of the coefficients for general usage, e.g. Bean (1962), Hasegawa & Stokesberry (1975) and Bevis et al. (1994). The use of 'best average' rather than 'best available' coefficients provides a certain robustness against unmodelled systematic errors and increases the reliability of the values, particularly if data from different laboratories and researchers can be averaged. As an alternative to the formula based on the 'best available' coefficients (see above), the available data have been revisited, and new 'best average' coefficients computed

(Rüeger 2002). Using the newly derived weighted mean coefficients (Rüeger 2002, Tables 4 to 6), the following four-term formula can be constructed:

$$N_r = 77.6681 \frac{p_{d-c}}{T} + 71.2952 \frac{p_w}{T} + 375463 \frac{p_w}{T^2} + 133.4800 \frac{p_c}{T} \qquad (8)$$

Using Eq. (5b), the older value of 300 ppm (0.03%) for the content of carbon dioxide and p_c as 0.0003 p_d, the final three-term equation for the radio refractivity N_r (in ppm) with weighted mean coefficients for air with 0.03% (300 ppm) content of CO_2 becomes:

$$N_r = 77.6848 \frac{p_d}{T} + 71.2952 \frac{p_w}{T} + 375463 \frac{p_w}{T^2} \qquad (9)$$

where the dry air (including carbon dioxide) pressure p_d (= p_{tot} - p_w) and the partial water vapour pressure p_w are taken in hPa, and the temperature T in K. The accuracy of the dry air refractivity component N_d is nominally 0.012% of N_d or, more conservatively (see Rüeger 2002, Table 2), 0.02% of N_d. A realistic value for the accuracy of the water vapour component (N_w) is about 0.15% of N_w (Rüeger, 2002, bottom part of Table 5) or, more conservatively (see Rüeger 2002, Table 3), 0.20% of N_w. A comparison of this with other formulae can be found in Tables 1 and 2. Recomputing the K_1 term for a carbon dioxide content of 375 ppm (0.0375%), expected around the year 2004, gives the final form of the 'weighted mean' formula for the non-dispersive radio wave refractivity N_r (in units of ppm, K and hPa):

$$N_r = 77.6890 \frac{p_d}{T} + 71.2952 \frac{p_w}{T} + 375463 \frac{p_w}{T^2} \qquad (10)$$

4. COMPUTER ROUTINE (1 Hz to about 1 THz, ∞ m to 0.3 mm)

A practical model for the complex refractive index for the propagation calculation of electromagnetic waves through the atmosphere has been developed by Liebe et al. over many years (Liebe 1996 1989 1985, Liebe et al. 1992 1993). The **M**illimetre-Wave **P**ropagation **M**odel (MPM) is as a computer program (for personal (IBM compatible) computers) for frequencies below 1000 GHz in the atmosphere. The MPM 'consists of 44 oxygen and 30 local water resonance lines, of non-resonant spectra for dry air and of an empirical water vapour continuum that reconciles experimental discrepancies' (Liebe et al. 1992). The model is applicable for barometric pressures between 0 and 1200 hPa, ambient temperatures between -100 and +50°C, relative humidity between 0 and 100% and suspended water droplets and ice particle densities between 0 and >5 g/m^3. The MPM makes use of spectral data and is supported by many laboratory measurements to validate and enhance the overall performance of the model. The authors note that MPM dry-air absorption values agree with measured ones at the 1% level. 'Model predictions involving water vapour and water droplets are estimated to lie in the 10 percent range' (Liebe et al. 1992). Presently, MPM does not provide an input for the CO_2 content. Presumably, a carbon dioxide content of 0.03% is included in the dry-air non-dispersive term. Rüeger (2002, Table 1) has investigated the historical development of the coefficients K_1, K_2 and K_3 used by MPM, and their likely origin.

