Precise Point Positioning (PPP) with carrier phase

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Precise Point Positioning (PPP)

1. Introduction
2. Orbits and Clocks: Broadcast and Precise
3. Code and carrier measurements and modelling errors
4. Linear observation model for PPP
5. Parameter estimation: Floating Ambiguities
6. Carrier Ambiguity fixing concept: DD and undifferenced
7. Accelerating Filter convergence with accurate ionosphere
The PPP technique allows **centimetre-level** accuracy to be achieved for **static** positioning and **decimetre level, or better**, for **kinematic positioning**, after the best part of one hour.

This high accuracy requires the use of code and **carrier measurements** and an accurate measurement modelling up to centimetre level or better.
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Satellite Orbit and clock accuracies

**Broadcast:**
Few metres of accuracy for broadcast orbits and clocks

**Precise:**
Few centimetres of accuracy for precise orbits and clocks

Precise clocks for GPS satellites can be found on the International GNSS Service (IGS) server [http://igscb.jpl.nasa.gov](http://igscb.jpl.nasa.gov)
# IGS Precise orbit and clock products: RMS accuracy, latency and sampling

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<td></td>
<td></td>
<td></td>
<td>~ TBD (15 min)</td>
</tr>
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</table>

Computation of satellite coordinates from precise products.

Precise orbits for GPS satellites can be found on the International GNSS Service (IGS) server http://igscb.jpl.nasa.gov

Orbits are given by \((x,y,z)\) coordinates with a sampling rate of 15 minutes. The satellite coordinates between epochs can be computed by polynomial interpolation. A 10th-order polynomial is enough for a centimetre level of accuracy with 15 min data.

\[
P_n(x) = \sum_{i=1}^{n} y_i \frac{\prod_{j \neq i} (x - x_j)}{\prod_{j \neq i} (x_i - x_j)}
= y_1 \frac{x - x_2}{x_1 - x_2} \cdots \frac{x - x_n}{x_1 - x_n} + \cdots
+ y_i \frac{x - x_1}{x_i - x_1} \cdots \frac{x - x_{i+1}}{x_i - x_{i+1}} \cdots \frac{x - x_n}{x_i - x_n} + \cdots
+ y_n \frac{x - x_1}{x_n - x_1} \cdots \frac{x - x_{n-1}}{x_n - x_{n-1}}
\]
IGS orbit and clock products (for PPP):
Discrepancy between the different centres
Computation of **satellite clocks** from precise products

Precise clocks for GPS satellites can be found on the International GNSS Service (IGS) server [http://igscb.jpl.nasa.gov](http://igscb.jpl.nasa.gov)

IGS is providing precise **orbits and clock files** with a sampling rate of **15 min**, as well as precise **clock files with a sample rate of 5 min and 30 sec**, in SP3 and CLK formats.

Some centres also provide GPS satellite clocks with a 5 sec sampling rate, like the ones obtained from the Crustal Dynamics Data Information System (CDDIS) site.

Stable clocks with a sampling rate of 30 sec or higher can be interpolated with a first-order polynomial to a few centimetres of accuracy. Clocks with a lower sampling rate should not be interpolated, because clocks evolve as random walk processes.
Session 3.2, Ex7b: Precise 30s clock interpolation error

@ J. Sanz & J.M. Juan
Session 3.2, Ex7a: Precise 300s clock interpolation error

time (s)

metres

Clock
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For high-accuracy positioning, the carrier phase must be used, besides the code pseudorange.

The carrier measurements are very precise, typically at the level of a few millimetres, but contain unknown ambiguities which change every time the receiver locks the signal after a cycle slip.

Nevertheless, such ambiguities can be estimated in the navigation solution, together with the coordinates and other parameters.
PPP Model components

We are going to review first the measurements modelling for the Standard Point Positioning (SPP). A brief summary is given next.

After this summary, we will focus on the additional modelling need for Precise Point Positioning.

