Computation of Curve Staking out Coordinates on the Excel Spreadsheet

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SUMMARY

A procedure for the computation of curve setting out data on the Excel spreadsheet is outlined. An Excel worksheet with five interrelated sheets computes the curve setting out data. In sheet one the included angles are computed from the coordinates entries of the intersection points of the straights. In sheet two curve elements for the various curves are computed from the entries of the radii and lengths of the transition curves and the included angles computed in sheet one. Sheet three computes the chainages of the principal points of the curves and the coordinates of these points. Sheet four and sheet five computes the coordinates of the center-line of the points along the straights and curves respectively.
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1. INTRODUCTION

When working in an engineering environment, the surveyor is called upon time and again to set out engineering works. This is the case in road construction. Curve setting out may be necessary either, during the actual road construction or for determining the extent of road reserve for purposes of land acquisition where necessary. The need may also become apparent for purposes of showing any encroachments on the road reserve. In the latter case, where cadastral information is in cassini coordinates, while the engineering data is in UTM coordinates, conversion between the two systems would be necessary. This conversion was demonstrated in [1] on the Excel spreadsheet.

The design engineer provides setting out data, usually being generated via engineering software. The data is accompanied by a design drawing showing the various elements of the design data. However, where the print out of the staking out coordinates for the center line is unavailable and the surveyor has the possession of the design drawing showing the various elements of the curves, staking out data can be conveniently computed by use of the Excel spreadsheet.

Presented here are steps and procedures necessary for the computation of the staking out coordinates of the centerline of the curve data. The steps are simple since no prior knowledge of programming is necessary as the functions used are in-built in the Excel spreadsheet. Furthermore, the computation of curve setting coordinates is repetitive, and can therefore be conveniently carried out on the Excel spreadsheet. Also presented is the computation of running chainages through a road project.

2. ELEMENTS OF THE CIRCULAR CURVE

The horizontal alignment of a road consists of a series of straights and curves. Where two straights meet, they create an intersection point. The designed road consists of sections of straights and curves between straights for the road under design. The curves are defined by the deflection angles (difference in bearing) between straights and the radius of the curve. It therefore follows that the initial requirements for setting out any curve are the location of the straights and their intersection points. The intersection angles are either provided by the design engineer or through direct field measurements. Once the intersection angles are provided together with the radius the curve can be set out.

3. ELEMENTS OF THE TRANSITION CURVE

A transition curve is one in which the curvature varies uniformly with respect to arc, in order to allow a gradual change from one radius to another (a straight being a curve of infinite radius) to

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Gacoki Thomas Gicira (Kenya)

FIG Working Week 2017
Surveying the world of tomorrow - From digitalisation to augmented reality
Helsinki, Finland, May 29–June 2, 2017
permit a gradual change in the super elevation. It must of course have the same radius of curvature at its ends, as the circular curve it links. The transition curve must have a constant rate of change of curvature with respect to arc. The solution presented here assumes a curve with two equal transitions and a circular curve in the middle.

The various elements of the circular and transition curves are shown in appendix 2, with the formulae for the various elements as shown in [2] and the figure 2 is as shown in [3].

4. THE SOLUTION

As an example, we start with seven intersection points with known coordinates are used to compute the six included angles formed from the seven intersection points. There after, the various elements (radii, transition curve length) are entered to compute curve parameters like the shifts (for transition curves), tangent lengths, curve lengths etc. Subsequently, the coordinates of the principal points and running chainages are computed.

The solution is illustrated by the use of five different sheets on the same worksheet. The first three sheets compute the various elements including the coordinates of the principal points. Thereafter, the fourth worksheet is used to compute the coordinates of the straights, while the fifth worksheet computes the coordinates of the two spirals and the circular curve.

