A New Method to Check the Angle Precision of Total Stations

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Keywords: Total Station, Metrological Check, Uncertainty Equation, Directions Measurement, "Monte Carlo" Method, Scale of the Total Station

SUMMARY

This paper proposes a new method to check the directions’ (angles’) precision of a total station (TS) against its manufacturer specification. It can be performed in less than half the time of the existing ISO 17123-3 test. Additionally it is innovative as provides both the random and the systematic errors that a TS may hold. The application of the method shows that, it is reliable, advantageous and time-efficient. Only half an hour is required for a TS’s check.

TS must be checked for systematic or random errors that contribute to the angles measurements. In many countries, by law, an accreditation certification should accompany every TS which is used at the modern infrastructure constructions. The high level of the demanded accuracy leads to the imperative need to ensure the proper TSs operation.

The directions (angle) measurements are sensitive, because parameters like the nominal precision of the TS, the observer, the target and the environmental conditions are involved. More analytically the personal perception of the observer for the correct targeting, the type of target, the quality of the telescope’s lenses, the quality of the reading sensors of the TS, the sighting distance, the TS maintenance, the user treatment and the transportation conditions make the direction measurements enough sensitive.

The proposed procedure requires readings of directions (horizontal and vertical) with the inspected TS at a series of twenty targets which are established at an indoor laboratory hall. These readings are being compared with their "true" values which had been acquired by a first class (reference) TS. The number of the proper targets is resulted by a simulation of the control field, by using the "Monte Carlo" method.

Both, the systematic and random errors of the inspected TS, are being calculated by using the appropriate uncertainty equation by means of the least squares method. Moreover, the “scale” of the TS is calculated as an equation coefficient. The “scale” presents the grade of the TS’s identification with the prototype or even its regularity. Thus, not only the certification but also the accreditation of TS could be achieved. Likewise, the suitability of TS for use is confirmed, for every specific application. Also, a comparison of the proposed method with the recommended tests for TSs by ISO (ISO17123, 2001) is given.
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1. INTRODUCTION

The aim of this work is to present a method to check the angle precision of total station (TS) at an indoor control field in order to determine its systematic and random errors. The modern TSs consist of an extremely complicated system of electronic and mechanical parts (hardware) which is combined with advanced software. The structure of TSs has at least three components. The first pertains to simple checks such as centering and leveling, the second is concerned with the distance measurement and the third deals with the horizontal and vertical directions (angles) reading (Lambrou & Pantazis, 2010). These delicate parts make TSs more sensitive but ensure high measurement quality and accuracy, under the consideration of its proper operation. The proper running of TS must be checked periodically, especially today, that the nominal uncertainty of the direction reading is increased and fluctuates from ±0.5" to ±9".

The fundamental idea for the majority of the metrological tests is the comparison of a measured value with an instrument under test with the "true" value from a first order reference instrument (Woschitz et al., 2002).

It is known that the "true" value of a measurement could never be known. Therefore, values provided by reference devices or instruments include by default an uncertainty. Such instruments are, therefore, fundamentally incapable to provide the "true" values. (Doeblin, 1995)

Practically, when a measurement is repeated so many times by using the standard instrument, the measurement number is considered infinite. Then the average value of this measurement will be considered as "true" and its standard deviation approaches the zero value (Leica Geosystems, 2010b).

The standard values of the control fields are considered "true" or completely accurate (without uncertainty). Usually these values are computed by many more measurements with higher order instrumentation or by another system, which gives an order of magnitude (or more) better accuracy than the instruments under check.

The accuracy of the standard instrument must be about 10 times better than the instrument under inspection in order to ensure the credibility of the procedure (Doeblin, 1995). This is a fundamental rule, but practically often too strict, and not always applied.

In the specific case of the direction (angle) measurement the following procedure is proposed. An indoor control field is being established. The standard measurements ("true" values) of directions to specific targets are being acquired using a standard TS. These values are compared with the corresponding direction measurements which were observed by TS under examination.

The proper number of targets is calculated by using a Monte Carlo simulation. Also it is constrained by the full coverage of the TS’s disk, the time efficiency of the measurements, either by the standard and the checked TS and the satisfactory degrees of freedom for the adjustment. An appropriate uncertainty equation is used for the measurements adjustment by means of the least square method.
2. THE PARAMETERS OF THE CONTROL FIELD

In order to define the correct checking procedure it is indispensable to take into consideration or to eliminate the influence of some parameters such as the environmental conditions and the observer, the TS features and the target to be used.

