# Triple Frequency Multi-GNSS Cycle Slip Detection Using Ionospheric Residuals

**Gethin Wyn ROBERTS, UK**

**Key words**: GNSS, Cycle Slips

## SUMMARY

Cycle Slips are a limiting factor when using carrier phase GNSS. These are caused by several factors, both physical and electronic in nature. This paper investigates the use of the ionospheric residuals to detect cycle slips. The ionospheric residual value is the residual, caused by the ionosphere, when using the frequency combination multiplier to convert from one frequency carrier phase value into another. In a vacuum, this value would be zero. However, due to the ionosphere, this value exists. This value changes relatively slowly over time, due to the moving satellites' GNSS data passing through changing ionosphere. By analysing the change in ionospheric residual from individual satellites between high rate epochs, the change is close to zero. However, when a cycle slip occurs, the residual value jumps.

Data from triple frequency GPS, Galileo and BeiDou satellites are used to illustrate the precision of such measurements, as well as the ability to detect cycle slips introduced into real data.

# Triple Frequency Multi-GNSS Cycle Slip Detection Using Ionospheric Residuals

**Gethin Wyn ROBERTS, UK**

## 1. INTRODUCTION

Carrier phase positioning techniques, whether using relative positioning or stand-alone Precise Point Positioning (PPP), are usually prone to cycle slips. Cycle slips occur in the carrier phase data, and affect the integer ambiguity value. It is important to be able to detect cycle slips, and preferably fix these slips for effective carrier phase positioning to take place. With the introduction of multi GNSS systems, such as BeiDou, GPS, GLONASS and Galileo, as well as the introduction of triple frequencies, or more, calculating the integer ambiguity values has moved towards being instantaneous. However, there are still real-life scenarios where instantaneous integer ambiguity resolution is not fail safe, such as in built up areas with the reduction of the number of satellites available, and the more traditional approach of calculating the integer ambiguities can take several epochs, which then needs to remain fixed for precise positioning. During such process, cycle slips are possible, which need to be addressed for efficient carrier phase positioning.

## 2. CYCLE SLIP DETECTION

There are several techniques that can be used to detect and fix cycle slips. These include:

- Comparing the changes in range between a specific satellite and a specific GNSS receiver using both the carrier phase and pseurorange observations over successive epochs.
- Comparing the change in the value of the Melbourne Equation [Melbourne, 1985], which uses both the pseudoranges and carrier phase values on dual frequency data to estimate the wide lane integer ambiguity value, and hence any wide lane cycle sips.
- Investigating the triple difference values when differencing carrier phase observables between two GNSS receivers. Any cycle slips will be jumps in the data.

There are other techniques too. The first two listed above have the inherent problem that the pseudorange observable is noisier than the carrier phase, and is more prone to multipath noise. Therefore, these techniques are not suitable to detect cycle slips of the order of a single or a few cycles, and they are not suitable for correcting cycle slips to a single cycle precision.

The triple differencing technique works well when considering static GNSS data, but not so well when kinematic data is considered.

An alternative method is known as the Ionospheric Residua (IR)l.

## 3. IONOSPHERIC RESIDUAL

The various carrier phase frequencies, ie L1, L2 and L5 for GPS, B1, B2 and B3 for BeiDou and E1, E5, E5a and E5b for Galileo, measure the change in range between respective satellites and the

GNSS receiver. Consequently, a phase observation from a specific satellite to a specific GNSS receiver can be converted to an equivalent phase measurement on another frequency using the frequencies of the carrier waves. Equation 1 illustrates the relationship between the speed of light c and the frequency of the electromagnetic wave f and the corresponding wavelength λwhen travelling through a vacuum.

$$c = f \, . \, \lambda \qquad (1)$$

Consequently, if multiple frequencies are considered e.g. for GPS, then equation 2 can be derived

$$c = f_{L1} \, . \, \lambda_{L1} = f_{L2} \, . \, \lambda_{L2} = f_{L5} \, . \, \lambda_{L5} \qquad (2)$$

Equations 1 and 2 are only applicable when the radio frequencies travel through a vacuum. Systematic error due to the ionosphere are frequency dependent, and will not be completely removed. This error is known as the ionospheric residual, and will change slowly over time, as the satellite moves, or the user changes location, causing the radio wave to pass through different parts of the ionosphere. The electromagnetic characteristics of the ionosphere itself will also change over time [Goad, 1986]. The Ionospheric Residual value is defined in equation 3 [Roberts, 1997].