Remember that the error component most difficult to model is the ionosphere. But, in the PPP technique the ionosphere error is removed (more than 99.9%) using dual-frequency measurements in the ionosphere-free combination (Lc,Pc). This combination also removes the Differential Code Bias (or TGD).
Satellite coordinates computation at signal emission time

- Coeff. at emission
- Earth rotation
Signal propagation errors on the Atmosphere

- Ionospheric delay: 2 - 30 m
- Tropospheric delay: 2 - 30 m
- Receiver clock offset: < 300 km
- Receiver instrumental delay: ~ m

Model: Ion. corrections [SPP]

Vertical positioning error [SPP]

Horizontal positioning error [SPP]

Troposphere

Ionosphere error is removed using dual-frequency measurements
Satellite clocks and Total Group Delay (TGD)

Satellite clocks and Relativity sat clock

TGDs cancel in the ionosphere-free combination
Additional Modelling for PPP

The PPP technique allows centimetre-level of accuracy to be achieved for static positioning and decimetre level, or better, for kinematic navigation. This high accuracy requires an accurate modelling “up to the centimetre level or better”, where all previous model terms must be taken into account (except ionosphere and TGD [*]), plus some additional terms given next:

[*] Remember that in the PPP technique the ionosphere error is removed (more than 99.9%) using dual-frequency measurements in the ionosphere-free combination (Lc,Pc). This combination also removes the Differential Code Bias (or TGD).

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Broadcast orbits are referred to the antenna phase center, but IGS precise orbits are referred to the satellite mass center.
GNSS measurements are referred to the APC. This is not necessarily the geometric center of the antenna, and it depends on the signal frequency and the incoming radio signal direction. For geodetic positioning a reference tied to the antenna (ARP) or to monument is used.

**Receiver APC:**

The antenna used for this experiment, has the APC position vertically shifted regarding ARP. Thence, neglecting this correction, an error on the vertical component occurs, but not in the horizontal one.
Additional Modelling for PPP: APC

**Antenna biases and orientation:**

- The satellite and receiver **antenna phase centres (APCs)** can be found in the IGS ANTEX files, after GPS week 1400 (Nov. 2006).
- The **carrier phase wind-up** effect due to the satellite's motion must be taken into account.
Wind-up affects only carrier phase. It is due to the electromagnetic nature of circularly polarized waves of GNSS signals. As the satellite moves along its orbital path, it performs a rotation to keep its solar panels pointing to the Sun direction. This rotation causes a carrier variation, and thence, a range measurement variation.

Wind-Up

Wind-up changes smoothly along continuous carrier phase arcs. In the position domain, wind-up affects both vertical and horizontal components.
Additional Modelling for PPP: **Eclipse condition**

High-accuracy GNSS positioning degrades during the GNSS satellites' eclipse seasons.

- Once the satellite goes into shadow, the radiation pressure vanishes. This effect introduces errors in the satellite dynamics due to the difficulty of properly modelling the solar radiation pressure.
- On the other hand, the satellite's solar sensors lose sight of the Sun and the attitude control (trying to keep the panels facing the Sun).

As a consequence, the orbit during shadow and eclipse periods may be considerably degraded and the removal of satellites under such conditions can improve the high-accuracy positioning results.
Additional Modelling for PPP: **Atmospheric Effects**

- **The ionospheric refraction and TGDs are removed** using the ionosphere-free combination of code and carrier measurements (PC, LC).

- **The tropospheric refraction can be modelled by Dry and Wet components** (and different mappings are usually used for both components, e.g. mapping of Niell).

- A residual tropospheric delay is estimated (as wet ZTD delay) in the Kalman filter, together with the coordinates, clock and carrier phase ambiguities.

The troposphere is estimated as a Random Walk process in the Kalman Filter. A process noise of 1cm/\sqrt{h} has been taken.
Earth deformation effects:

- **Solid tides** must be modelled by equations
- **Ocean loading and pole tides** are second-order effects and can be neglected for PPP accuracies at the centimetre level

**Solid Tides** concern the movement of Earth's crust (and thus the variation in the receiver's location coordinates) due to gravitational attractive forces produced by external bodies, mainly the Sun and Moon. Solid tides produce vertical and horizontal displacements that can be expressed by the spherical harmonics expansion \((m, n)\), characterised by the Love and Shida numbers \(h_{mn}\) and \(l_{mn}\).
Solid Tides
It comprises the Earth’s crust movement (and thence receiver coordinates variations) due to the gravitational attraction forces produced by external bodies, mainly the Sun and the Moon.

These effects do not affect the GNSS signals, but if they were not considered, the station coordinates would oscillate with relation to a mean value. They produce vertical (mainly) and horizontal displacements.