The MPM93 is freely available, fully documented and includes an executable file for IBM PCs as well as the FORTRAN source code. It can be downloaded from the web site of the Institute of Telecommunication Sciences in Boulder, Colorado, USA (http://www.its.bldrdoc.gov -> Resources -> Anonymous ftp -> pub/ -> mpm93/ -> refrac/ or directly from ftp://ftp.its.bldrdoc.gov/pub/mpm93/refrac). The REFRAC software module is the most useful variant of MPM93 for use in geodesy and surveying. Hill (2000) prepared FORTRAN77 software routines (IR_N) for the calculation of the phase and group refractive indices of air and its gaseous constituents in connection with a JPL/NASA research project. These subroutines are applicable to wavelengths from the visible through to infinite (radio waves). The effects of absorption resonances of H_2O, CO_2, O_2, O_3, CH_4, CO and N_2O are modelled using the HITRAN data base. The source code can be obtained (on CD) from J. M. Rüeger. It should be noted that there is, presently, no manual for, nor a description of, IR_N and that some programming by the user is required before IR_N can be used.

5. COMPARISON OF FORMULAE

Tables 1 and 2 show a comparison of the **M**illimetre-Wave **P**ropagation **M**odel 1993 (MPM93, non-dispersive refractivity (N_o) only) with the simple formulae by Essen & Froome (1951, as adopted by IUGG in 1960 and 1963), Boudouris (1963), a formula based on coefficients determined by Liebe et al. (1977), the formula recommended by the International Telecommunication Union in 1986 (CCIR 1986, after Bean & Dutton 1968 and Smith & Weintraub (1952)), a new formula based on the best available coefficients (Eq. (6), after Boudouris (1963) and Newell & Baird (1965)) and a new formula based on the best average coefficients (Eq. 9).

T	PWVP	MPM93	E&F51 IUGG'60 Eq.(2)	Liebe 1977	Boud 1963	Best Available Eq.(6)	Best Average Eq.(9)	CCIR'86
[°C]	p_w [hPa]	N_r [ppm]	N_r [ppm]	N_r [ppm]	N_r [ppm]	N_r [ppm]	N_r [ppm]	N_r [ppm]
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
60	199.26	902.2	892.8	902.2	903.5	903.7	903.4	903.0
45	95.85	597.1	592.2	597.1	597.7	598.0	597.8	597.4
30	42.43	428.3	426.0	428.4	428.5	428.8	428.7	428.3
15	17.04	346.0	345.0	346.1	346.0	346.3	346.3	345.9
0	6.10	314.8	314.3	314.9	314.6	315.0	315.0	314.6
-15	0.00	300.8	300.8	300.9	300.6	301.0	300.9	300.6
-30	0.00	319.3	319.3	319.5	319.1	319.5	319.5	319.1

Table 1: Radio wave refractivity N_r (in parts per million) computed with the **M**illimetre-Wave **P**ropagation **M**odel and a number of simple formulae at selected temperatures, 1000 hPa total pressure, 300 ppm CO_2 content and 100% relative humidity.

A total pressure (p_{tot}) of 1000 hPa exactly was used for the comparison. To be consistent with the historical equations, the Eqs. (6) and (9) for a CO2 content of 0.03% (300 ppm) were used (rather than Eqs. (7) and (10) for 375 ppm carbon dioxide). The relative humidity was set at 100% for temperatures between 0°C and 60°C. The saturation water vapour pressures used for the computations of Columns 4 to 9 are shown in Column 2 and were taken from

Rüeger (1996, App. B). MPM uses relative humidity as input and converts it to partial water vapour pressure using the equations of Goff & Gratch (1946).

The precision of the new 'best available' formula (Eqs. 6 and 7) was predicted using the propagation law of variances and the given precisions of the constants K_1, K_2 and K_3. Column 10 in Table 2 gives the precision without consideration of the correlation between the constants K_2 and K_3 (correlation coefficient $\rho = 0.0$). Column 11 in Table 2 uses a correlation coefficient of -0.995 between the two constants to compute the covariance between them. This correlation coefficient was obtained from a repetition of Boudouris' curve fit.