$$IR_a = \phi_a - \phi_b \, . \left(\frac{f_a}{f_b}\right) + \varepsilon \qquad (3)$$

Where $\phi_a$ and $\phi_b$ are the carrier phase value for two frequency combinations being compared, e.g. L1 and L2, or L1 and L5 or L2 and L5 for GPS, $f_a$ and $f_b$ are the corresponding frequencies of the two combinations, and $\varepsilon$ illustrates the error values due to the ionosphere, troposphere, receiver noise as well as the integer ambiguity. These values can also be those using any of the GNSS constellations. The ionospheric residual will change slowly over time, of the order of 1 cycle per minute, and any cycle slips will result in a sudden jump in the ionospheric residual value. Equation 4 is used, which looks at the change in ionospheric residual from one epoch i to the previous epoch i-1. This also eliminates other error sources or unknowns $\varepsilon$, as this value will change very slightly from one epoch to the next. When considering data with no cycle slips, then the Ionospheric Residual values at subsequent epochs will be very similar in value.

$$\delta IR = \left(\phi_a - \phi_b \, . (\frac{f_a}{f_b})\right)_{(i)} - \left(\phi_a - \phi_b \, . (\frac{f_a}{f_b})\right)_{(i-1)} \qquad (4)$$

If a cycle slips occurs between two epochs, epochs i and i-1, then either one or both of the $\phi_a \; or \; \phi_b$ will be affected. There are, therefore, two unknowns in this equation ie the cycle slip values on $\phi_a \; or \; \phi_b$. One approach to their solution involves the generation of all the possible values for the unknowns, which are used to calculate the corresponding Ionospheric Residual values. These values are known as the Refinement Integers. Different combinations of these refinement integers $RmL_a$ and $RmL_b$ are then used to find the solution, or possible solutions, which gives the closest agreement between the observed and computed Ionospheric Residual values. A search is conducted, using various pairs of refinement integers combinations to calculate which pair corresponds to the calculated ionospheric residual value.

Table 1 illustrates an example of the ionospheric residual refinement integer values for the GPS L1 and L5 combinations, to a limit of ± 5 cycles. It can be seen here that if a value of 0 cycles is obtained for the ionospheric residual, this corresponds to values of zero L1 cycles and zero L5 cycles. Similarly, if a value of, for example, 8.696 cycles is obtained for the ionospheric residual, then the corresponding values for the L1 and L5 cycle slips are 2 cycles and 5 cycles respectively. The difference in adjacent numerically ordered values for the ionospheric residuals values within the ± 5 cycles range are illustrated in Figure 1. Figure 1 gives an illustration of how precise the ionospheric residual measurements need to be to distinguish correctly between possible L1 and L5 combinations, in this ±5 cycle example. Similar graphs and results are possible for all combinations for each satellite system. However, if the search range increases, then so does the number of L1 L5 combinations. If, for example, a search range of ±10 cycles is considered, Figure 2, it can be seen that there are 306 possible combinations that result in ionospheric residual values that are different by 0.0174 cycles from another combination. However, still with the ±10 cycles search, there are always unique solutions, as the difference in numerically ordered ionospheric residual refinement integer values is not equal to zero. So, if the noise of the real Ionospheric Residual value is within the 0.0174 cycles value, for the GPS L1 L5 example, or within a statistically accepted tolerance, then the L1 and L5 cycle slips can be accurately detected and repaired.

Table 1, An example of the ±5 cycles Refinement Integer values.

| | | RML1 | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | **-5** | **-4** | **-3** | **-2** | **-1** | **0** | **1** | **2** | **3** | **4** | **5** |
| RML5 | **-5** | 1.696 | 2.696 | 3.696 | 4.696 | 5.696 | 6.696 | 7.696 | 8.696 | 9.696 | 10.696 | 11.696 |
| | **-4** | 0.357 | 1.357 | 2.357 | 3.357 | 4.357 | 5.357 | 6.357 | 7.357 | 8.357 | 9.357 | 10.357 |
| | **-3** | -0.983 | 0.017 | 1.017 | 2.017 | 3.017 | 4.017 | 5.017 | 6.017 | 7.017 | 8.017 | 9.017 |
| | **-2** | -2.322 | -1.322 | -0.322 | 0.678 | 1.678 | 2.678 | 3.678 | 4.678 | 5.678 | 6.678 | 7.678 |
| | **-1** | -3.661 | -2.661 | -1.661 | -0.661 | 0.339 | 1.339 | 2.339 | 3.339 | 4.339 | 5.339 | 6.339 |
| | **0** | -5.000 | -4.000 | -3.000 | -2.000 | -1.000 | 0.000 | 1.000 | 2.000 | 3.000 | 4.000 | 5.000 |
| | **1** | -6.339 | -5.339 | -4.339 | -3.339 | -2.339 | -1.339 | -0.339 | 0.661 | 1.661 | 2.661 | 3.661 |
| | **2** | -7.678 | -6.678 | -5.678 | -4.678 | -3.678 | -2.678 | -1.678 | -0.678 | 0.322 | 1.322 | 2.322 |
| | **3** | -9.017 | -8.017 | -7.017 | -6.017 | -5.017 | -4.017 | -3.017 | -2.017 | -1.017 | -0.017 | 0.983 |
| | **4** | -10.357 | -9.357 | -8.357 | -7.357 | -6.357 | -5.357 | -4.357 | -3.357 | -2.357 | -1.357 | -0.357 |
| | **5** | -11.696 | -10.696 | -9.696 | -8.696 | -7.696 | -6.696 | -5.696 | -4.696 | -3.696 | -2.696 | -1.696 |

