Improved Practical Mechanism for Reconstruction of Old Cadastral Boundaries by Coordinate Transformation

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Key words: Coordinate Transformation, Reconstruction of Cadastral Boundaries, Rejection of Outliers

SUMMARY

Reconstruction of old cadastral boundaries is done according to the Survey of Israel regulations by means of coordinate transformation from the old to the new Israeli grid. The transformation is based on few original marks, which survived the fast development of Israel and are capable for re-measuring in the new grid. In order to establish new transformation instructions attached to new regulations, a research was conducted for the Survey of Israel in order to improve the results of cadastral coordinate transformations. The paper describes and presents an improved practical mechanism for cadastral coordinate-transformation. The new mechanism was tested at the beginning on simulated synthetic cases and then on many of real cases. The proposed mechanism includes a uniform automatic choice of the preferred transformation type, in the case shown between shift transformation and conformal, as well as built in outlier rejection process. A special, quite surprising, unorthodox idea concerning the weighting of the original points on which the transformation is based, is discussed. This weighting is used in the least square adjustment of the transformation. Another idea is suggested in order to estimate the accuracy of the transformed coordinates of boundary points to be reconstructed. Where necessary the proposed solutions presented in the paper are simple and practical.
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1. INTRODUCTION

Coordinated Based Cadaster (CBC) is one of the most important tasks of the Survey of Israel (SOI) (Srebro, 2009). The ultimate goal is that all the cadastral boundaries will be defined by accurate coordinates for their reconstruction when necessary. Reconstruction of old cadastral boundaries is one of the main difficult tasks of the surveyors in Israel. The Israeli cadaster was established in 1920 by the British mandate in Palestine, and was based on Torrens principles. The accuracy in which the boundaries were defined is quite heterogeneous as a result of the survey methods and equipment that were in use along the years. Most of the basic cadastral boundaries were measured using the orthogonal method. New survey regulations in 1987 improved the accuracy by introducing EDM, but until 1996 the old Israeli grid (that followed the Palestine grid from the early nineteenth) prevailed. This grid was based on the old Israeli geodetic horizontal control which suffered severe problems (for details see Adler and Papo (1984) and Steinberg (2001, 2012)). The old Israeli grid was formally replaced by the new Israeli grid (Adler and Papo, 1997) with the Survey Regulations published in 1998 which was improved in 2007 by Israeli grid 2005 (IG05) based on the CORS of Israel (Steinberg and Even Tzur, 2004, 2005, Even Tzur, 2005). According to the 1998 Israeli survey regulations, reconstruction of the old boundaries should be done by coordinates shifting transformation from the old Israeli grid to the new one, based on measuring (in the new system) of at least 3 old control points, or boundary marks, or authentic objects, that were measured in the same coordinate system with the other old boundaries. A root mean square error (RMSE) of 15 cm in each direction (y or x) is acceptable. The condition for rejecting a transformation point in case that the RMSE is greater than 15 cm is that its residual is more than twice the RMSE. This condition practically means that in order to reject a point, the transformation should be based on more than 5 points. Although the boundaries were marked in the field, the surveyors can rarely find enough authentic boundary marks or a near-by control points that were used for the original measurements. It is more likely to find some objects as old buildings that survived the accelerated urban and agriculture development of Israel. After a very long time (see Steinberg, 2006) new Israeli survey regulations were published in June 2016 (Fishbein et al, 2017). During 2014 the authors conducted for SOI a research named “Optimal transform methods for achieving approximate coordinated based cadaster” (Steinberg and Even-Tzur, 2015). By approximate CBC (or ACBC), we mean that the results of the transformed coordinates should not be used for CBC without additional check related to its accuracy. As in Steinberg (1999 and 2001) and Steinberg et al (2011) we believe that a non-authentic fence should be considered as an authentic one if its measured coordinates are within the error ellipse, at a significant level of 95%, of the transformed coordinates of the original boundary mark. The results of the research were
delivered to SOI in order to be used in the new survey regulations and in the technical directions of the General Director of SOI for reconstructions of cadastral boundaries.