T [°C]	PWVP p_w [hPa]	Prec Eq.6 ρ=0.0 [ppm]	Prec Eq.6 ρ=-0.995 [ppm]	(4-3) E&F51 minus MPM93 [ppm]	(5-3) Lie77 minus MPM93 [ppm]	(6-3) Bou63 minus MPM93 [ppm]	(7-3) Eq.6 minus MPM93 [ppm]	(8-3) Eq.9 minus MPM93 [ppm]	(9-3) CCIR'86 minus MPM93 [ppm]
(1)	(2)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
60	199.26	±8.3	±1.2	-9.4	0.0	1.4	1.6	1.3	0.9
45	95.85	±4.2	±0.5	-4.8	0.0	0.6	0.9	0.7	0.3
30	42.43	±2.0	±0.2	-2.3	0.1	0.2	0.5	0.4	0.0
15	17.04	±0.9	±0.1	-1.0	0.1	0.0	0.3	0.3	-0.1
0	6.10	±0.3	±0.0	0.0	0.1	-0.1	0.2	0.2	-0.2
-15	0.00	±0.0	±0.0	0.0	0.1	-0.2	0.2	0.2	-0.2
-30	0.00	±0.0	±0.0	0.0	0.1	-0.2	0.2	0.2	-0.2

Table 2: Comparison of the non-dispersive part of the radio wave refractivity N_r (in parts per million) from the Millimetre-Wave Propagation Model and a number of simple formulae at selected temperatures, 1000 hPa total pressure, 300 ppm CO_2 content and 100% relative humidity.

Column 12 of Table 2 shows clearly that the Essen & Froome equation (1951) differs significantly from the other three models at high temperature and humidity. Not unexpectedly, the formula ('Liebe77') based on the K_1, K_2 and K_3 terms by Liebe et al. (1977) agrees very well with the MPM (see Column 13, Table 2); the differences do not exceed 0.15 ppm between -30°C and +60°C. Boudouris' formula agrees slightly better with the MPM93 than Eq. (6) derived above (from the 'best available' coefficients). The differences are, however, smaller than 1 ppm at temperatures below 50°C. Eq. (6) gives values that are, on average, 0.3 ppm higher than those of Boudouris. This is expected since Newell & Baird's K_1 constant is slightly larger than that of Boudouris and since Eq. (6) (see Column 7 in Table 1) uses the same K_2 and K_3 constants as Boudouris' equation (Column 6, Table 1).

The new 'best average' Eq. (9) (see Column 16 in Table 2) shows deviations from MPM93 that are similar to those of Boudouris' equation and Eq. (6). At negative temperatures and zero humidity, the Boudouris' formula and the new Eqs. (6) and (9) have offsets from the MPM93 that are of the same magnitude but different sign. The differences in Columns 14 to 17 of Table 2 compare better with the precision values in Column (11), which take account of the (mathematical) correlation between K_2 and K_3, than with those in Column (10), which do not. This confirms that the correlation between K_2 and K_3 should not be ignored when predicting the precision of computed refractivity. Columns 14 to 16 in Table 2 further show that the MPM93 seems to underestimate the radio refractive index at high humidities and high temperatures when compared to the four alternative formulae tabled.

6. CONCLUSIONS

The radio refractive index formula recommended in Resolution No. 1 of the 13th General Assembly of IUGG (Berkley 1963) is now clearly out of date. With new absolute and relative measurements of the mid-infrared to radio wave refractive index of air being available, and with considerable advances having been made with the computation of anomalous refractivity in the mid-infrared to radio wave spectrum, there is a clear need to rescind the 1963 IUGG resolutions and to propose a more appropriate alternative.

Since a number of authors, particularly in the field of geodesy (e.g. Herring 1992, Jarlemark 1994, Mendes 1999), have started using the radio refractive index formula and coefficients of Thayer (1974), it is stressed again that the coefficients K_1, K_2 and K_3 proposed by Thayer (1974) should **not** be used. As explained before, Thayer's K_1 and K_2 have been derived, at least in part, from optical data, which is inappropriate.