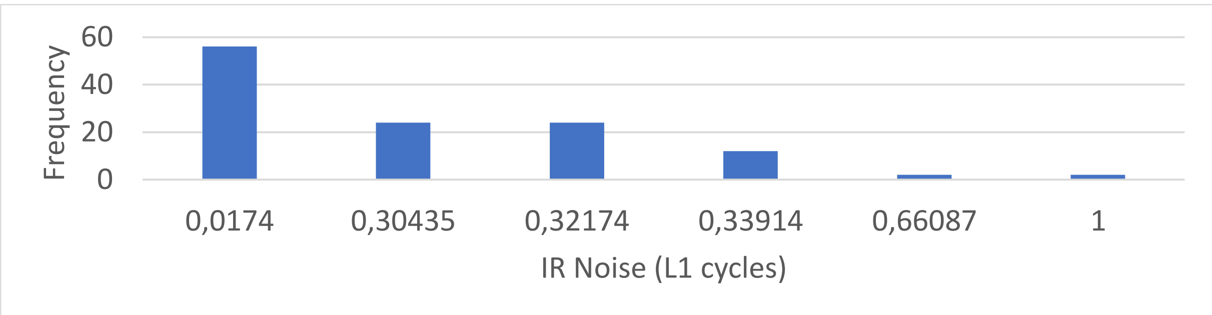

Figure 1, Frequency of the differences between numerically ordered Ionospheric Residual values in the ±5 L1 L5 cycles range.

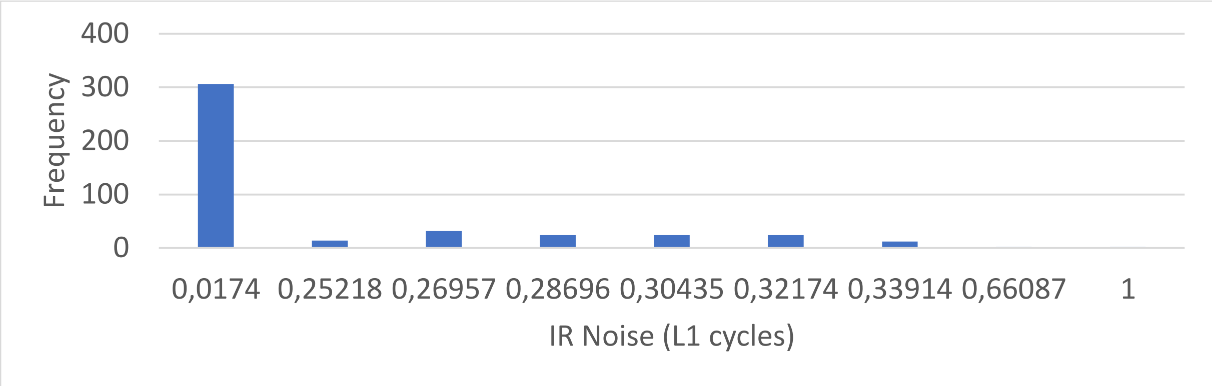

Figure 2, Frequency of the differences between numerically ordered Ionospheric Residual values in the ±10 L1 L5 cycles range.