"Solids Tides:
Sun
Moon"
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Linear observation model for PPP

It is based code and carrier measurements in the ionosphere-free combination (Pc, Lc), which are modelled as follows:

\[
P_{C_{\text{rec}}}^{\text{sat}} = \rho_{\text{rec}}^{\text{sat}} + c \cdot (dt_{\text{rec}} - dt_{\text{sat}}) + Trop_{\text{rec}}^{\text{sat}} + M_{\text{Pc}} + \varepsilon_{\text{Pc}}
\]

\[
L_{C_{\text{rec}}}^{\text{sat}} = \rho_{\text{rec}}^{\text{sat}} + c \cdot (dt_{\text{rec}} - dt_{\text{sat}}) + Trop_{\text{rec}}^{\text{sat}} + \lambda_{N} \omega_{\text{rec}}^{\text{sat}} + B_{C_{\text{rec}}}^{\text{sat}} + m_{\text{LC}} + \varepsilon_{\text{LC}}
\]

where

\[
P_{C} = \frac{f_{1}^{2} P_{1} - f_{2}^{2} P_{2}}{f_{1}^{2} - f_{2}^{2}}; \quad L_{C} = \frac{f_{1}^{2} L_{1} - f_{2}^{2} L_{2}}{f_{1}^{2} - f_{2}^{2}}
\]

Remark, \(\rho\) is referred to the Antenna Phase Centres (APC) of satellite and receiver antennas in the ionosphere free combination.

\[
B_{C} = \lambda_{N} \left( B_{1} + \frac{\lambda_{W}}{\lambda_{2}} B_{W} \right)
\]

\[
B_{W} = B_{1} - B_{2}
\]

\[
\lambda_{N} = 10.7 \text{ cm, } \lambda_{W} = 86.2 \text{ cm}
\]
**Linear model:** For each satellite in view

\[
P_{C_{sat}}^{rec} = \rho_{rec}^{sat} + c \cdot (dt_{rec} - dt^{sat}) + Trop + \varepsilon
\]

Linearising \( \rho \) around an ‘a priori’ receiver position \((x_{rec,0}, y_{rec,0}, z_{rec,0})\)

\[
= \rho_{rec,o}^{sat} + \frac{x_{rec,o} - x^{sat}}{\rho_{rec,o}^{sat}} \Delta x_{rec} + \frac{y_{rec,o} - y^{sat}}{\rho_{rec,o}^{sat}} \Delta y_{rec} + \frac{z_{rec,o} - z^{sat}}{\rho_{rec,o}^{sat}} \Delta z_{rec} + c \left(dt_{rec} - dt^{sat}\right) + Trop
\]

where:

\[
\Delta x_{rec} = x_{rec} - x_{rec,o} \quad ; \quad \Delta y_{rec} = y_{rec} - y_{rec,o} \quad ; \quad \Delta z_{rec} = z_{rec} - z_{rec,o}
\]

and taking:

\[
Trop_{rec}^{sat} = Trop_{0_{rec}}^{sat} + M_{wet,rec}^{sat} \Delta T_{Z,wet}
\]

**Prefit-residuals (Prefit)**

\[
P_{C_{sat}}^{rec} - \rho_{rec,o}^{sat} + c dt^{sat} - Trop_{0_{rec}}^{sat} = \Delta x_{rec} + \frac{x_{rec,o} - x^{sat}}{\rho_{rec,o}^{sat}} \Delta x_{rec} + \Delta y_{rec} + \frac{y_{rec,o} - y^{sat}}{\rho_{rec,o}^{sat}} \Delta y_{rec} + \Delta z_{rec} + \frac{z_{rec,o} - z^{sat}}{\rho_{rec,o}^{sat}} \Delta z_{rec} + c dt_{rec} + M_{wet}^{sat} \Delta T_{Z,wet}
\]

The same for carrier, but adding the ambiguity as an unknown
Remark

\[ P_{C_{rec}}^{sat} = \rho_{rec}^{sat} + c \cdot (dt_{rec} - dt_{sat}) + Trop + \varepsilon \]

Linearising \( \rho \) around an ‘a priori’ receiver position \((x_{rec,0}, y_{rec,0}, z_{rec,0})\)

\[
\begin{align*}
\rho_{rec,o}^{sat} + \frac{x_{rec,o}^{sat} - x_{sat}^{sat}}{\rho_{rec,o}^{sat}} \Delta x_{rec} + \frac{y_{rec,o}^{sat} - y_{sat}^{sat}}{\rho_{rec,o}^{sat}} \Delta y_{rec} + \frac{z_{rec,o}^{sat} - z_{sat}^{sat}}{\rho_{rec,o}^{sat}} \Delta z_{rec} + c (dt_{rec} - dt_{sat}) + Trop
\end{align*}
\]

Of course, receiver coordinates \((x_{rec}, y_{rec}, z_{rec})\) are not known (they are the target of this problem). But, we can always assume that an “approximate position \((x_{0,rec}, y_{0,rec}, z_{0,rec})\) is known”.