2. CADASTRAL TRANSFORMATION METHODS

2.1 General

Any choice of coordinate transformation should fit the nature of the distortion between the two coordinate systems. Due to the heterogeneity of the old cadastral measurements it is not right to decide and adopt just one transformation method. In order to improve results of coordinate transformation, provided that there are enough authentic points to base the transformation, it is possible to use sophisticated transformations like Aktuğ (2012) or Li et al (2013). The authors believe that due to the poor accuracy of the original measurements it is not useful to try an artificial improvement which will be very difficult to the surveyors. A slight artificial improvement is not helpful, since, as mentioned above, we believe that the cadastral boundaries reconstruction should anyhow be based on the accuracy estimation of the transformed coordinates. However, it might be helpful to improve a transformation results with regard to the residuals at the base points, the distances of transformed points from the base points and other relevant cadastral material like registered distances or geometric conditions. These ideas of improving transformation results were not part of the conducted research. Two transformation methods are relevant in those conditions: shifting transformation of 2 parameters and conformal (Helmert) transformation of 4 parameters (shifting, rotation and scale).

2.2. Criterion for choosing the preferred transformation method

It is well known that using the same number of base transformation points, the RMSE of the transformation results is lower as the number of parameters is higher. Due to this reason, an automatic preferring of conformal transformation over the shifting one is not acceptable.

In order to achieve uniform criterion for selecting the “best” transformation model, we can use the Akaike's Information Criterion (AIC) (Akaike, 1974) suggested by Felus and Felus (2009). Since in coordinate transformation we usually deal with small sample size, the second-order Akaike Information Criterion (AICc) should be used instead AIC (Burnham and Anderson, 2002). AICc is defined as
\[
\text{AICc} = n \cdot \log(v^T \cdot P_v) + 2k\left(n/(n-k-1)\right)
\]  

(1)

where \(n\) is the number of observations, \(v^T \cdot P_v\) is the weighted sum of squared residuals and \(k\) is the number of parameters. The AICc penalizes for the addition of parameters, and thus selects a model that fits well but has a minimum number of parameters. A good mathematical model is one that has the smallest AICc score.

We made mathematical experiments with real cadastral material as well as with artificial simulations. The recommendation to use the AICc in every case was based also on the results of those experiments. Although in most of the experiments the shifting transformation was preferred, there were some real and some artificial cases in which we could see a clear preference to use the conformal transformation.

3. REJECTION OF OUTLIERS

3.1 The existing rejection mechanism

As mentioned above the 1998 regulations enable just shifting transformation. There are three conditions for rejection of outliers:

a. The RMSE of a single coordinate difference (between the old and new grid) in \(y\) or \(x\) direction is more than 15 cm.

b. The residual (the difference between the average and a single coordinate difference at a base point) in any direction is more than twice the above mentioned RMSE in that direction.

c. The transformation should be based on at least 3 points.

The actual outcome of those conditions is that a base point can be rejected only if it’s residual in any direction is more than 30 cm. The first condition was arbitrary decided based on a rough accuracy estimation of the original measurements. The second condition reflects 95% statistic confidence while using many base transformation points. Since usually one rarely finds a lot of authentic points to base the transformation on, it is difficult to reject points considering that the second condition dictates mathematically that at least 6 points are needed in order to reject one.

3.2 Examination of improved rejection mechanism.

In order to improve the existing mechanism, a substitute mechanism based on the \(w\) statistic test (Baarda, 1968; Kok, 1984) was examined.
The Least Square (LS) estimation of the residuals based on the Gauss-Markov linear model is

\[ v = Ax - L = A(A^T PA)^{-1} A^T PL - L \]  \hspace{1cm} (2)

and their cofactor matrix is

\[ Q_v = P^{-1} - A(A^T PA)^{-1} A^T \]  \hspace{1cm} (3)

Were \( L \) is \( n \times 1 \) vector of observations, \( x \) is the \( u \times 1 \) unknown parameters vector, \( A \) is \( n \times u \) design matrix and \( P \) is \( n \times n \) weight matrix.