If, however, a search range of ± 100 cycles is considered, there are 3,428 non-unique combinations out of the possible 4,004,001 combinations, as well as many combinations that have very small differences between numerically ordered values of the solutions. Figure 3 illustrates the differences in numerically ordered ionospheric residual values within this range. This is a log based graph. For example, an ionospheric residual value of -64.2 for the L1 L5 GPS combination can have two possible solutions, within the ± 100 cycles range. These are -95L1 and -23L5, or 59L1 and 92L5. Both sets will result in a value of -64.2 L1 cycles when used in equation 4. However, there is only one unique solution for IR = 0, which is 0 L1 and 0 L5, within the ± 100 cycle range. The smallest value for the ionospheric residual when considering the ±100 L1 and L5 cycles Rm values are ±0.0087 L1 cycles. Therefore, to accurately detect cycle slips, when restricting the search range to ± 100 cycles, the noise of the ionospheric residual must be smaller than 0.0087 L1 cycles. This noise value can also be calculated for all the constellations types, and all their frequencies.

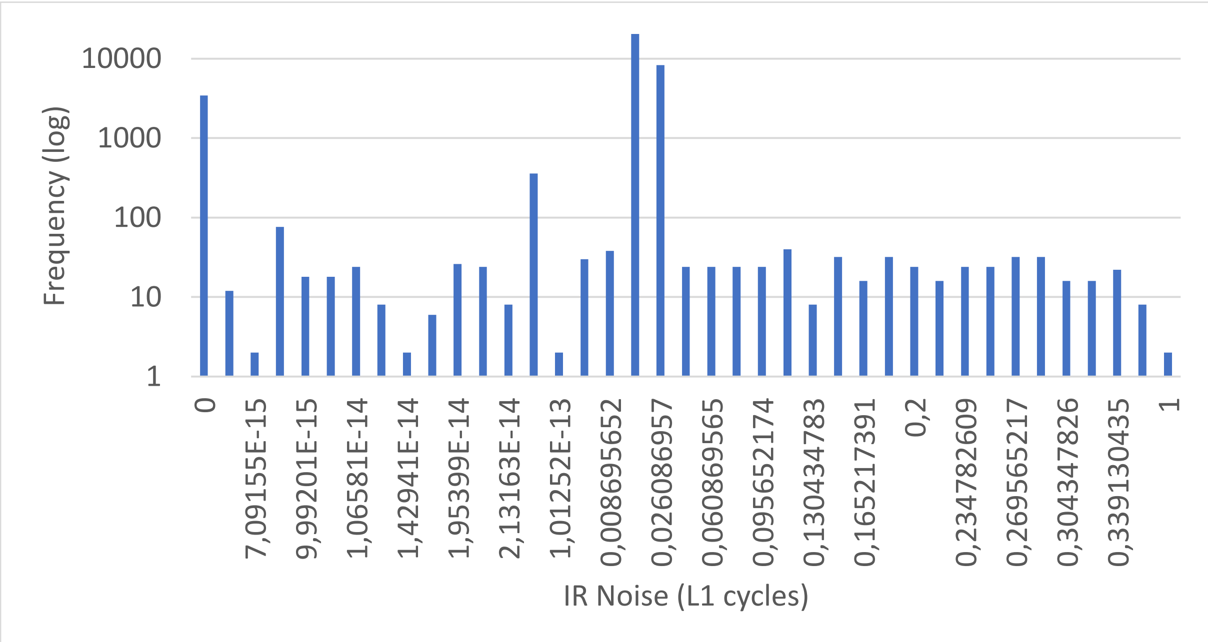

Figure 3, Frequency of the differences between numerically ordered Ionospheric Residual values in the ±100 L1 L5 cycles range.

If a range of ± 1000 cycles are considered for the L1 L5 combination, then there are 13 solutions that exist that will result in the IR = 0. These are listed, for illustration, in table 2. Similarly, as the search range increases, so do the number of non-unique solutions.

Table 2, The L1 and L5 Values that result in the combined ionospheric residual equal to zero, within a search range of ±1000 cycles.

| **L1** | **L5** | **IR L1L5** |
|---|---|---|
| -924 | -690 | 0 |
| -770 | -575 | 0 |
| -616 | -460 | 0 |
| -462 | -345 | 0 |
| -308 | -230 | 0 |
| -154 | -115 | 0 |
| 0 | 0 | 0 |
| 154 | 115 | 0 |
| 308 | 230 | 0 |
| 462 | 345 | 0 |
| 616 | 460 | 0 |
| 770 | 575 | 0 |
| 924 | 690 | 0 |

Table 3 illustrates the percentage of unique solutions in the ± 10, ± 100 and ±1000 cycle search range. This is for both GPS and BeiDou. The table also illustrates the number of repeating combinations. This is number of ionospheric residual solutions that have more than one corresponding possible pairs of refinement integer values. The table also illustrates the number of ionospheric residual solutions that are equal to zero for the various pair combinations.