Sheet 1: Computation of intersection angles

The coordinates of the intersection points are entered in this sheet and are used to calculate the intersection angles for the curves formed by these points. The coordinates of the intersection points (IPs) are entered in cell C6:D13 for 7 intersection points as shown in Table 1. This is used as an example; otherwise the number of points to be entered is not limited. From these entries the distances and the bearings of the straights, followed by the intersection angles are computed in columns E, F and G respectively as follows:

Distances

The distance between the intersection points is computed from the following:

\[ E7 := ((C7-C6)^2+(D7-D6)^2)^{0.5} \]

This formulae is then copied to cells E8:E13

Bearing

The bearings between the straights are computed from the following:

\[ F7 := IF(ATAN2((C7-C6),(D7-D6))<0,(ATAN2((C7-C6),(D7-D6))+2*PI())*180/PI(),(ATAN2((C7-C6),(D7-D6))*180/PI())) \]

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Gacoki Thomas Gicira (Kenya)

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Helsinki, Finland, May 29–June 2, 2017
Table 1: Computation of total deflection angles from the intersection points

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>njo ro turnoff to timboroa - calculation of total deflection angles</td>
<td>UNIT METRES</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>intersection points</td>
<td>curve no.</td>
<td>NORTINGS</td>
<td>EASTINGS</td>
<td>dist</td>
<td>brg</td>
<td>delta (total deflection angle)</td>
<td>deg</td>
<td>min</td>
<td>sec</td>
<td>delta (total deflection angle)</td>
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<td>9968890.58</td>
<td>841709.08</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>7</td>
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<td>839814.42</td>
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<td>914.421</td>
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<td>1</td>
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<td>8</td>
<td>IP2</td>
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<td>835710.29</td>
<td>3571.145</td>
<td>290.5324531</td>
<td>4.03722</td>
<td>4</td>
<td>2</td>
<td>13.99</td>
<td>4.0372196</td>
<td>R</td>
</tr>
<tr>
<td>9</td>
<td>IP3</td>
<td>4</td>
<td>9969980.21</td>
<td>835052.89</td>
<td>657.449</td>
<td>294.5696728</td>
<td>30.84782</td>
<td>25</td>
<td>16</td>
<td>52.15</td>
<td>30.847819</td>
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<tr>
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<td>834529.00</td>
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<td>31.83</td>
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<td>20.52</td>
<td>15.122366</td>
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<td></td>
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<tr>
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<td>837362.56</td>
<td>4680.544</td>
<td>328.9163989</td>
<td>38.24795</td>
<td>38</td>
<td>14</td>
<td>52.62</td>
<td>-38.24795</td>
<td>L</td>
</tr>
</tbody>
</table>

Sheet 1

The copy command is used to copy this formula to cells F8:F12. The explanation for the derivation of this formula is shown in appendix 1. In cell G7, the total deflection angles are computed by entering the following formula.

\[ G7: =IF(ABS(F7-F8)>180,360-ABS(F7-F8),ABS(F7-F8)) \]

This is then copied to cells F8:F12. The angles obtained are the total deflection angles irrespective of direction (either right or left); Columns H, I, and J computes the degrees, minutes and seconds of the included angle as follows:

\[ H7: =INT(G7) \]
\[ I7: =INT((G7-H7)*60) \]
\[ J7: =((G7-H7)*60-I7)*60 \]

The formulae are the copied to cells H8:J12.

Lastly, the total deflection angles are computed showing the direction (either left or right). This is accomplished by entering the following formula:

\[ K7:= IF(F8-F7>180,(F8-F7)-360,IF(F8-F7<-180,(F8-F7)+360,F8-F7)) \]

This is then copied to cells K8:K12. A positive angle means the curve formed from the intersection of the two straights is a right hand curve and vice versa.

Sheet 2: Computation of Curve Elements
The total deflection angles computed in Table 1 are used to compute the coordinates of the principal points of the curves. Prior to this however, certain curve elements need to be computed. These are the shifts, transition deflection angles, deflection angles for the circular curve, and cartesian coordinates for the transition curves. Other necessary elements include the total deflection distance between the origin and the end of the transition curve, the tangent distances for the curves, length of the circular curves and the distances of straights between the curves.