No final proposal for a 'best' simple formula for hand calculations is presented. If the MPM is taken as reference, then a formula of the type of Eq. (10), with the constants K_1, K_2 and K_3 determined by Liebe et al. (1977), produces the best agreement. On the other hand, if the newly derived 'best available' and 'best average' formulae (see Eqs. 7 and 10) are taken as reference, then the MPM needs some fine tuning. The relative merits of the K_2 and K_3 values by Boudouris (1963) and Liebe et al. (1977) as well as of the K_1 values by Newell & Baird (1965) and Liebe et al. (1977) need to be established before a final decision on an appropriate course of action can be made.

For frequencies above 1 GHz and, particularly, for frequencies close and beyond the relevant resonance lines (e.g. between 53.59 and 66.30 GHz and at 118.75 GHz for oxygen, at 22.23 GHz, 67.81 GHz and 119.99 GHz for water vapour) anomalous refractivity has to be modelled. Liebe's Millimetre-Wave Propagation Model (MPM) for frequencies below 1000 GHz is, presently, the most accessible computer routine that models anomalous refractivity of oxygen and water vapour. Hill's software routines (IR_N) use a much more complete set of resonance lines (HITRAN), but require some FORTRAN programming by the user.

It is shown in Rüeger (2002) that the most meaningful precision information is quoted in percent of the refractivity N_i or in percent of the refractivity coefficient K_i. Rüeger (2002) used a relative precision of ±0.02% for the dry air refractivity (N_d) and of ±0.2% for the water vapour refractivity (N_w) and considers these values realistic and, possibly, somewhat conservative. Readers requiring an in-depth review of the radio refractive index should refer to Rüeger (2002) which gives much more information than the technical paper at hand. Rüeger (2002) comes bundled with earlier reports on the refractive index of air.

REFERENCES

Barrell, H. 1951. The Dispersion of Air Between 250 nm and 650 nm. *Journal of the Optical Society of America*, **41**(5): 295-299.

Bean, B. R. 1962. The Radio Refractive Index of Air. *Proceedings of the IRE*, **50**(3): 260-273, **50**(4): 1520.

Bean, B. R., Dutton, E.J. 1968. *Radio Meteorology*. Dover Publications Inc., New York, 423 pages.

Bevis, M., Businger, S., Chiswell, S., Herring, T. A., Anthes, R. A., Rocken, C., Ware, R. H. 1994. GPS Meteorology: Mapping Zenith Wet Delays onto Precipitable Water. *Journal of Applied Meteorology*, **33**(3): 379-386.

Birnbaum, G., Chatterjee, S. K. 1952. The Dielectric Constant of Water Vapour in the Microwave Region. *J. App. Phys.*, **23**: 220-223.

Boudouris, G. 1963. On the Index of Refraction of Air, the Absorption and Dispersion of Centimetre Waves by Gases. *J. of Res. of the Nat. Bureau of Standards – D. Radio Propagation*, **67D**(6): 631–684.

CCIR. 1986. *Recommendations and Reports of the CCIR 1986.* Vol. 5: Propagation in Non-Ionised Media. XVIth Plenary Assembly Dubrovnik 1986, International Radio Consultative Committee, International Telecommunication Union (ITU), Geneva.

Deichl, K. 1984. Der Brechungsindex für Licht- und Mikrowellen. *Allgemeine Vermessungsnachrichten (AVN)*, **91**:85-100.

Edge, R. C. A. 1962. Report of IAG Special Study Group No. 19 on Electronic Distance Measurement using Ground Instruments. *In:* Levallois, J. J. (ed.) *Travaux de L'Association Internationale de Géodésie*, Vol. 21, Rapport Généraux et Rapport Techniques établis á l'Occasion de la douzième Assemblée Générale, Helsinki, July 1960, 59-99.