Table 3, Various characteristics of repetition within the ionospheric residual solutions for both GPS and BeiDou combinations for search ranges of ± 10, ± 100 and ±1000 cycles.

| | | ± 10 cycles | | | ± 100 cycles | | | ± 1000 cycles | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Ionospheric Residual Combination | % Unique Solutions | Number of "zero" solutions | Number of repeating combinations | % Unique Solutions | Number of "zero" solutions | Number of repeating combinations | % Unique Solutions | Number of "zero" solutions | Number of repeating combinations |
| GPS | L1L2 | 100 | 1 | 0 | 75.23 | 3 | 2 | 1.9 | 11 | 22 |
| | L1L5 | 100 | 1 | 0 | 89.54 | 9 | 1 | 3.26 | 13 | 12 |
| | L2L5 | 100 | 1 | 0 | 9.8 | 1 | 8 | 0.5 | 83 | 83 |
| BeiDou | B1B2 | 100 | 1 | 0 | 100 | 1 | 0 | 58.43 | 3 | 2 |
| | B1B3 | 100 | 1 | 0 | 100 | 1 | 0 | 57.94 | 3 | 2 |
| | B2B3 | 100 | 1 | 0 | 46.85 | 3 | 3 | 0.83 | 33 | 34 |

The results so far, have illustrated the resulting ionospheric residual values resulting from the various refinement integers. It has been shown that as the search range expands, then the number of non-unique solutions increase, and the differences between adjacent numerically ordered solutions becomes smaller. It is also shown that the number of solutions resulting in a zero value increases, and is not only the case when both the carrier phase values considered are equal to zero. To gain an understanding of the expected noise for the calculated ionospheric residual values, using the real carrier phase data, clean data was analysed. This results in the noise calculated when the ionospheric residual value is equal to zero. Figures 4, 5, 6 and 7 illustrate examples of the ionospheric residual solutions for GPS L1L2, GPS L1L5, BeiDou B1B2 and Galileo E5aE5. These data represent the whole cycle of the GPS PRN 25, BeiDou PRN 12 and Galileo PRN 18 satellites. The vertical lines drawn over the graphs illustrate the times when the satellites are at 10° to 55° at 5° steps, and then vice versa on the second half of the graphs.

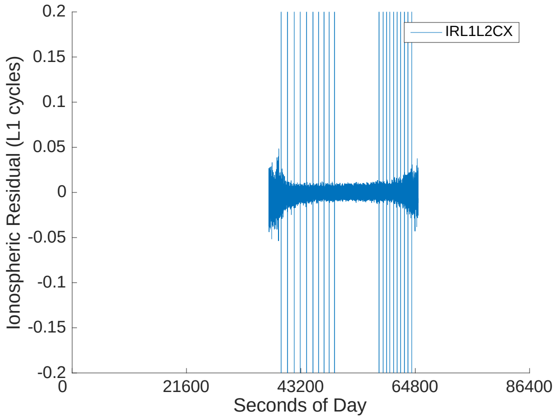


Figure 4, Ionospheric Residual for GPS PRN 25, L1L2 combination.

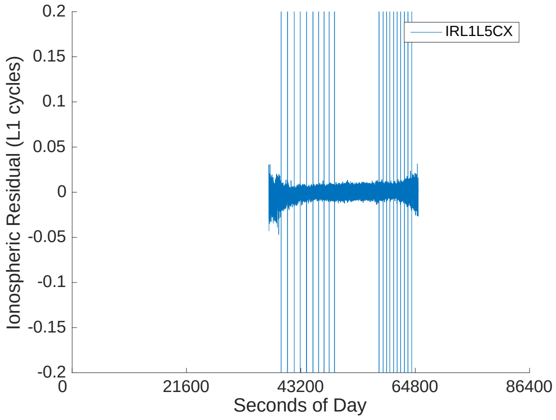


Figure 5, Ionospheric Residual for GPS PRN 25, L1L5 combination.

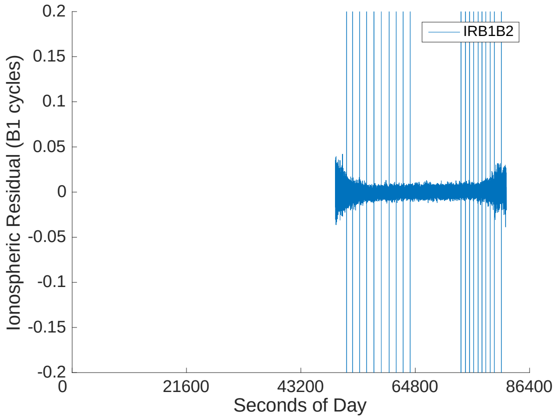

Figure 6, Ionospheric Residual for BeiDou PRN 12, B1B2 combination.