For the computations to be possible the radii of the various curves and the lengths of the transition curves must be provided. For the purpose of this paper it will be assumed that the transition curves are identical on both sides of the circular curve. The computation of these elements are illustrated in Table 2

Table 2: Computation of curve elements

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>curve no.</td>
<td>curve &quot;R&quot; or &quot;L&quot;</td>
<td>Radius</td>
<td>Lt</td>
<td>s (shifts)</td>
<td>transition deflection angle</td>
<td>deflection angle for circular curve</td>
<td>Total X</td>
<td>Total Y</td>
<td>Total deflection distance for the transition curve L</td>
<td>Total tangent length T</td>
<td>circular curve length Lc</td>
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<tr>
<td>2</td>
<td>6</td>
<td>1</td>
<td>R</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>2</td>
<td>R</td>
<td>4973.02</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>4.037220</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>3</td>
<td>L</td>
<td>870</td>
<td>80</td>
<td>0.30651</td>
<td>2.63428871</td>
<td>20.006930</td>
<td>79.983</td>
<td>1.226</td>
<td>79.992</td>
<td>175.28</td>
</tr>
<tr>
<td>5</td>
<td>9</td>
<td>4</td>
<td>R</td>
<td>870</td>
<td>80</td>
<td>0.30651</td>
<td>2.63428871</td>
<td>35.479241</td>
<td>79.983</td>
<td>1.226</td>
<td>79.992</td>
<td>175.28</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>5</td>
<td>R</td>
<td>870</td>
<td>80</td>
<td>0.30651</td>
<td>2.63428871</td>
<td>9.853789</td>
<td>79.983</td>
<td>1.226</td>
<td>79.992</td>
<td>175.28</td>
</tr>
<tr>
<td>7</td>
<td>11</td>
<td>6</td>
<td>L</td>
<td>870</td>
<td>70</td>
<td>0.23467</td>
<td>2.30500262</td>
<td>33.637946</td>
<td>69.989</td>
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<td>69.995</td>
<td>334.56</td>
</tr>
</tbody>
</table>

The formulae necessary for the computation of these elements are shown in appendix 2 and are computed as follows:

**Shift**

\[ E7 := \frac{D7^2}{24C7} \]

This formula is copied to cells E8:E12

**Transition deflection angle**

\[ F7 := \frac{(D7/(2C7)) \times 180}{\pi} \]

This formula is copied to cells F8:F12

**Deflection angle for the circular curve**

\[ G7 := \text{Sheet1!G7-2*F7} \]

This formula is copied to cells G8:G12

**Cartesian coordinates**

\[ H7 := D7-(D7^3/(40C7^2)) \]

Computation of Curve Staking out Coordinates on the Excel Spreadsheet (8671)

Gacoki Thomas Gicira (Kenya)

FIG Working Week 2017

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Helsinki, Finland, May 29–June 2, 2017
I7: \(=(D7^2)/(6*C7) -(D7^4)/(336*C7^3)\)
This formula is copied to cells H8:H12 and I8:I12 respectively

Total deflection distance for the transition curve

J7:= IF(D7=0,"0",D7-(D7^5/(90*(D7*C7)^2))+(D7^9/(22680*(D7*C7)^4)))
This formula is copied to cells J8:J12

Tangent length

K7: =H7-C7*SIN(F7*PI()/180)+((C7+E7)*TAN((Sheet1!G7/2)*PI()/180))
This formula is copied to cells K8:K12

Length of circular curve

L7: =C7*((Sheet1!G7-2*F7)*PI()/180)
This formula is copied to cells M8:M12

Length of straights between curves

M7: =Sheet1!E7-(K6+K7)
This formula is copied to cells N8:N12

Sheet 3: Computing coordinates of the principal points of the curves

The curve elements computed in Table 2 are used to compute the running chainages of the principal points and their coordinates. Eventually, these coordinates of the principal points are used to compute the setting out data for the horizontal alignment. The principal points are as follows:

<table>
<thead>
<tr>
<th>Curve no.</th>
<th>Curve &quot;R&quot; or &quot;L&quot;</th>
<th>Chainage</th>
<th>Coordinates (SC/BCC)</th>
<th>Coordinates (CS/ECC)</th>
<th>Chainage CT</th>
<th>Coordinates CT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>R</td>
<td>8657.240</td>
<td>9968243.891</td>
<td>839978.179</td>
<td>10505.001</td>
<td>9968809.580</td>
</tr>
<tr>
<td>2</td>
<td>R</td>
<td>11403.893</td>
<td>9968441.954</td>
<td>839122.256</td>
<td>11403.893</td>
<td>9969650.716</td>
</tr>
<tr>
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<td>R</td>
<td>14915.036</td>
<td>9969890.542</td>
<td>839592.063</td>
<td>14995.036</td>
<td>9970576.312</td>
</tr>
<tr>
<td>4</td>
<td>R</td>
<td>15437.931</td>
<td>9970163.364</td>
<td>839436.851</td>
<td>15517.931</td>
<td>9970857.312</td>
</tr>
<tr>
<td>5</td>
<td>R</td>
<td>16302.244</td>
<td>9971798.496</td>
<td>839530.796</td>
<td>16382.244</td>
<td>9971998.496</td>
</tr>
</tbody>
</table>