The quadratic form of the residuals, \( R = v^T P v \), follows central \( \chi^2 \) distribution if the measurements are without gross errors with \( f \) degrees of freedom, \( f = n - \text{rank}(A) \). Consider we have \( n_2 \) measurements with gross errors in the system. Let us define \( \Delta R = R - R' \), where \( R' \) is the quadratic form of the residuals of those measurements without gross errors. The quantity \( \Delta R \) is equal to (Chen et al, 1987; Ethrog, 1991)

\[ \Delta R = v^T P (E^T P Q, PE)^{-1} E^T P v \]  \hspace{1cm} (4)

and follows a non-central \( \chi^2 \) distribution with \( n_2 \) degrees of freedom. If there are no other gross errors, then \( R - \Delta R \) follows a central \( \chi^2 \) distribution with \( f - n_2 \) degrees of freedom. \( E \) is \( n \times n_2 \) zero matrix, with 1 in each column in the \( i \)th row relative to the suspected measurement that contains the gross error.

To detect gross errors by statistical test, at a certain confidence level \((1-\alpha)\), we use \( \Delta R \) in the following way (Chen et al, 1987),

\[ \frac{\Delta R}{n_2 \sigma_0^2} \geq F(\alpha, n_2, \infty) \]  \hspace{1cm} (5)

when the a-priori variance factor \( \sigma_0^2 \) is given.

If only one gross error is assumed and \( P \) is diagonal then \( \Delta R \) get the simplified form

\[ \Delta R_i = \frac{v_i^2}{\sigma_{v_i}^2} \]  \hspace{1cm} (6)

where \( \sigma_{v_i}^2 \) is the \( i \)th diagonal element of \( Q_v \) and \( v_i^2 \) is the \( i \)th component of \( v \). The above statistical tests become the well-known \( w \)-test

\[ w_i = \sqrt{\Delta R_i / \sigma_0} \geq \sqrt{F(\alpha, 1, \infty)} = N(\alpha / 2) \]  \hspace{1cm} (7)
By means of the Normal distribution or Fisher distribution tabulates we can get the threshold values for determining gross error using w-test,

\[
\begin{align*}
\sqrt{F(0.1, 1, \infty)} &= N(0.1/2) = \sqrt{2.705} = 1.645 \\
\sqrt{F(0.05, 1, \infty)} &= N(0.05/2) = \sqrt{3.841} = 1.960 \\
\sqrt{F(0.025, 1, \infty)} &= N(0.025/2) = \sqrt{5.024} = 2.241 \\
\sqrt{F(0.001, 1, \infty)} &= N(0.001/2) = \sqrt{6.635} = 2.576
\end{align*}
\] (8)

Therefore, we can use \( v_i / \sigma_{v_i} \) and Normal distribution for the w-test. If the calculated value for a certain measurement is greater than 1.96, then at a significance level of 5%, for example, there is a gross error in the tested measurement.

### 3.3 Conclusions and recommendations for rejection mechanism

Based on the mathematical experiments with real cadastral material as well as with artificial simulations, the authors recommended using the rejecting mechanism based on the described above w statistic test. However, two reservations are requested:

a. There is a need for determination of minimal RMSE which suits a rough estimation of the measuring accuracy (old and new) of the base points. This is due to the fact that usually the transformation is based on few points, and in that case, a point with a reasonable RMSE might be rejected. If the coordinates of the base points in the old grid are calculated through least square adjustment, its accepted accuracy estimation can be used in order to determine the minimal value. An alternative way is to determine the minimal RMSE according to the measurements methods and the area type. In that case reasonable values of RMSE are 10 to 20 cm for the orthogonal method, and 5 to 10 cm for the polar measurements. This reservation is similar to condition a described in 3.1 above.

b. The w criterion should be determined regarding the number of base points. For a small number of base points there is a need for larger value of w. The authors recommended using the table below which is based on the executed experiments. The w values in this table enable rejecting a point also in the case that there are less than 6 base points.
4. WEIGHTING THE BASE POINTS