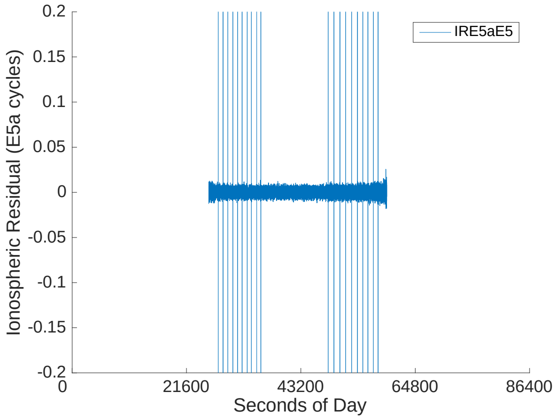

Figure 7, Ionospheric Residual for Galileo PRN 18, E5aE5 combination.

The RMS values of the noise are illustrated in table 4. These values are calculated using the data within the various elevation mask angles, ranging from 0°, and then 10° to 55° at 5° intervals. There is an increase in noise when the lower elevation mask angles are used. The various GPS, BeiDou and Galileo combinations are illustrated, including the data available for GPS using the carrier phase values derived from the L1 C/A code, the L2 C-code and that derived from the P-code, as well as the L5 carrier. There is less noise in the results for both the L1L2 and L2L5 when using the direct L2 C-code, as opposed to the derived P-code. It can be seen that there is an increase in noise when including the lower elevation data. It can also be seen that through comparing these results with Figures 1, 2 and 3, the Rm values' acceptable noise values can be estimated.

Table 4, The RMS noise of the Ionospheric Residual using various elevation cut off masks.

| | | Elevation Cut Off Angle (°) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 0 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 |
| GPS SV25 | L1L2CW | 0.0107 | 0.0065 | 0.0054 | 0.0048 | 0.0044 | 0.0042 | 0.0041 | 0.0041 | 0.0041 | 0.0042 | 0.0042 |
| | L1L2CX | 0.0063 | 0.0050 | 0.0045 | 0.0043 | 0.0042 | 0.0041 | 0.0040 | 0.0041 | 0.0041 | 0.0041 | 0.0041 |
| | L1L5CX | 0.0057 | 0.0046 | 0.0043 | 0.0042 | 0.0041 | 0.0041 | 0.0041 | 0.0042 | 0.0042 | 0.0043 | 0.0043 |
| | L2L5WX | 0.0082 | 0.0054 | 0.0048 | 0.0045 | 0.0044 | 0.0043 | 0.0043 | 0.0044 | 0.0044 | 0.0045 | 0.0045 |
| | L2L5XX | 0.0050 | 0.0044 | 0.0043 | 0.0042 | 0.0042 | 0.0043 | 0.0043 | 0.0043 | 0.0044 | 0.0045 | 0.0045 |
| *No Samples* | | *28,210* | *24,660* | *22,740* | *20,820* | *18,960* | *17,100* | *15,240* | *13,440* | *11,820* | *10,200* | *8,400* |
| BeiDou SV12 | B1B2 | 0.0054 | 0.0045 | 0.0040 | 0.0038 | 0.0036 | 0.0036 | 0.0036 | 0.0036 | 0.0035 | 0.0036 | 0.0036 |
| | B1B3 | 0.0052 | 0.0045 | 0.0042 | 0.0042 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0041 | 0.0042 |
| | B2B3 | 0.0047 | 0.0042 | 0.0040 | 0.0039 | 0.0039 | 0.0039 | 0.0039 | 0.0039 | 0.0040 | 0.0040 | 0.0041 |
| *No Samples* | | *32,326* | *29,220* | *26,760* | *24,660* | *22,500* | *20,400* | *18,300* | *15,960* | *13,860* | *11,760* | *9,600* |
| Galileo SV18 | E1E5 | 0.0046 | 0.0038 | 0.0037 | 0.0036 | 0.0036 | 0.0035 | 0.0034 | 0.0034 | 0.0034 | 0.0033 | 0.0033 |
| | E1E5a | 0.0048 | 0.0039 | 0.0038 | 0.0037 | 0.0036 | 0.0035 | 0.0035 | 0.0034 | 0.0034 | 0.0033 | 0.0033 |
| | E1E5b | 0.0047 | 0.0038 | 0.0037 | 0.0036 | 0.0035 | 0.0035 | 0.0034 | 0.0033 | 0.0033 | 0.0033 | 0.0032 |
| | E5aE5 | 0.0036 | 0.0035 | 0.0035 | 0.0034 | 0.0034 | 0.0034 | 0.0034 | 0.0033 | 0.0033 | 0.0033 | 0.0033 |
| | E5aE5b | 0.0039 | 0.0035 | 0.0035 | 0.0034 | 0.0034 | 0.0034 | 0.0033 | 0.0033 | 0.0033 | 0.0033 | 0.0033 |
| | E5bE5 | 0.0037 | 0.0036 | 0.0035 | 0.0035 | 0.0035 | 0.0034 | 0.0034 | 0.0034 | 0.0034 | 0.0033 | 0.0033 |
| *No Samples* | | *33,650* | *30,180* | *28,380* | *26,460* | *24,540* | *22,620* | *20,700* | *18,600* | *16,740* | *14,580* | *12,720* |