Sheet 3

TS The tangent to the transition curve (transition curve origin).
SC/BCC The beginning of the circular curve
CS/ECC The end of the circular curve (circular curve to transition).
CT Transition to tangent

Computation of Curve Staking out Coordinates on the Excel Spreadsheet (8671)
Gacoki Thomas Gicira (Kenya)

FIG Working Week 2017
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Helsinki, Finland, May 29–June 2, 2017
The running chainages and the coordinates of these principal points are computed as is illustrated in Table 3. The starting chainage is entered in cell L8 and the corresponding coordinates in cell M8:N8

**TS**

Chainage:  \( C9 := L8 + \text{Sheet2!M7} \)

Northings:  \( D9 := \text{Sheet1!C7} + \text{Sheet2!K7} \times \cos((\text{Sheet1!F7} - 180) \times \pi / 180) \)

Eastings:  \( E9 := \text{Sheet1!D7} + \text{Sheet2!K7} \times \sin((\text{Sheet1!F7} - 180) \times \pi / 180) \)

These formulae are copied to cells C10:E14

**SC/BCC**

Chainage:  \( F9 := C8 + \text{Sheet2!D7} \)

Northings:  \( G9 := \text{IF}(B9 = "R", D9 + \text{Sheet2!J7} \times \cos((\text{Sheet1!F7} + (\text{Sheet2!F7} / 3)) \times \pi / 180), D9 + \text{Sheet2!J7} \times \cos((\text{Sheet1!F7} - (\text{Sheet2!F7} / 3)) \times \pi / 180)) \)

Eastings:  \( H9 := \text{IF}(B9 = "R", E9 + \text{Sheet2!J7} \times \sin((\text{Sheet1!F7} + (\text{Sheet2!F7} / 3)) \times \pi / 180), E9 + \text{Sheet2!J7} \times \sin((\text{Sheet1!F7} - (\text{Sheet2!F7} / 3)) \times \pi / 180)) \)

These formulae are copied to cells F10:H14

**CS/ECC**

Chainage:  \( I9 := F9 + \text{Sheet2!L7} \)

Northings:  \( J9 := \text{IF}(B9 = "R", G9 + (\text{Sheet2!C7} \times 2 \times \sin((\text{Sheet2!G7} / 2) \times \pi / 180)) \times \cos((\text{Sheet1!F7} + \text{Sheet2!F7} + \text{Sheet2!G7} / 2) \times \pi / 180)), G9 + (\text{Sheet2!C7} \times 2 \times \sin((\text{Sheet2!G7} / 2) \times \pi / 180)) \times \cos((\text{Sheet1!F7} - \text{Sheet2!F7} - \text{Sheet2!G7} / 2) \times \pi / 180)) \times \cos((\text{Sheet1!F7} - \text{Sheet2!F7} - \text{Sheet2!G7} / 2) \times \pi / 180)) \)

Eastings:  \( K9 := \text{IF}(B9 = "R", H9 + (\text{Sheet2!C7} \times 2 \times \sin((\text{Sheet2!G7} / 2) \times \pi / 180)) \times \sin((\text{Sheet1!F7} + \text{Sheet2!F7} + \text{Sheet2!G7} / 2) \times \pi / 180)), H9 + (\text{Sheet2!C7} \times 2 \times \sin((\text{Sheet2!G7} / 2) \times \pi / 180)) \times \sin((\text{Sheet1!F7} - \text{Sheet2!F7} - \text{Sheet2!G7} / 2) \times \pi / 180)) \)

These formulae are copied to cells I10:K14

**CT**

Chainage:  \( L9 := L8 + \text{Sheet2!M7} + 2 \times \text{Sheet2!D7} + \text{Sheet2!L7} \)

