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VOLUME CALCULATION THROUGH USING DIGITAL ELEVATION MODELS CREATED BY DIFFERENT INTERPOLATION METHODS

Nazan YILMAZ
Karadeniz Technical University, Department of Geomatics Engineering,
Trabzon, Turkey

SUMMARY

The digital elevation model is a model which defines the surface of land three-dimensional and has been created through the elevation data of the land. The digital elevation model has been widely used in the fields of application such as, preparation of road projects, excavation-filling-related volume calculations, land arrangement studies, etc. The volume calculations which is the subject of this study and have been used in a variety of engineering services, have often been used in the reserve determination of mine sites, in the determination of splitting and filling soil removal works of the sites such as, road, airport, tunnels etc. Since the amount of the calculated volume burdened financially great expenses to employer, the calculations must be made in a precise manner. The aim of this study is to make and compare the volume calculations with different grid ranges and different interpolation methods. In this study, grid ranges were selected as 50 m, 100 m, 150 m and 200 m. The interpolation methods used are Inverse Distance to a Power (k=1 and k=2), Point Kriging, Minimum Curvature, Modified Shepard's Method, Natural Neighbor, Nearest Neighbor, Polynomial Regression (simple planar surface), Multiquadratic Radial Basis Function, Triangulation with Linear Interpolation. The volume calculation methods used are Trapezoidal rule, Simpson's rule, Simpson's 3/8 rule. The digital elevation models were prepared in the "Surfer 8" program. The surface modelling of the land is made through the chosen different interpolation methods and the grid extended files of these resulting surfaces were created. Afterwards, the volumes of these surfaces with reference to the selected reference surface, Z = 0, were determined with different methods and were compared.

VOLUME CALCULATIONS

In surfer, three methods are used to determine volumes: Trapezoidal Rule, Simpson's Rule, and Simpson's 3/8 Rule. Mathematically, the volume under a function $f(x, y)$ is defined by a double integral

$$V = \int_{X_{\min}}^{X_{\max}} \int_{Y_{\min}}^{Y_{\max}} f(x, y) dx dy \qquad (1)$$

In Surfer, this is computed by first integrating over X (the columns) to get the areas under the individual rows, and then integrating over Y (the rows) to get the final volume.
Surfer approximates the necessary one-dimensional integrals using three classical numerical integration algorithms: extended trapezoidal rule, extended Simpson's rule, and extended Simpson's 3/8 rule. In the following formula, Δx represents the grid column spacing, Δy represents the grid row spacing and H_{ij} represents the grid node value in row i and column j.
Extended Trapezoidal Rule;

$$A_i = \frac{\Delta x}{2}\left[H_{i,1} + 2H_{i,2} + 2H_{i,3} + \ldots + 2H_{i,ncol-1} + H_{i,ncol}\right] \qquad (2)$$

$$V \cong \frac{\Delta y}{2}\left[A_1 + 2A_2 + 2A_3 + \ldots + 2A_{ncol-1} + A_{ncol}\right] \qquad (3)$$

The pattern of the coefficients is {1,2,2,2,...,2,2,1}.
Extended Simpson's Rule;

$$A_i = \frac{\Delta x}{3}\left[H_{i,1} + 4H_{i,2} + 2H_{i,3} + 4H_{i,4} + \ldots + 2H_{i,ncol-1} + H_{i,ncol}\right] \qquad (4)$$

$$V \cong \frac{\Delta y}{3}\left[A_1 + 4A_2 + 2A_3 + 4A_3 + \ldots + 2A_{ncol-1} + A_{ncol}\right] \qquad (5)$$

The pattern of the coefficients is {1,4,2,4,2,4,2,...,4,2,1}.