Essen, L., Froome, K. D. 1951. The Refractive Indices and Dielectric Constants of Air and its Principal Constituents at 24 GHz. Proc. of the Physical Society (London), Section B, **64**:862-875.

Goff, J. A., Gratch, S. 1946. Low Pressure Properties of Water from -160 to 212°F. Transactions, American Society of Heating and Ventilating Engineers, **52**:95-122.

Hartmann, G. K. 1993. The Determination of Tropospheric Conditions (especially H_2O) Affecting the Accuracy of Position Measurements. *In:* Geophysical Monograph 73, IUGG Volume 13, *Environmental Effects on Spacecraft Trajectories and Positioning*), Am. Geophys. Union, 73-82.

Hartmann, G. K., Leitinger, R. 1984. Range Errors due to Ionospheric and Tropospheric Effects for Signal Frequencies above 100 MHz. *Bulletin Géodésique*, **58**: 109-136.

Hasegawa, S., Stokesberry, D. P. 1975. Automatic Digital Microwave Hygrometer. *Review of Scientific Instruments*, **46**(7): 867-873.

Herring, T. A. 1992. Modelling Atmospheric Delays in the Analysis of Space Geodetic Data. In: DeMunck, J. C., Spoelstra, T. A. Th. (Eds.), Proceedings of the Symposium on Refraction of Transatmospheric Signals in Geodesy, 19-22 May 1992, The Hague, Publications in Geodesy, Netherlands Geodetic Commission, New Series, No. 36, 157-164.

Hill, R. J. 2000. Personal communication on the release of FORTRAN77 subroutines and supporting data (IR_N) for the calculation of the phase and group refractive indices of air and its gaseous constituents. 10 October 2000.

Hill, R. J. 1996. Refractive Index of Atmospheric Gases. Chapter II.3.1.1 in: Dieminger-Hartmann-Leitinger (eds), *The Upper Atmosphere*, Springer-Verlag, Berlin-Heidelberg-New York, 261-270.

Hill, R. J. 1995. Personal communication.

Hill, R. J., Lawrence, R. S., Priestley, J. T. 1982. Theoretical and Calculational Aspects of the Radio Refractive Index of Water Vapour. *Radio Science*, **17**(5): 1251-1257.

IAG (International Association of Geodesy). 1999. Resolutions, 22nd General Assembly (see http://www.gfu.ku.dk/~iag/resolutions), 19 - 30 July 1999, Birmingham, U.K.

IUGG (International Union of Geodesy and Geophysics). 1960. Resolutions, 12th General Assembly, 26 July - 6 August 1960, Helsinki, Finland. *Bulletin Géodésique*, **58**: 413.

IUGG (International Union of Geodesy and Geophysics). 1963. Resolutions, 13th General Assembly, 19-31 August 1963, Berkeley, California, USA. *Bulletin Géodésique*, **70**: 390.

Jarlemark, O. J. 1994. Studies of Wet Delay Algorithms for Three European Sites Utilising New Attenuation Models. Research Report No. 171, Chalmers Tekniska Högskola, Göteborg, Sweden.

Liebe, H. J. 1996. Models for the Refractive Index of the Neutral Atmosphere at Frequencies Below 1000 GHz. Chapter II.3.1.2 in: Dieminger-Hartmann-Leitinger (eds.), *The Upper Atmosphere – Data Analysis and Interpretation*. Springer-Verlag, Berlin-Heidelberg-New York, 1996, 270-287.

Liebe, H. J. 1989. MPM – An Atmospheric Millimetre Wave Propagation Model. *Int. Journ. Infrared and Millimetre Waves*, **10**(6): 631-650.

Liebe, H. J. 1985. An Updated Model for Millimetre-Wave Propagation in Moist Air. *Radio Science*, **20**(5): 1069-1089.