Thence, the navigation problem will consist on:

1.- To start from an approximate value for receiver position \((x_{0,rec}, y_{0,rec}, z_{0,rec})\) e.g. the Earth’s centre) to linearise the equations.

2.- With the pseudorange measurements and the navigation equations, compute the correction \((\Delta x_{rec}, \Delta y_{rec}, \Delta z_{rec})\) to have improved estimates:

\[
(x_{rec}, y_{rec}, z_{rec}) = (x_{0,rec}, y_{0,rec}, z_{0,rec}) + (\Delta x_{rec}, \Delta y_{rec}, \Delta z_{rec})
\]

3.- Linearise the equations again, about the new improved estimates, and iterate until the change in the solution estimates is small enough.
**Linear model:** For each satellite in view

\[ P_{C_{sat}}^{rec} = \rho_{rec}^{sat} + c \cdot (dt_{rec} - dt_{sat}) + Trop + \varepsilon \]

Linearising \( \rho \) around an ‘a priori’ receiver position \((x_{rec,0}, y_{rec,0}, z_{rec,0})\)

\[ \rho_{rec,0}^{sat} + \frac{x_{rec,o} - x^{sat}}{\rho_{rec,o}^{sat}} \Delta x_{rec} + \frac{y_{rec,o} - y^{sat}}{\rho_{rec,o}^{sat}} \Delta y_{rec} + \frac{z_{rec,o} - z^{sat}}{\rho_{rec,o}^{sat}} \Delta z_{rec} + c(dt_{rec} - dt_{sat}) + Trop \]

where:

\[ \Delta x_{rec} = x_{rec} - x_{rec,o} \; ; \; \Delta y_{rec} = y_{rec} - y_{rec,o} \; ; \; \Delta z_{rec} = z_{rec} - z_{rec,o} \]

and taking:

\[ Trop_{rec}^{sat} = Trop_{0_{rec}}^{sat} + M_{wet,rec}^{sat} \Delta Tr_{Z,wet} \]

**Prefit-residuals (Prefit)**

\[ P_{C_{sat}}^{rec} - \rho_{rec,0}^{sat} + c dt_{sat} - Trop_{0_{rec}}^{sat} = \]

measurement

computed

unknown

The same for carrier, but adding the ambiguity as an unknown
**Linear model:** For each satellite in view

\[
P_{c}^{sat}_{rec} = \rho_{rec}^{sat} + c \cdot (dt_{rec} - dt^{sat}_{rec}) + Trop + \epsilon
\]

Linearising \( \rho \) around an ‘a priori’ receiver position \((x_{rec,0}, y_{rec,0}, z_{rec,0})\)

\[
= \rho_{rec,0}^{sat} + \frac{x_{rec,0} - x^{sat}}{\rho_{rec,0}^{sat}} \Delta x_{rec} + \frac{y_{rec,0} - y^{sat}}{\rho_{rec,0}^{sat}} \Delta y_{rec} + \frac{z_{rec,0} - z^{sat}}{\rho_{rec,0}^{sat}} \Delta z_{rec} + c \left( dt_{rec} - dt^{sat}_{rec} \right) + Trop
\]

where:

\[
\Delta x_{rec} = x_{rec} - x_{rec,0} \quad ; \quad \Delta y_{rec} = y_{rec} - y_{rec,0} \quad ; \quad \Delta z_{rec} = z_{rec} - z_{rec,0}
\]

and taking:

\[
Trop^{sat}_{rec} = Trop_{0rec}^{sat} + M_{wet,rec}^{sat} \Delta T_{r,\text{wet}}
\]