Northings:  \( M9 := \text{Sheet1!C7} + \text{Sheet2!K7} \times \cos((\text{Sheet1!F8} \times \pi / 180)) \)

Eastings:  \( N9 := \text{Sheet1!D7} + \text{Sheet2!K7} \times \sin((\text{Sheet1!F8} \times \pi / 180)) \)

Computation of Curve Staking out Coordinates on the Excel Spreadsheet (8671)
Gacoki Thomas Gicira (Kenya)

FIG Working Week 2017
Surveying the world of tomorrow - From digitalisation to augmented reality
Helsinki, Finland, May 29–June 2, 2017
These formulae are copied to cells L10:N14

Sheet 4: Computation of center-line coordinates for the straights

This is a straightforward computation as only the bearing of the straight is used to compute the coordinates of the preceding chainages. However, the starting chainage, bearing and coordinates for the particular straight are copied from sheet 1 and 3. Table 4. As an example we will compute the coordinates of the centerline of the straights between curves 4 and 5.

The starting chainage for the particular straight in this case given by: -

B6:=Sheet3!L12

The subsequent chainages are obtained by inserting the following formula:-

B7:= IF(B6+((INT(B6/20))+1-(B6/20))*20<$B$6+Sheet2!$M$11,B6+((INT(B6/20))+1-(B6/20))*20,IF(B6=$B$6+Sheet2!$M$11,“,IF(B6+((INT(B6/20))+1-(B6/20))*20>$B$6+Sheet2!$M$11,$B$6+Sheet2!$M$11,”))

This formula returns a null value “” or #value when the length of straight is exceeded, acting as a check.

The bearing for the particular straight is inserted from sheet 1 as follows: -

G6:=Sheet1!$F$11

Since this bearing is constant it is simply copied to the other cells i.e G7:G15. The coordinates of the starting chainage are copied from sheet 3 as follows: -

Northings: I6=Sheet3!M12
Eastings: J6=Sheet3!N12

Finally, the coordinates of the chainages of the straights are computed as follows:-

Northings: I7:= I6+C7*COS(G7*PI()/180)
Eastings: J7:= J6+C7*SIN(G7*PI()/180)

These formulae are then copied to cells I7:J15.

Table 4: Computation of center-line coordinates for the straights
Straight 5: Straight between curves 4 and 5

Sheet 5: Computation of chainages and staking out coordinates for the first transition curve

The computation of the staking out coordinates of the first transition curve starts from the last chainage of the preceding straight. The computation is illustrated in Table 5 and is computed as follows:

Table 5: Computation of center-line coordinates for the first transition curve

Curve 5: first transition curve

The northings (N) and easting (E) of the TS point are known (having already been computed in Table 3). In addition from the equations in appendix 2, various points on the transition curve can be computed. Using these points we could extract (N,E) values of various chainages of the spiral. The Northings and Eastings values are computed at 20m intervals along the spiral as shown in Table 5.

Computation of Curve Staking out Coordinates on the Excel Spreadsheet (8671)
Gacoki Thomas Gicira (Kenya)

FIG Working Week 2017
Surveying the world of tomorrow - From digitalisation to augmented reality
Helsinki, Finland, May 29–June 2, 2017
The starting chainage for the TS in this case, TS5 given by:

\[ B7 := \text{Sheet3!C13} \]

The subsequent chainages are obtained by inserting the formula:

\[ B8 := \text{IF}(B7+((\text{INT}(B6/20))+1-(B7/20))*20<$\text{Sheet2!$D$11},B7+((\text{INT}(B7/20))+1-(B7/20))*20,$\text{IF}(B7+((\text{INT}(B7/20))+1-(B7/20))*20>$\text{Sheet2!$D$11},B7+\text{Sheet2!$D$11,}"")) \]

The formula returns a null value “” or #value when the length of the transition curve is exceeded acting as a check.