Liebe, H. J., Hufford, G. A., Cotton, M. G. 1993. Propagation Modelling of Moist Air and Suspended Water/Ice Particles at Frequencies below 1000 GHz. AGARD, 52nd Specialists' Meeting of the Electromag. Wave Propag. Panel, Palma da Mallorca, Spain, 17-21 May 1993, 3-1 to 3-10.

Liebe, H. J., Hufford, G. A., Cotton, M. G. 1992. Progress in Atmospheric Propagation Modelling at Frequencies below 1000 GHz. Proc., 1992 Battlefield Atmospherics Conference, 1-3 December 1992, Fort Bliss, El Paso, Texas, 247-256.

Liebe, H. J., Gimmestad, G. G., Hopponen, J. D. 1977. Atmospheric Oxygen Microwave Spectrum – Experiment versus Theory. *IEEE Trans. on Antennas and Propagation,* **AP-26**(3): 327-335.

Mendes, V. B. 1999. *Modelling the Neutral-Atmosphere Propagation Delay in Radiometric Space Techniques*. PhD dissertation, Department of Geodesy and Geomatics Engineering Technical Report No. 199, University of New Brunswick, Fredericton, New Brunswick, Canada, 353 pages.

Newell, A. C., Baird, R. C. 1965. Absolute Determination of Refractive Indices of Gases at 47.7 GHz. *Journal of Applied Physics*, **36**(12): 3751-3759.

Owens, J. C. 1967. Optical Refractive Index of Air: Dependence on Pressure, Temperature and Composition. *Applied Optics,* **6**(1): 51-59.

Rüeger, J. M. 2002. Refractive Index Formulae for Electronic Distance Measurement with Radio and Millimetre Waves, Unisurv Report S-68, School of Surveying and Spatial Information Systems, University of New South Wales, UNSW SYDNEY NSW 2052, Australia, 1-52.

Rüeger, J. M. 1999. Report of the Ad-Hoc Working Party on Refractive Indices of Light, Infrared and Radio Waves in the Atmosphere of the IAG Special Commission SC3 - Fundamental Constants (SCFC) for the period 1993 - 1999. Presented at the 22nd General Assembly of IUGG, 18-30 July 1999, Birmingham, UK, 25 pages.

Rüeger, J. M. 1996. *Electronic Distance Measurement – An Introduction*. 4th ed. Springer-Verlag, Berlin-Heidelberg-New York, 276 + xix pages.

Smith, E. K., Weintraub, S. 1953. The Constants in the Equation for Atmospheric Refractive Index at Radio Frequencies. *Proceedings of the Institute of Radio Engineers (I.R.E.),* **41**: 1035-1037.

Thayer, G. D. 1974. An Improved Equation for the Radio Refractive Index of Air. *Radio Science,* **9**(10): 803-807.

Zhevakin, S. A., Naumov, A. P. 1967. Refraction of Millimetre and Submillimetre Radio Waves in the lower Atmosphere. *Radio Engineering and Electronic Physics* (translated and published by IEEE), **12**(1): 885-894.

BIOGRAPHICAL NOTES

Jean M. Rüeger graduated from the Swiss Federal Institute of Technology (Zürich) and then worked a few years in private practice and at Swiss Federal Institute of Technology. In 1975, he was appointed to the staff of the University of New South Wales, Sydney, Australia, where he is an Associate Professor in the School of Surveying and Spatial Information Systems. He wrote extensively on the subject of EDM, EDM instrument calibration and legal traceability, EDM heighting, the use and calibration of digital levels as well as a book on EDM. He is the convener of ad-hoc working group on *Refractive Indices of Light, Infrared and Radio Waves in the Atmosphere* of the Special Commission SC3 on Fundamental Constants (SCFC) of the International Association of Geodesy (IAG). He is a member of the FIG Working Groups WG-5.12 (Digital Levels), WG-5.15 (Procedures for Laboratory Calibrations) and WG-5.22 (Height Determination using Terrestrial Methods).