Extended Simpson's 3/8 Rule:

$$A_i = \frac{3\Delta x}{8}\left[H_{i,1} + 3H_{i,2} + 3H_{i,3} + 2H_{i,4} + \ldots + 2H_{i,ncol-1} + H_{i,ncol}\right] \tag{6}$$

$$V \cong \frac{3\Delta y}{8}\left[A_1 + 3A_2 + 3A_3 + 2A_3 + \ldots + 2A_{ncol-1} + A_{ncol}\right] \tag{7}$$

The pattern of the coefficients is {1,3,3,2,3,3,2,...,3,3,2,1}

MATERIALS AND METHODS

Study area

In practice, geodetic network with 1175 points, which was established in Konya, was used. x,y coordinates of these points were measured by GPS and the orthometric heights of which, were measured by levelling method. Minimum and maximum orthometric heights of these reference points were 1034.541 m and 1671.294 m, moreover, the topographical structure of the land was indicated in the form of three dimensional surfaces in Figure 1 and in the form of wireframe in Figure 2. The range in x and y directions of Gauss-Krüger projection system coordinates of these reference points were determined as $\Delta x = 486.562$ m and $\Delta y = 487.116$ m.

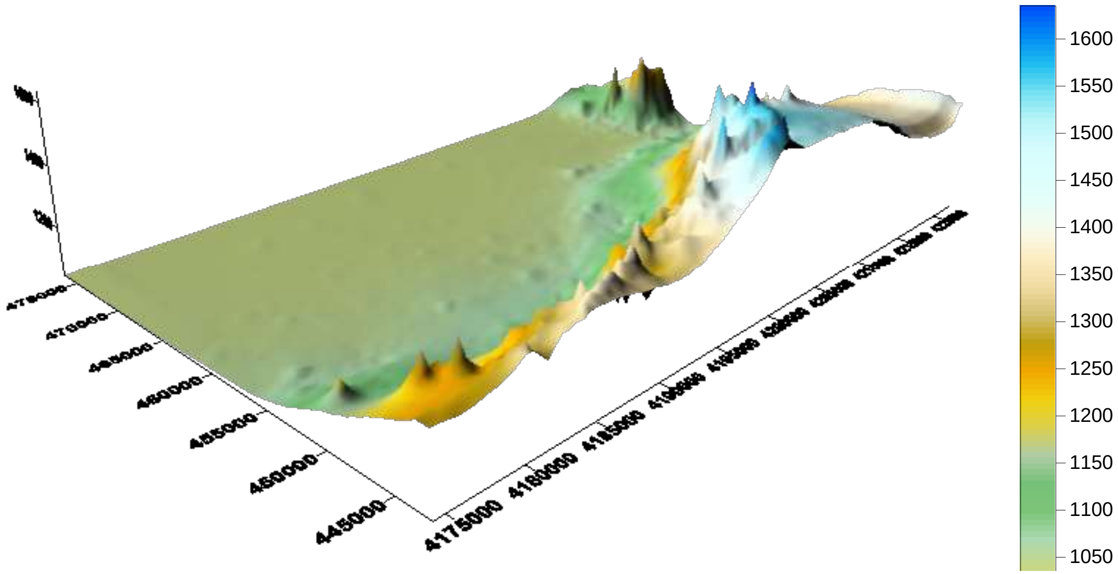

Fig. 1. Three-dimensional surface view of the study area

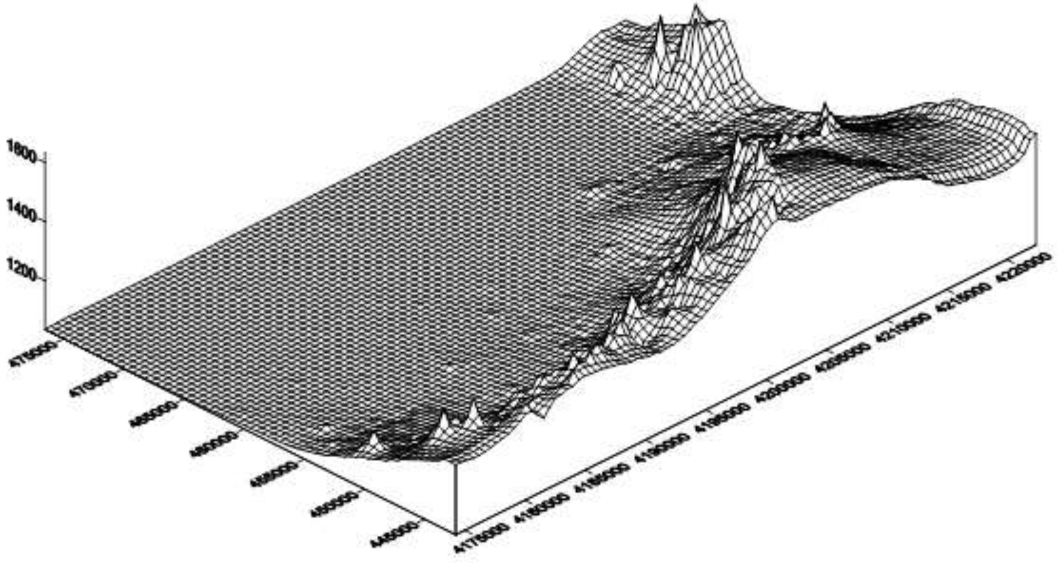

Fig. 2. Three-dimensional view of the study area by the wireframe method

Volume Calculations

The real volume of the land was calculated through NETCAD5.0 software package program. Following the triangulation process covering the land had been performed through AP program, regarding the volume calculation made by using Delaunay triangles, since the volume was calculated directly by using the coordinates of the reference points without applying interpolation, the result was accepted as the value of the actual volume. Real volume was determined as 1436639831721.4 m^3.

Then, without making any change in the standard settings of the Surfer program;

- The grid ranges were selected as 50 m, 100 m, 150 m and 200 m.
- Interpolation methods: The surface modeling was carried out within the limits we specified using the interpolation methods including Inverse Distance to a Power (k=1 and k=2), Point Kriging, Minimum Curvature, Modified Shepard's Method, Natural Neighbor, Nearest Neighbor, Polynomial Regression (simple planar surface), Multiquadratic Radial Basis Function, Triangulation with Linear Interpolation

and *.grid extended files of these resulting surfaces were created. The volume values of these surfaces, which were resulted by making surface modeling, with respect to specific reference surface, Z=0, were calculated according to the rules given below :

Trapeze (Terminal areas method) Rule, Simpson's Rule, Simpson's 3/8 Rule

The results of the volumes calculated with different volume calculation methods depending on the different interpolation methods and grid ranges applied are given in Table 1, and the differences of these results from the actual volume and the calculated relative errors wıth respect to these differences are given in Table 2.