\[
P_{c}^{sat}_{rec} - \rho_{rec,0}^{sat} + c dt^{sat}_{rec} - Trop_{0rec}^{sat} = \frac{x_{rec,0} - x^{sat}}{\rho_{rec,0}^{sat}} \Delta x_{rec} + \frac{y_{rec,0} - y^{sat}}{\rho_{rec,0}^{sat}} \Delta y_{rec} + \frac{z_{rec,0} - z^{sat}}{\rho_{rec,0}^{sat}} \Delta z_{rec} + c dt_{rec} + M_{wet}^{sat} \Delta T_{r,\text{wet}}
\]

\[
L_{c}^{sat}_{rec} - \rho_{rec,0}^{sat} + c dt^{sat}_{rec} - Trop_{0rec}^{sat} - \lambda_{N} \omega^{sat}_{rec} = \frac{x_{rec,0} - x^{sat}}{\rho_{rec,0}^{sat}} \Delta x_{rec} + \frac{y_{rec,0} - y^{sat}}{\rho_{rec,0}^{sat}} \Delta y_{rec} + \frac{z_{rec,0} - z^{sat}}{\rho_{rec,0}^{sat}} \Delta z_{rec} + c dt_{rec} + M_{wet}^{sat} \Delta T_{r,\text{wet}} + B_{C}^{sat}
\]

**unknowns**
Following the same procedure as with the code based positioning (SPP), the linear observation model \( y = G \mathbf{x} \) for the code and carrier measurements can be written as follows:

\[
\begin{pmatrix}
\begin{array}{c}
\Delta x_{\text{rec}} \\
\Delta y_{\text{rec}} \\
\Delta z_{\text{rec}} \\
\Delta T_{rZ,\text{wet}} \\
\mathbf{B}_C^1 \\
\mathbf{B}_C^n
\end{array}
\end{pmatrix} =
\begin{pmatrix}
\mathbf{P}(P_c)^k - \rho_0^k + cdt^k - Trop_0^k \\
\mathbf{P}(L_c)^k - \rho_0^k + cdt^k - Trop_0^k - \lambda_N \omega^k
\end{pmatrix}
\]
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Parameter estimation PPP: Floating Ambiguities

The linear observation model $\mathbf{y} = \mathbf{G} \mathbf{x}$ can be solved using the Kalman filter. The next stochastic model can be used

- **Carrier phase ambiguities** ($B_C$) are taken as ‘constant’ along continuous phase arcs, and as ‘white noise’ when a cycle slip happens ($\sigma = 10^4 \text{ m}$ can be taken, for instance) → **FLOATED AMB.**

- **Wet tropospheric delay** ($Tr_{z;wet}$) is taken as a random walk process (with $d\sigma^2/dt = 1 \text{ cm}^2/\text{h}$, for instance).

- **Receiver clock** ($cdt$) is taken as a white-noise process
  
  (with $\sigma = 3.10^5 \text{ m}$, i.e. 1 ms for instance).

- **Receiver coordinates** ($\Delta x, \Delta y, \Delta z$)
  
  - For **static positioning** the coordinates are taken as constants.
  - For **kinematic positioning** the coordinates are taken as white noise or a random walk process.
This solution procedure is called **floating ambiguities**. ‘Floating’ in the sense that the ambiguities are estimated by the filter ‘as real numbers’.

The bias estimations $\hat{B}_C$ will converge to a solution after a transition time that depends on the observation geometry, model quality and data noise.

In general, one must expect errors at the decimetre level, or better, in pure kinematic positioning (after the best part of one hour) and at the centimetre level in static PPP over 24h data.

See exercises in the laboratory session.
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Carrier Ambiguity Fixing concept

In the previous formulation, the ambiguities have been estimated as real numbers in the Kalman filter, without exploiting its integer nature.

That is, the ambiguities in $L_1$, $L_2$ or $L_W$ signals are an integer number ($N$) of its associated wavelength ($\lambda$) plus a fractional part associated to the satellite and to the receiver.

$$B_{1,\text{rec}}^{\text{sat}} = \lambda_1 N_{1,\text{rec}}^{\text{sat}} + b_{1,\text{rec}} + b_1^{\text{sat}}$$

$$B_{2,\text{rec}}^{\text{sat}} = \lambda_2 N_{2,\text{rec}}^{\text{sat}} + b_{2,\text{rec}} + b_2^{\text{sat}}$$

$$N_W = N_1 - N_2$$

$$B_C = \lambda_N \left( N_1 + \frac{\lambda_W}{\lambda_2} N_W \right) + b_{C,\text{rec}} + b_C^{\text{sat}}$$

$B_C$ is not an integer number of $\lambda_N$ but can be related with the integers $N_1$, $N_W$.