This section has shown how the ionospheric residual approach can be used with all the frequency combinations on the various satellite constellations. Any jumps in the ionospheric residual values can be investigated to see whether this is due to cycle slips. The resulting ionospheric residual can be compared to the solution when using refinement integers, and the corresponding refinement integer pair being the cause of the cycle slips. This section, however, has shown that as the search size of the refinement integers becomes larger, then the non-unique number of solution grows, as well as the number of combinations that equal to zero, including the 0,0 combination. Further to this, the differences between numerically ordered ionospheric residual solutions becomes smaller, making it more difficult to distinguish the results.

## 4. SIMULATED CYCLE SLIPS

This section illustrates the introduction of cycle slips into carrier phase data. The previous section has, in effect, illustrated the results of zero value cycle slips in the data, and the resulting ionospheric residuals. A selection of cycle slips was introduced into real GPS L1 and BeiDou B1 data, through changing the carrier phase values in the RINEX data files. Figure 8 and 9 illustrate samples of the resulting ionospheric residuals, illustrating the presence of cycle slips. Cycle slips of the order of up to 10 cycles were introduced, but larger cycle slips could also be detected. It can be seen from Figures 8 and 9 that the value of the ionospheric residual in the presence of cycle slips is far greater than when the cycle slip value is equal to zero.

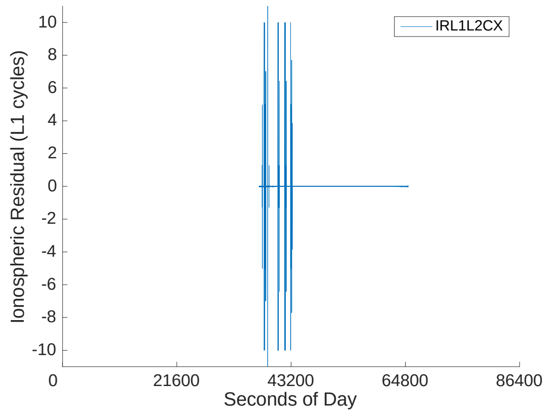

Figure 8, Simulated cycle slips introduced onto L1 GPS data, PRN 25, and the resulting L1L5 Ionospheric Residual value.

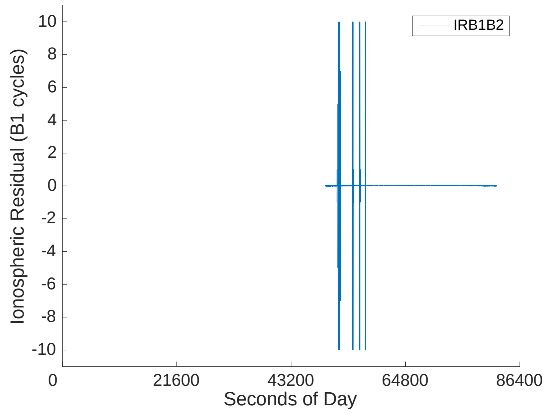

Figure 9, Simulated cycle slips introduced onto B1 BeiDou data, PRN 12, and the resulting B1B2 Ionospheric Residual value.

Table 5 illustrates the values of the cycle slips introduced, as well as the calculated ionospheric residual values for these combinations.