Table 1. The results of the volumes calculated with different volume calculation methods depending on the different interpolation methods and grid ranges

Interpolation methods	Grid ranges (m)	Volume calculation methods		
		Trapezoidal rule (m^3)	Simpson's rule (m^3)	Simpson 's 3/8 rule (m^3)
Inverse Distance to a Power (k=1)	50	1873063892573.60	1873053304065.40	1873064832869.50
	100	1873059712549.10	1873031819106.40	1873063854823.70
	150	1873047910977.70	1873044834373.50	1873038277428.80
	200	1873118855689.20	1873077533053.40	1873075626208.90
Inverse Distance to a Power (k=2)	50	1874988732022.50	1874983517357.80	1874989438476.00
	100	1874983336793.00	1874968392886.10	1874983630882.30
	150	1874979095000.20	1874975664864.80	1874975645025.50
	200	1875021088515.50	1875008182482.70	1875012230840.70
Kriging	50	1890669126848.40	1890664648539.00	1890669499593.70
	100	1890660938849.80	1890665785794.20	1890658667976.40
	150	1890663981903.90	1890651355131.90	1890660994133.80
	200	1890656510689.50	1890650019559.00	1890637786303.80
Minimum Curvature	50	1887255486394.60	1887252201199.40	1887252202665.40
	100	1883970464527.90	1883963925645.10	1883963896059.80
	150	1876261658283.20	1876254349189.40	1876255619304.20
	200	1880519658141.10	1880506202368.30	1880504208000.60
Modified Shepard's Method	50	1695389648358.00	1695414183124.80	1695397784210.80
	100	1695419879930.40	1695457080993.70	1695419061048.00
	150	1695394176404.30	1695271395651.00	1695503792596.10
	200	1695347759194.40	1695585380221.40	1695293885257.30
Natural Neighbor	50	1431232295188.70	1431264700544.10	1431270420902.90
	100	1426287826183.50	1426384070515.50	1426252849094.70
	150	1421137309559.90	1421355816146.20	1421183594979.40
	200	1416440820929.10	1416514253713.80	1416561413218.10
Nearest Neighbor	50	1882348910460.40	1882347606946.50	1882348121508.00
	100	1882364418326.70	1882348305723.30	1882389499454.90
	150	1882351613301.90	1882435927709.10	1882371998262.00
	200	1882398585456.00	1882361662497.80	1882364937768.50
Polynomial Regression	50	1848148187924.90	1848148187924.90	1848148187924.90
	100	1848148187924.90	1848148187924.90	1848148187924.90
	150	1848148187924.90	1848148187924.90	1848148187924.90
	200	1848148187924.90	1848148187924.90	1848148187924.90
Radial Basis Function	50	1895047179544.60	1895038765860.20	1895046695774.50
	100	1895051043047.00	1895058190116.80	1895046343354.30
	150	1895046555132.40	1895019253479.40	1895047703609.50
	200	1895036682491.70	1895031171674.10	1895007245275.10
Triangulation with Linear Interpolation	50	1436555123600.20	1436634127195.00	1436616258308.90
	100	1436336475464.00	1436560750795.30	1436491325418.90
	150	1436145696855.70	1436454584369.30	1436620300762.60
	200	1435565412824.90	1436354350787.70	1436015289880.00