The deflection angle i.e. the angle subtended at TS5 by the extended tangent and the chord connecting TS and any arbitrary point on the spiral (positive if angle is right hand and negative if left hand) is computed from the following:

\[ E7 := ((D7^2/(6*$E$6))-((0.0762*(D7^2/(6*$E$6))^3)+(0.0166*(D7^2/(6*$E$6))^5)))\times180/\pi() \]

The length of the spiral chord from TS5 to any given point on the spiral is computed from:

\[ F7 := \text{IF}(D7=0,"0",D7-(D7^5/(90*(E$6)^2))+(D7^9/(22680*(E$6)^4))) \]

The deflection bearing is given by

\begin{align*}
\text{Bearing from preceding straight:} & \quad G7 := \text{Sheet1!F11} \\
\text{Deflection bearing:} & \quad G8 := \text{IF}($\text{Sheet1!$L$11}="R",$G$7+E8,$G$7-E8) \\
\end{align*}

The tangential bearing is given by

\begin{align*}
\text{Bearing from preceding straight:} & \quad H7 := \text{Sheet1!F11} \\
\text{Tangential bearing:} & \quad H8 := \text{IF}($\text{Sheet1!$L$11}="R",$H$7+((D8^2/(2*E$6))*180/\pi()),$H$7-((D8^2/(2*E$6))*180/\pi())) \\
\end{align*}

The coordinates of TS5 are copied from sheet 3 as follows:

\begin{align*}
\text{Northings:} & \quad I7 := \text{Sheet1!D13} \\
\text{Eastings:} & \quad J7 := \text{Sheet1!E13} \\
\end{align*}

Finally the coordinates of a point on the spiral are given by

\begin{align*}
\text{Northings:} & \quad I8 := \$I$7+(F8*COS(G8*PI()/180)) \\
\text{Eastings:} & \quad J8 := \$J$7+(F8*SIN(G8*PI()/180)) \\
\end{align*}
The starting chainage for the particular circular curve in this case given by:

\[
M_7 := \text{Sheet3!F13}
\]

The subsequent chainages are obtained by inserting the formula:

\[
M_8 := \begin{cases} 
\text{IF}(M_7 + ((\text{INT}(M_7/20)) + 1 - (M_7/20)) \times 20 < \text{Sheet2!$L$11}, M_7 + ((\text{INT}(M_7/20)) + 1 - (M_7/20)) \times 20, \text{Sheet2!$L$11}) & \text{if length exceeded} \\
M_7 + ((\text{INT}(M_7/20)) + 1 - (M_7/20)) \times 20, \text{Sheet2!$L$11}) & \text{otherwise}
\end{cases}
\]

The starting chainage for the particular circular curve in this case given by:

\[
P_7 := ((O7/$P$6)*180/\pi())/2
\]

The distance from the beginning of the circular curve to a point on the circular curve

\[
Q_7 := 2*\text{Sheet2!$L$6}*\sin(P_7*\pi())/180
\]

The deflection bearing is given

\[
\text{Start bearing: } R_7 := \text{IF(Sheet2!$F11$="R",Sheet1!$F11$+Sheet2!$F11$,Sheet1!$F11$-Sheet2!$F11$)
\]
Deflection bearing: \[ R8 := \text{IF}(\text{Sheet1!$L$11} = "R", R7 + P8, R7 - P8) \]

The tangential bearing is given by

Start bearing: \[ S7 := \text{IF}(\text{Sheet2!B11} = "R", \text{Sheet1!F11} + \text{Sheet2!F11}, \text{Sheet1!F11} - \text{Sheet2!F11}) \]

Tangential bearing: \[ S8 := \text{IF}(\text{Sheet1!$L$11} = "R", S7 + 2*P8, S7 - 2*P8) \]

The coordinates of TS5 are copied from sheet 3 as follows:

Northings: \[ T7 := \text{Sheet3!G1} \]
Eastings: \[ U7 := \text{Sheet3!H1} \]

Finally the coordinates of a point on the circular curve are given by

Northings: \[ T8 := T7 + (Q8 * \text{COS}(R8 * \text{PI()}/180)) \]
Eastings: \[ U8 := U7 + (Q8 * \text{SIN}(R8 * \text{PI()}/180)) \]

This formula is copied to cells T9:U12

**Sheet 5: Computation of chainages and staking out coordinates for the second transition curve**

The starting chainage for the particular second transition curve in this case given by:- X7:=Sheet3!I13

**Table 7: Computation of center-line coordinates for the second transition curve**

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<th>Z</th>
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<th>AD</th>
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</table>

**Curve 5: Second transition curve**

The subsequent chainages are obtained by inserting the formula:
The formula returns a null value “ ” or #value when the length of the second transition curve is exceeded acting as a check.