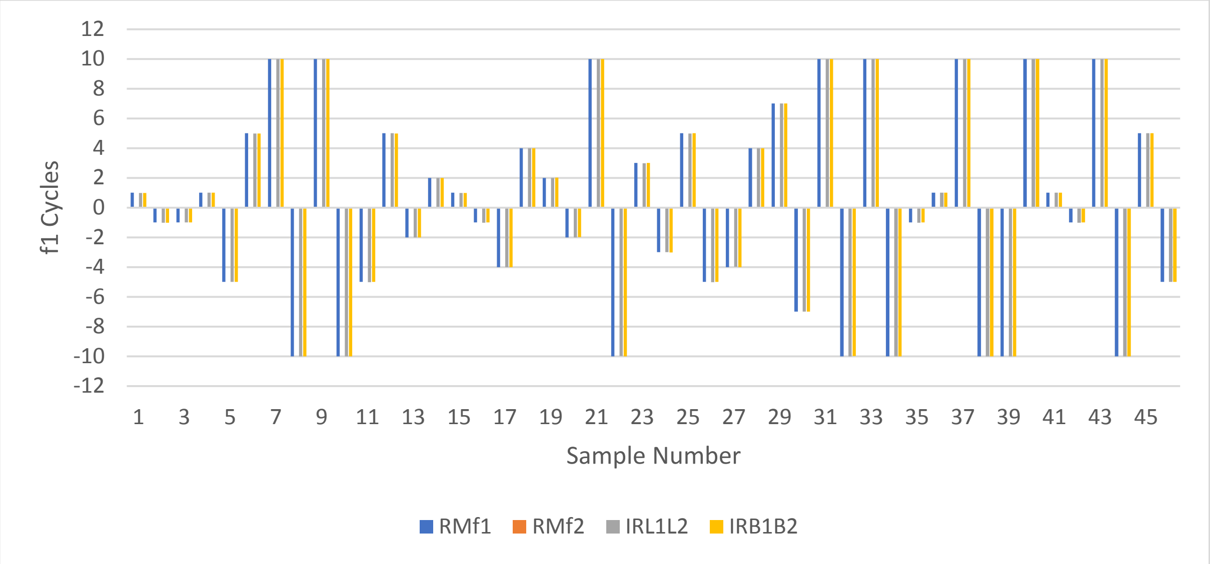

Figure 10, Simulated cycle slips introduced into L1 and B1 carrier phase data

## 5. CONCLUSIONS

The ionospheric residual approach to detect and fix cycle slips can be applied to all GNSS frequency pair combinations. There are, however, a few problems with this approach. The non-unique solutions increase as the number of refinement integers considered increase. The number of solutions resulting in a zero value for the ionospheric residual increase, as the search number increases. The differences between adjacent numerically ordered ionospheric residual values become smaller as the number considered increases. However, the technique is good at detecting cycle slips, and if the search range is limited, to say ±10 cycles, then it works well. This means that a coarse technique is also required to bring the cycle slip value to this smaller range, and then the ionospheric residual technique used to finely tune the cycle slip correction.

## REFERENCES

Goad, C., (1986). Precise Positioning with the GPS. Proceedings of the CERN Accelerator School of Particle Acceleration, CERN, Geneva.

MELBOURNE, W. G., 1985, The Case for Ranging in GPS-Based Geodetic Systems, Proc. First International Symposium on Precise Positioning with the Global Positioning System, 'Positioning with GPS, 1985, vol 1, pp 373 - 386, Rockville, Maryland.

Roberts, G. W., (1997). Real Time On The Fly Kinematic GPS. PhD Thesis, the University of Nottingham.

## ACKNOWLEDGEMENTS

The work in this paper is supported by the Ningbo Science and Technology Bureau as part of the project ‘Structural Health Monitoring of Infrastructure in the Logistics Cycle (2014A35008)’. The

GNSS data used for this paper were obtained from the Curtin GNSS Research Centre's at Curtin University http://saegnss2.curtin.edu.au/ldc/.

## BIOGRAPHICAL NOTES

Gethin Wyn Roberts has worked and studied at the university of Nottingham for over 26 years. He has a degree in Mining Engineering and a PhD in Engineering Surveying and Geodesy. He has co-authored over 250 publications, and supervised 25 PhD students. Gethin is a Fellow of the Chartered Institution of Civil Engineering Surveyors, and past Chairman of FIG's Commission 6 "Engineering Surveys". Gethin is also Co-Editor in Chief of the Springer Journal of Applied Geomatics.

## CONTACTS

Dr Gethin Wyn Roberts
The University of Nottingham
University Park
Nottingham
UK
Email: Gethin.Roberts@nottingham.ac.uk
Web site: www.nottingham.ac.uk