Relative Error is given by (8)

$$E_{relative} = \frac{V_{estimated} - V_{actual}}{V_{actual}} \quad (8)$$

where $E_{relative}$ is relative error; $V_{estimated}$ is estimated volume; V_{actual} is actual volume.

Table 2. Differences between actual volume and volumes calculated with different volume calculation methods depending on the different interpolation methods and grid ranges, and the calculated relative errors wıth respect to these differences

Interpolation methods	Grid ranges (m)	Trapezoidal rule		Simpson's rule		Simpson 's 3/8 rule	
		Differences (m^3)	Relative error	Differences (m^3)	Relative error	Differences (m^3)	Relative error
Inverse Distance to a Power (k=1)	50	436424060852.20	0.304	436413472344.00	0.304	436425001148.10	0.304
	100	436419880827.70	0.304	436391987385.00	0.304	436424023102.30	0.304
	150	436408079256.30	0.304	436405002652.10	0.304	436398445707.40	0.304
	200	436479023967.80	0.304	436437701332.00	0.304	436435794487.50	0.304
Inverse Distance to a Power (k=2)	50	438348900301.10	0.305	438343685636.40	0.305	438349606754.60	0.305
	100	438343505071.60	0.305	438328561164.70	0.305	438343799160.90	0.305
	150	438339263278.80	0.305	438335833143.40	0.305	438335813304.10	0.305
	200	438381256794.10	0.305	438368350761.30	0.305	438372399119.30	0.305
Kriging	50	454029295127.00	0.316	454024816817.60	0.316	454029667872.30	0.316
	100	454021107128.40	0.316	454025954072.80	0.316	454018836255.00	0.316
	150	454024150182.50	0.316	454011523410.50	0.316	454021162412.40	0.316
	200	454016678968.10	0.316	454010187837.60	0.316	453997954582.40	0.316
Minimum Curvature	50	450615654673.20	0.314	450612369478.00	0.314	450612370944.00	0.314
	100	447330632806.50	0.311	447324093923.70	0.311	447324064338.40	0.311
	150	439621826561.80	0.306	439614517468.00	0.306	439615787582.80	0.306
	200	443879826419.70	0.309	443866370646.90	0.309	443864376279.20	0.309
Modified Shepard's Method	50	258749816636.60	0.180	258774351403.40	0.180	258757952489.40	0.180
	100	258780048209.00	0.180	258817249272.30	0.180	258779229326.60	0.180
	150	258754344682.90	0.180	258631563929.60	0.180	258863960874.70	0.180
	200	258707927473.00	0.180	258945548500.00	0.180	258654053535.90	0.180
Natural Neighbor	50	-5407536532.70	-0.004	-5375131177.30	-0.004	-5369410818.50	-0.004
	100	-10352005537.90	-0.007	-10255761205.90	-0.007	-10386982626.70	-0.007
	150	-15502522161.50	-0.011	-15284015575.20	-0.011	-15456236742.00	-0.011
	200	-20199010792.30	-0.014	-20125578007.60	-0.014	-20078418503.30	-0.014
Nearest Neighbor	50	445709078739.00	0.310	445707775225.10	0.310	445708289786.60	0.310
	100	445724586605.30	0.310	445708474001.90	0.310	445749667733.50	0.310
	150	445711781580.50	0.310	445796095987.70	0.310	445732166540.60	0.310
	200	445758753734.60	0.310	445721830776.40	0.310	445725106047.10	0.310
Polynomial Regression	50	411508356203.50	0.286	411508356203.50	0.286	411508356203.50	0.286
	100	411508356203.50	0.286	411508356203.50	0.286	411508356203.50	0.286
	150	411508356203.50	0.286	411508356203.50	0.286	411508356203.50	0.286
	200	411508356203.50	0.286	411508356203.50	0.286	411508356203.50	0.286
Radial Basis Function	50	458407347823.20	0.319	458398934138.80	0.319	458406864053.10	0.319
	100	458411211325.60	0.319	458418358395.40	0.319	458406511632.90	0.319
	150	458406723411.00	0.319	458379421758.00	0.319	458407871888.10	0.319
	200	458396850770.30	0.319	458391339952.70	0.319	458367413553.70	0.319
Triangulation with Linear Interpolation	50	-84708121.20	0.000	-5704526.40	0.000	-23573412.50	0.000
	100	-303356257.40	0.000	-79080926.10	0.000	-148506302.50	0.000
	150	-494134865.70	0.000	-185247352.10	0.000	-19530958.80	0.000
	200	-1074418896.50	-0.001	-285480933.70	0.000	-624541841.40	0.000