4.1. Theoretic weighting principles

The usual way in a least square adjustment is to weigh the observations reciprocally to their MSE. In the case of coordinate transformation, the observations are the coordinate differences between the two grids. The MSE of this difference is composed of the sum of the MSE in the two grids. The MSE (or in other words the accuracy) of the measured coordinates in the new grid is dictated by the new regulations. The modern measuring instruments and facilities (CORS as an example) enable MSE of about 2-3 cm relatively to the nominal coordinates of the Israeli CORS. Prima facie, the accuracy estimation of the coordinates of every measured point in the old grid can be achieved within the results of a least square adjustment of the old measurements done by existing commercial software. This adjustment is done in two steps. In the first step (which is not a least square adjustment) the coordinates are computed based on scaling the measured length of every main line to the computed distance from the coordinates of its end control points. This step contains also a check of the measured distances (“fronts”) between the boundary points against their computed distances from the orthogonal measurement. The coordinates achieved in the first step use as initial (approximate) coordinates in the least square adjustment of step 2. The weights of the observations (fronts, running distance, orthogonality, and the orthogonal length) in this adjustment are given according to some previous knowledge about their accuracy. The adjustment software can consider also the accuracy of the straightness of the straight lines, and to add some constraints for parallel lines and for the width of roads. In order to get a reliable estimation of the coordinates accuracy (boundary or detail points) the accuracy of the main basic control points on which their measurement is based should be added too. In addition to the above-mentioned previous knowledge about the old measurements accuracy (based on the old regulations and equipment), the initial estimation of the distances can and should relate also to the differences between the “known”

<table>
<thead>
<tr>
<th>Number of base points</th>
<th>Rejecting criterion (w greater than)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-5</td>
<td>25</td>
</tr>
<tr>
<td>6-10</td>
<td>15</td>
</tr>
<tr>
<td>11-20</td>
<td>10</td>
</tr>
<tr>
<td>21-30</td>
<td>5</td>
</tr>
<tr>
<td>more than 30</td>
<td>3.8</td>
</tr>
</tbody>
</table>

Table 1- values of w criterion according to the number of base points
(computed) and the measured distances in step 1. It is worth noting that there are not regulations or other formal directions for computing the coordinates in the old grid, and usually the surveyors stop after the first step. As well, the existing commercial softwares do not have all the features described above.

4.2. Incompatibility (paradox) of the usual theoretic weighting principles to our goal

The purpose of the coordinate transformation is the reconstruction of the old cadastral boundary marks which were not found in the field, in their original place. An authentic mark which was found in the field will continue to use as a boundary mark. This is correct even if its old coordinates were found to be wrong (in that case the meaning of a rejected point is only that it can’t be a part of the base transformation points). If the weight of a point is given according to the accuracy of its original measuring, the higher weight will be given to the control points on which the measurement is based even if they are far away. On the other hand, an authentic boundary mark shall get a too low weight due to its poor accuracy relatively to those control points. As mentioned in the introduction it is more likely to find some objects as old buildings that survived the accelerated urban development of Israel than to find authentic boundary marks. Using the usual accuracy estimation, those base points will get the minimal weights because their accuracy (which is determined relatively to the control points) is the poorest. But, what really matters is their accuracy relatively to their close boundary points and not relatively to the far control points. The effect of this paradox is especially powerful when the surveyor can’t find enough authentic boundary marks or objects within or close to his work area, and far control (or other) points are added to the base points. In that cases, the residuals (v) at the important authentic base points might be high. The coordinates of those points in the new grid do not change by the transformation, but their relative position to their close transformed boundary points is changing. This phenomenon actually means that the reconstruction of the boundary points is wrong. Understanding that it is not right to change the position of the boundary points, which should be reconstructed as close as possible to its original place, means that the usual theoretic weighting principles are incompatible with our goal. That insight leads to the conclusion that the weight of authentic boundary points should be the highest. As well, the weight of the authentic objects should be high too with reference to the accuracy of the relative measurement to their nearby boundaries.
4.3. Practical initial weighting recommendations

Due to the above conclusion and to the difficulties of accuracy estimation described in 4.1 we recommended on a simple practical attitude for the initial weighting of the base points with regard to their identity and their location. The initial weighting is reciprocal to the square of the recommended following accuracies.

a. Base points in the transformed zone:
   - Original control point or original boundary point: accuracy of 3 cm in each direction (y or x).
   - None authentic boundary walls or fences (“compatible points”): accuracy of 5 cm in each direction.
   - Authentic objects (“details”): accuracy of 10 cm in each direction.

b. Base points out of the transformed zone: The same basic accuracy with an addition of 5 cm error per every 100 meters distance from its closer transformed boundary point to be reconstructed.