The deflection angle from CT5- end of transition curve

\[ AA7 := \left( \frac{(Z7^2)/(6*AA5)}{((0.0762*Z7^2/(6*AA5))^3)+(0.0166*Z7^2/(6*AA5))^5)} \right)^{*180/PI()} \]

Distance from CT5

\[ AB7 := \begin{align*} &\text{IF}(Z7=0,"0",Z7-(Z7^5/(90*($AA5)^2))+(Z7^9/(22680*($AA5)^4))) 
\end{align*} \]

\[ Phi := AC7 := (Z7^2/(2*$AA5))*180/PI() \]

Distance from CS5- end of circular curve

\[ AD7 := (SAB^2+AB^2-2*SAB*AB*COS((AA5-AA5)*PI()/180))^0.5 \]

The deflection bearing is given by

\[ \begin{align*} \text{Start bearing:} \quad AE7 := &\begin{align*} &\text{IF}(\text{Sheet2!B11}="R",\text{Sheet1!F11+Sheet2!F11+Sheet2!G11,Sheet1!F11-Sheet2!F11-Sheet2!G11)} 
\end{align*} \]

\[ \text{Deflection bearing:} \quad AE8 := \begin{align*} &\text{IF}(\text{Sheet2!BSB11}="R",SAB7+(SAC7-SSS7)-(ACOS((AD8^2+SAB^2-AB8^2)/(2*AD8*SAB7))*180/PI())),SAB7-(SAC7-SSS7)-(ACOS((AD8^2+SAB^2-AB8^2)/(2*AD8*SAB7))*180/PI()))) 
\end{align*} \]

The tangential bearing is given by

\[ \begin{align*} \text{Start bearing:} \quad AF7 := &\begin{align*} &\text{IF}(\text{Sheet2!B11}="R",\text{Sheet1!F11+Sheet2!F11+Sheet2!G11,Sheet1!F11-Sheet2!F11-Sheet2!G11)} 
\end{align*} \]

\[ \text{Tangential bearing:} \quad AF8 := \begin{align*} &\text{IF}(\text{Sheet1!SL$11}="R", SAE7+(SAC7-AC7),$AE7-(SAC7-AC8)) 
\end{align*} \]

The coordinates of TS5 are copied from sheet 3 as follows:-
Finally the coordinates of a point on the spiral are given by

Northings: \( AG_8 := AG_7 + AD_8 \cos(\alpha_8 \pi / 180) \)

Eastings: \( AH_8 := AH_7 + AD_8 \sin(\alpha_8 \pi / 180) \)

**5. CONCLUSION**

The results obtained are the same as can be obtained from a computer program. This use of the Excel Spreadsheet can be used to illustrate the computation of curve setting out data. The final computed coordinates can be converted to text format from Excel and transferred to a GIS software (Arc GIS for example) and be used to show the road centerline. Within the GIS software buffers can be applied for the road reserve, which in turn can be used for land acquisition where this is necessary. Indeed, the same can be used to determine properties encroaching on the road reserve.

The results shown here were taken from a project which had been successfully used in the construction of a road project and the results obtained were similar to those generated from a computer program.

**6. APPENDIX 1**

**Computation of bearings between intersection points**

Given the coordinates of two intersection points (IP0, IP1) as shown on figure 1, the bearing, \( \alpha \) between the two points can be computed from the following:

\[
\alpha = 2 \times \arctan \left( \frac{\Delta E}{S + \Delta N} \right)
\]

Where

\( \Delta E \) is the difference in Eastings between the two points

\( \Delta N \) is the difference in Northings between the two points

\( S \) is the distance between the two points
This formula can be illustrated by use of half angle formulae, however, the same can be shown graphically as follows:-

![Diagram](image)

**FIGURE 1**

The line from IP0 is prolonged to C by distance $S$. Thereafter C is then joined to IP1 as shown in figure 1 above. It can be clearly seen from figure 1 that the angle formed at $C$ is $\frac{\alpha}{2}$. From figure 1,

\[
\tan \frac{\alpha}{2} = \frac{\Delta E}{S + \Delta N}
\]

\[
\frac{\alpha}{2} = \text{ARCTAN} \left( \frac{\Delta E}{S + \Delta N} \right)
\]

\[
\alpha = 2 \times \text{ARCTAN} \left( \frac{\Delta E}{S + \Delta N} \right)
\]