Relative errors regarding the differences between actual volume value and volume values obtained with different volume calculation methods according to the different interpolation methods and grid spacing are shown in Fig. 3, Fig. 4, Fig. 5 and Fig. 6 with bar graphs.

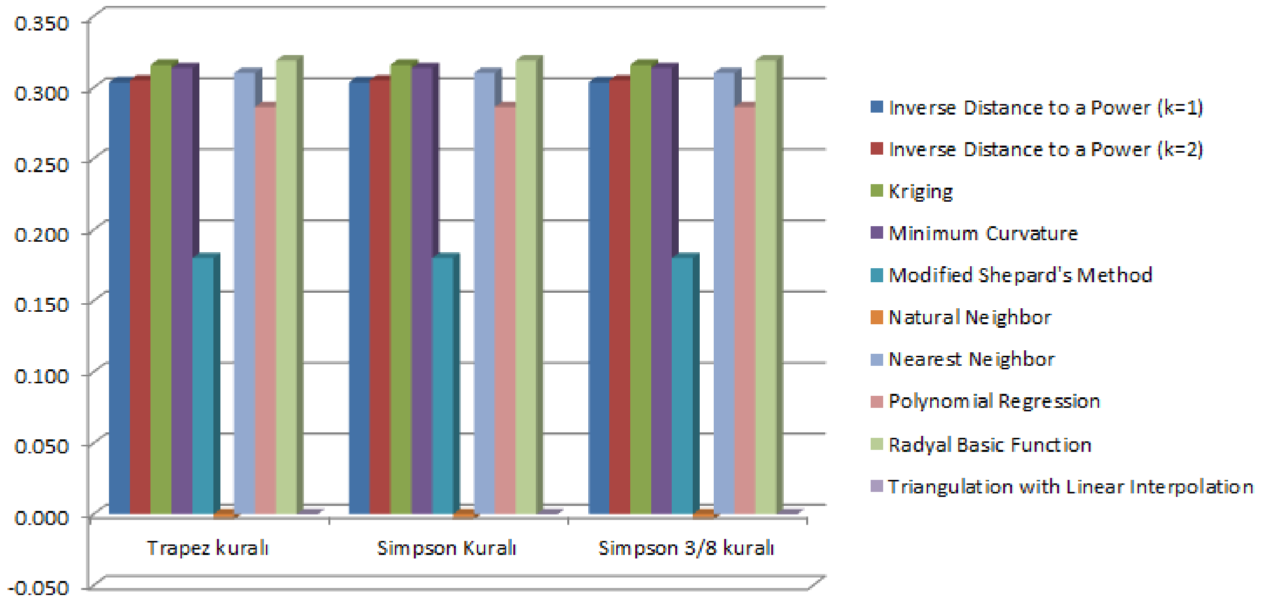

Fig. 3. Variation of relative errors according to 50 m grid range

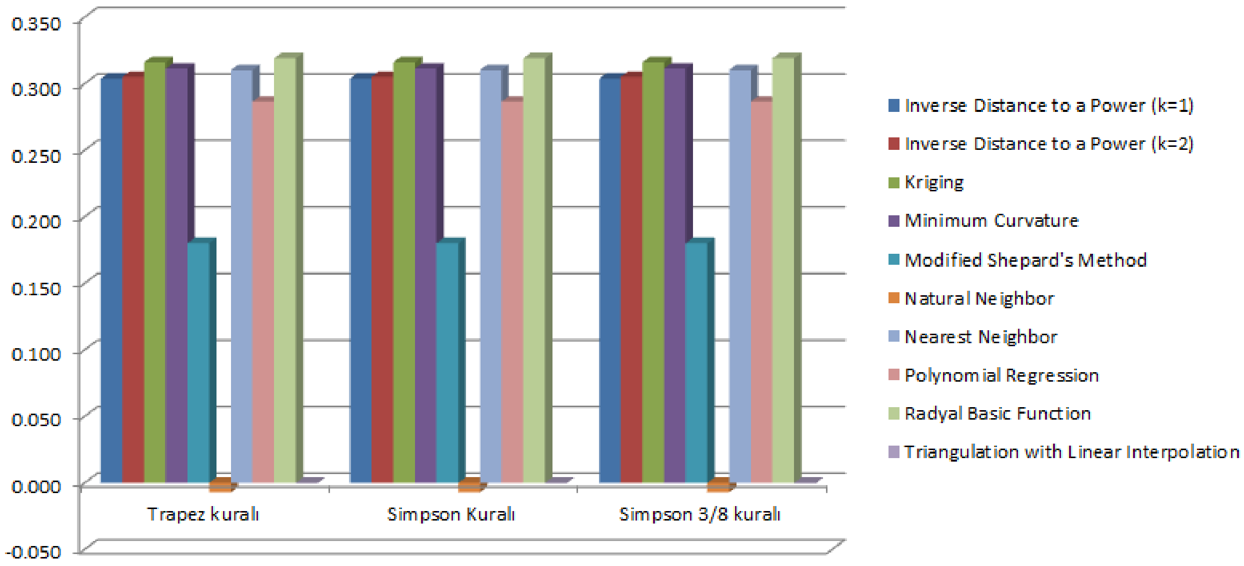

Fig. 4. Variation of relative errors according to 100 m grid range

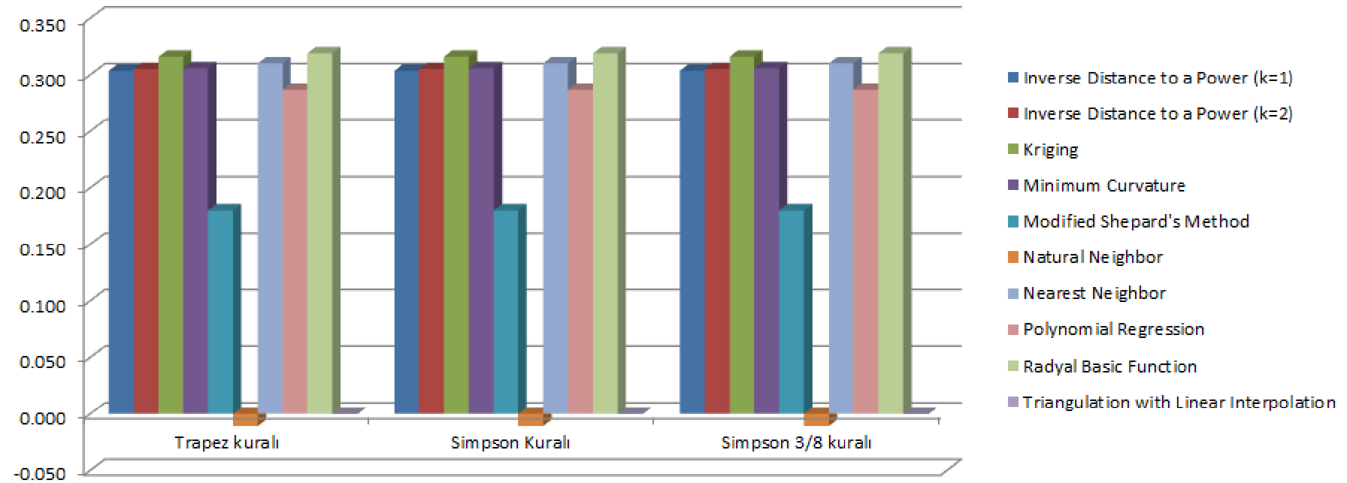

Fig.5. Variation of relative errors according to 150 m grid range

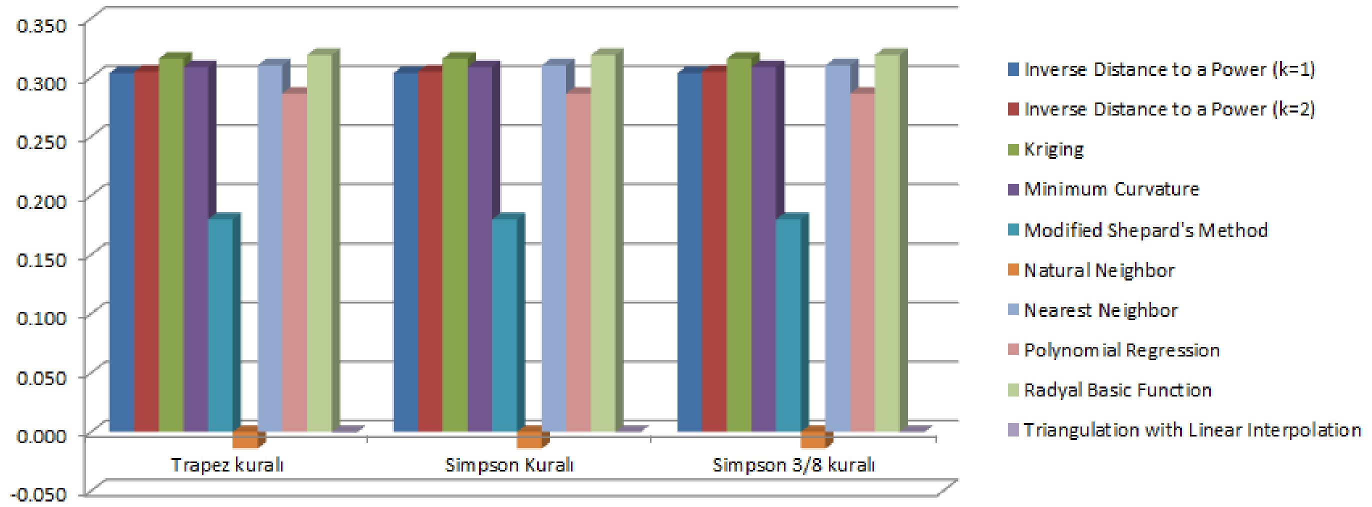

Fig. 6. Variation of relative errors according to 200 m grid range

CONCLUSIONS

The volume calculations were made through Surfer8 program. The effects of the parameters such as, grid range, interpolation methods, volume calculation methods on the volume calculation was investigated in the study. Moreover, the discussions made taking into account the amount of relative errors are as follows:

- When the relative errors examined (see Table 2, Figure 3-4-5-6), it was seen that the most appropriate interpolation model was triangulation with linear. When the relative errors calculated with other interpolation methods examined, it was seen that the most appropriate interpolation models were respectively, from smaller to larger, Natural Neighbor, Modified Shepard's Method, Polynomial Regression, Inverse Distance to a Power (k=1), Inverse Distance to a Power (k=2), Nearest Neighbor, Kriging, Radial Basis Function.
- It was seen in the interpolation methods used including minimum curvature, natural neighbor that the relative errors changed depending on the different grid ranges. In other methods, the change in grid ranges didn't affect the relative error.
- Changing the volume calculation methods didn't affect the relative error. The amount of relative error wasn't changed with changing the method of volume calculation.

THANK YOU VERY MUCH