The Ambiguity Fixing techniques take benefit of this INTEGER NATURE of $N_1$, $N_2$ ambiguities to properly fix them, reducing convergence time, and thence, achieving high accuracy quickly.
Two different approaches can be considered:

• **Double differenced Ambiguity Fixing (e.g. RTK):**

  This is the classical approach which relies in the fact that the fractional part of carrier ambiguities cancels when forming the double differences between receivers and satellites:

  ➔ That is, given:
  
  \[ B_{rec}^{sat} = \lambda N_{rec}^{sat} + b_{rec} + b_{sat} \]

  The double differences, regarding a reference receiver and satellite, yield:

  \[ \Delta \nabla B_{rec}^{sat} = B_{rec}^{sat} - B_{rec,R}^{sat} - \left( B_{rec}^{sat,R} - B_{rec,R}^{sat} \right) = \lambda \Delta \nabla N_{rec}^{sat} \]

  where the satellite and receiver ambiguity terms \((b_{rec}, b_{sat})\) cancel out.

• **Absolute (or undifferenced) ambiguity fixing (e.g. PPP AR):**
Example of Ambiguity Fixing by rounding
The LAMBDA METHOD

LAMBDA software package
Matlab implementation, Version 3.0

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Mathematical Geodesy and Positioning, Delft University of Technology

Curtin University
Two different approaches can be considered:

- **Double differenced Ambiguity Fixing (e.g. RTK):**

  This is the classical approach which relies in the fact that the fractional part of carrier ambiguities cancels when forming the double differences between receivers and satellites:

  That is, given:
  \[ B_{sat}^{rec} = \lambda N_{sat}^{rec} + b_{rec} + b_{sat} \]

  The double differences, regarding a reference receiver and satellite, yield:
  \[ \Delta \nabla B_{rec}^{sat} = B_{rec}^{sat} - B_{rec,R}^{sat} - \left( B_{rec}^{sat,R} - B_{rec,R}^{sat} \right) = \lambda \Delta \nabla N_{rec}^{sat} \]

- **Absolute (or undifferenced) ambiguity fixing (e.g. PPP AR):**

  This is a new approach, where the fractional part of the ambiguities are estimated from a global network of permanent stations and provided to the users. Thus, the user can remove this fractional part and fix the remaining ambiguity as an integer number.

  \[ B_{rec}^{sat} = \lambda N_{rec}^{sat} + b_{rec} + b_{sat} \]
Absolute (or undifferenced) ambiguity fixing

Nevertheless, these carrier hardware biases (or fractional part of carrier ambiguities) can be estimated together with the orbits, clocks, ionosphere and DCBs from measurements of a worldwide reference stations network.

![Graph showing carrier hardware biases over time](image)

These carrier hardware biases are slow varying parameters and can be broadcast to the user, together with the other differential corrections (orbits, clocks, ionosphere...) from a reference station network.

Then, the user can remove them and estimate the remaining ambiguities as integer numbers.

This can allow world-wide PPP users to perform ambiguity fixing, without baseline length limitation, improving accuracy and reducing convergence time.

**Note:** These carrier hardware biases are canceled in RTK when forming Double-Differences of measurements between pairs of satellites and receivers.
The main weakness of PPP is the large convergence time, which depend on satellites geometry, quality of data (code noise, cycle-slips...).
There is a sudden improvement when fixing carrier ambiguities, but still several tens of minutes are needed to achieve centimeter level of accuracy.

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Fast-PPP: Accelerating convergence with IONO

- During the early 2000s gAGE/UPC developed a two layers grid model, and afterwards assessed its suitability to help the PPP convergence.

This last study was done in the context of ESA project “Fast-PPP” and the technique is protected by an ESA patent since 2011. (PCT/EP2011/001512)

More recently, the accurate ionospheric model has been extended to worldwide (Rovira, et al 2014)

**MLVL** (252km), **EUSK** (170km) and **EIJS** (95km) (km from the nearest reference receiver)
Actual GNSS Data the entire Year 2014

Global Corrections Network

15 Rovers

World wide Iono. model

Rovers are located at distances are orders of magnitude greater than RTK baselines

Close to maximum Solar Cycle conditions

Moderate Geomagnetic Dst Values

36 Fast Filter: Satellite Clocks

150 Iono Filter: World-Wide Model, DCBs

172 Slow Filter: Orbits, Fract Ambigs

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• **How the ionosphere helps the filter convergence in Fast-PPP?**

  ➔ In the classical PPP, the navigation filter convergence is driven by the convergence of the **floated iono-free ambiguity** \((B_C)\), which, at the beginning, depends on the code noise.