In excel this is entered as

\[
\alpha = \text{ATAN2}(\Delta N, \Delta E)
\]

The function ATAN2 operates on $\Delta N$ and $\Delta E$ to give angles in radians in the range $-\pi$ to $\pi$, excluding $-\pi$. If both $\Delta N$ and $\Delta E$ are zero, Excel returns the message #DIV/0 or division by zero. Positive angles are in the first and second quadrant and are correct whole circle bearings. Negative angles are in the third and fourth quadrant and $2^*\pi$ is added to convert to whole circle bearings.
APPENDIX 2

Elements of Transition and Circular Curves

Following are the key parameters that explain this geometry:

- **Lt** Length of spiral from TS to SC - see table 2
- **IP** Point of horizontal intersection point (not shown on figure 2)
- **TS/TC** Point where spiral begins (Tangent to Spiral - spiral origin)
- **SC/BCC** Point where spiral ends and circular curve starts (Beginning of circle from spiral end)
- **φ** Spiral angle (or) deflection angle between tangent and tangential direction at end of spiral
- **CS/ECC** End of circle to spiral
- **CT** Spiral to Tangent (second spiral – not shown in figure 2)
- **K** TA Abscissa of the shifted curve PC referred to TS (or tangent) distance at shifted PC from TS
- **s** Shift of circular curve - see table 2

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Computation of Curve Staking out Coordinates on the Excel Spreadsheet (8671)
Gacoki Thomas Gicira (Kenya)

FIG Working Week 2017
Surveying the world of tomorrow - From digitalisation to augmented reality
Helsinki, Finland, May 29–June 2, 2017
O Center point of circular curve
\( \Delta c \) Angle subtended by circular curve
\( \Delta \) Total deflection angle between the two tangents \((2 \varphi + \Delta c)\)
\( R \) Radius of circular curve - see table 2
\( \delta \) Deflection angle for the circular curve for a chord of length \( a \) is given by \( \frac{a}{2R} \) in radians
\( L_c \) Circular curve length \( 2R \Delta c \) - see table 2
\( T_c \) Tangent length of circular curve \( R \tan \frac{\Delta c}{2} \)
\( T \) Total (extended) tangent length from TS to IP given by
\[ T = (R + s) \tan \frac{\Delta}{2} + K \] - see table 2

Where \( K = TotalX - Rs \sin \varphi \)

\( X \) Total X tangent distance at SC from TS

\[ TotalX = L - \frac{L^3}{40R^2} + \frac{L^5}{3456R^4} - \ldots \] as shown in [2], \( L \) is the full length of the transition curve. - see table 2

\( Y \) Total Y = D2D off set distance at SC from (Tangent at) TS

\[ TotalY = \frac{L^2}{6R - \frac{L^4}{336R^3} + \frac{L^5}{42240R^5} - \ldots \] as shown in [2], \( L \) is the full length of transition curve.

\( s \) AB The offset of initial tangent into the PC of shifted curve (shift of the circular curve).

\[ s = AB = AE - BE = TotalY - (R - R \cos \varphi) = \frac{L^3}{24LR} - \frac{L^7}{2668(LR)^3} \] as shown in [2].

\( \delta \) Deflection angle from TS for the spiral given by
\[ \varphi \frac{3}{3} - \left(0.0762\left(\frac{\varphi}{3}\right)^3 + 0.0166\left(\frac{\varphi}{5}\right)^5 + \ldots \right) \] where \( \varphi = \frac{c^2}{2LR} \) as shown in [2].

Computation of Curve Staking out Coordinates on the Excel Spreadsheet (8671)
Gacoki Thomas Gicira (Kenya)

FIG Working Week 2017
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Chord length from TS is given by \( l = c - \frac{c^5}{90(1R)^2} + \frac{c^9}{22680(1R)^4} \) for the spiral as shown in [2], where \( c \) is distance from the beginning of the spiral.

REFERENCES

http://www.eldoradosoft.com/curves.htm