The high accuracy of the compatible points is given due to the high probability that those fences were built as a substitute to the boundary mark in the original place.

4.4. Weighting of base points belongs to the same object

When using more than one base point belongs to the same object (like four corners of the same building), a high correlation between the measurements of those points should be considered. Relating to those points without considering the fact that they are highly correlated, the results of the transformation might be affected strongly to be wrong. In an extreme case, it may cause to a rejection of important “lonely” base point. On the other hand, we can’t consider them as just one point. Finding and using the “real” correlation is too difficult. In order to keep it simple, we recommended multiplying the weight of those points by \(1/\sqrt{n}\) when n is the number of base points belongs to the same object (1 for one point, 0.71 for two points etc.).

5. ACCURACY ESTIMATION OF THE TRANSFORMED BOUNDARY POINTS

COORDINATES
This part of the research was important because, as mentioned in the introduction, the authors believe that the accuracy estimation of the transformed boundary points coordinates should be used in order to decide whether to accept an existing boundary (like fence or wall) as legal. The transformation (shifting or conformal) is done through LS adjustment using the initial weights given to the base points according to chapter 4 above. The direct results of the transformation are the transformation parameters and their estimated accuracy. In order to get a more realistic accuracy estimation of the transformation parameters ($\Sigma_\alpha$), the inverse of the normal matrix is multiplied by the a-posteriori variance of unit weight, $\hat{\sigma}_0^2$ ($\Sigma_\alpha = \hat{\sigma}_0^2 (A^T PA)^{-1}$) when it is greater than 1 ($\hat{\sigma}_0^2 > 1$) and it is not multiplied by $\hat{\sigma}_0^2$ when it is smaller than 1 (assuming the a-priori variance of unit weight set as 1). The variance-covariance matrix of the transformed boundary points ($\Sigma_\alpha$) is obtained by using the law of variance-covariance propagation

$$\Sigma_\alpha = C \Sigma_\alpha C^T$$ (9)

where $C$ is a Jacobian matrix of the function $S$.

An analysis of the results of the mathematical experiments shows that the variances of the transformed points, reflected in the matrix $\Sigma_\alpha$, are too optimistic. We concluded that it is so since they do not reflect the local old measurement accuracy of the boundary points. In order to get results that are more appropriate, we recommended adding some local MSE to those variances.

Continuing our simple and practical attitude, we recommended using the measure $\Sigma v^2/(n-2)$ derived from the LS, when $n$ is the number of base points. This measure is the variance of a single coordinate difference between the old and the new grids that reflect the local accuracy of the points.

### 6. CONCLUSION AND FURTHER REMARKS

The paper describes shortly the main features included in a research done by the authors for the Survey of Israel. The research aimed to suggest improved technical instructions for cadastral coordinate transformations, to be annexed to new survey regulations. A uniform attitude for choosing the appropriate transformation method (shift or conformal) suited to the available transformation base points is presented. An improved rejection mechanism for inappropriate base points is discussed. An unorthodox (and quite surprising) idea for weighting the base points is discussed and presented. Following some previous papers, the authors emphasize the importance of the accuracy estimation of the transformed coordinates of the boundary points to be reconstructed.

*Improved Practical Mechanism for Reconstruction of Old Cadastral Boundaries by Coordinate Transformation (8510)*

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special thought and solution are given to that issue. Acknowledging the difficulties and time consuming of developing the most sophisticated solutions, the authors preferred simple and practical ones, which will be comfortable to the Israeli surveying community. New Israeli survey regulations were published in June 2016. Those regulations dictate that any cadastral coordinate transformation should be done according to a method approved by the Survey of Israel. New technical instructions for cadastral coordinate transformation were not published yet (Feb. 2017). The authors hope that their research results and recommendations presented partly in this paper will be helpful and will be included in the new technical instructions.

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