  1) There is a constraint between the **wide-lane ambiguity** \((B_w)\), the **iono-free ambiguity** \((B_C)\) and the **ionospheric refraction** \((STEC)\) and \((DCB)\).

     \[
     B_C = B_W - 1.98 (L_1 - L_2 - STEC - DCB)
     \]

  2) The **wide-lane ambiguity** \((B_w)\) is estimated/fixed quickly (~5 minutes with 2-freq and in single epoch with 3-freq. signals).

  3) The **ionosphere** \((STEC)\) is the bridge (through the mentioned constraint) to transfer the quick accuracy achieved in the **wide-lane ambiguity** \((B_w)\) computed value to the **iono-free ambiguity** \((B_C)\) estimation, accelerating in this way the filter convergence.

  ➔ Thence, in Fast-PPP the convergence time is strongly reduced thanks to the quick wide-lane ambiguity fixing and the accurate ionospheric corrections. It allows to achieve High Accuracy quickly (~5 minutes with 2-freq & single epoch with 3-freq).

The ionosphere will help provided that its quality (noise/error) is better than the code noise.

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This is a conceptual approach!
300 seconds smoothing
References


Thank you
GNSS Data Processing, Vol. 2: Laboratory exercises.
Backup slides
Fast-PPP Ionospheric Model

1.- Derived from the geometry-free combination of carrier-phases:

\[ LI = L1 - L2 \]

2.- Ambiguities are fixed (thanks to the CPF accurate geodetic modelling):

Unambiguous Carrier-phase noise (mm)

\[ LI_i - BI_i = STEC_j + DCB_i - DCB_j \]

3.- The Slant Total Electron Content (STEC) is estimated with Vertical STEC (VTEC) on each “k” Ionospheric Grid Point (IGP) at the two-layers:

\[ STEC_i = \sum \alpha_k \cdot VTEC_k \]

Plasmasphere: 1,600 km
Ionosphere: 270 km

<table>
<thead>
<tr>
<th>layer</th>
<th>IGPs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>7184</td>
</tr>
<tr>
<td>2nd</td>
<td>1800</td>
</tr>
</tbody>
</table>

Orbit errors (4cm) + sat Clocks (6cm)
Iono Accuracy \( \leq 1 \) TECU (16 cm in L1)
2-Layer Ionospheric Maps during Storm of DoY 058-2014

**2nd Layer**
- Plasmasphere 1600 km
- 2h UT
- 22h UT (storm)

**1st Layer**
- Ionosphere 270 km
- 2h UT
- 22h UT (storm)

Total Electron Content Units (TECU)
Agreement Electron Contents of dual-layers between GIM derived from “earth” data and “space-borne” data

Independent data


1 day of gAGE dual-layer GIM.

TEC ratio:

Neglecting this effect, introduce a miss-modelling degrading navigation.

- **Code** measurements are unambiguous but noisy (meter level noise).
- **Carrier** measurements are precise (few millimetres of noise) but ambiguous (the unknown ambiguities can reach thousands of km).
- **Carrier phase ambiguities are estimated in the navigation filter** along with the other parameters (coordinates, clock offsets, etc.). If these ambiguities were fixed, measurements accurate to the level of few millimetres would be available for positioning. However, some time is needed to decorrelate such ambiguities from the other parameters in the filter, and the estimated values are not fully unbiased.
Convergence: DoYs 169-200 of 2014
6 Positioning modes per rover, resets every 2h:
  a) Reference Iono-Free sol
  b) Iono Sol: IONEX (IGS-GIM):
  c) Iono Sol. Fast-PPP IONO:

Rover EIJS (5°E, 50°N): Mid Latitude, 76 km

Rover LKHU (95°W, 29°S): Low latitude, 455 km

Benefits of applying external ionospheric corrections are expected, provided its quality is superior to that of code noise.

Results from (Rovira et al., 2014)