The environmental conditions such as temperature, humidity and fog are parameters that influence both the sighting line and the observer. These parameters cannot be foreseen or modeled. Also the sighting distance influences the distinctness of the target and its apparent size. In the present work the influence of the environmental conditions are minimized as both the reference and the test measurements were carried out at an indoor laboratory hall.

The observer is also a variable parameter as induces systematic and random errors. The systematic part defined as the observer’s personal equation that depends on the site perception, the target recognition and the eye’s distinction for the correct pointing of the telescope’s crosshair on the target. The random part of the error depends on the observer’s diligence ability, the fatigue and finally the experience. These issues are taken into consideration for the target choice, in order to remove the main part of these errors. Additionally, it is obvious that a skilled observer should carry out the check.

2.1 The number of sightings

TS provides errors according to its manufacture defects (perpendicularity of its axis). Also a systematic error is due to the quality, the kind and the number of the electronic readers which are mounted on measuring disks as well as the software used for the measurements calculations. The modern TSs include up to 4 readers (Leica, 2009) in order to minimize this error.

A random error owed to the levelling error at the TS position. The improper levelling is corrected partially by the compensators (Leica 2010 c, Topcon 2010). The remaining error σ_t depends on the compensator’s setting accuracy (S), which fluctuates from 0.3” to 1.5”, upon TS nominal accuracy (Leica Geosystems, 2010a) is equal to

$$\sigma_t = 0.2 \cdot S$$

(1)

Thus the error on the horizontal directions reading σ_L, which depends also on the zenith angle z, is calculated according to the equation 2 (US Army, 2002).

$$\sigma_L = \sigma_t \cdot \tan z$$

(2)

Also the quality of the lenses, the crosshairs’ width and the magnification (M) that is rendered add a systematic error according to equation (3)

$$\sigma_m \approx 45''/M$$

(3)

The minimization of this error σ_d may be achieved by n repeatable measurements according to equation (4) (US Army, 2002)

$$\sigma_d = \frac{\sigma_m}{\sqrt{n}}$$

(4)

The repeatable measurements minimize all the above mentioned errors. Figure 1 presents the error σ_d in relation to the number of sightings for TSs which provide magnification 30×. Figure 1 shows that nine measurements are required in order to achieve the nominal accuracy for TS of ±0.5”and three measurements are required for TS of ±1”.

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2.2 The type of target

The type of the targets also influences the direction reading error. The target’s magnitude (relative to the sighting distance), the color, the shape, the definition of its center, the width of the incised crosshair lines are significant characteristics. The apparent width of the lines that define its center should be greater or equal to the TS’s crosshairs width. The apparent size of the target, at a distance D, is equal to its real size multiplied by the factor k (eq.5) (US Army, 2002).

\[ k = \frac{L_{tel}}{D} \]  

where \( L_{tel} \) is the focal length of the telescope.

So if the crosshair’s width is 3\( \mu \)m and the \( L_{tel} = 0.15 \)cm, then a target of 1mm width may be sighted at a maximum distance of 50m. Table 1 presents the apparent width of some targets at different distances.

<table>
<thead>
<tr>
<th>D</th>
<th>Real target line width</th>
<th>1mm</th>
<th>5mm</th>
<th>10mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>10m</td>
<td>15( \mu )m</td>
<td>75( \mu )m</td>
<td>150( \mu )m</td>
<td></td>
</tr>
<tr>
<td>30m</td>
<td>5( \mu )m</td>
<td>25( \mu )m</td>
<td>50( \mu )m</td>
<td></td>
</tr>
<tr>
<td>50m</td>
<td>3( \mu )m</td>
<td>15( \mu )m</td>
<td>30( \mu )m</td>
<td></td>
</tr>
</tbody>
</table>

**Table 1**: The apparent width of the targets’ lines at different distances

Thus, a suitable type of target must be designed to ensure the minimization of the pointing error, giving the observer a sense of uniqueness for the collimating point.

In an indoor location where the sight distances are of the order of 10m, such targets must be used so that the apparent thickness of its lines exceeds the thickness of the telescope’s crosshairs. After some experiments, the target illustrated in figure 2 was chosen. The enlargement of its center is also illustrated in the same figure. The spacing of 0.6mm between the target’s border lines at the distance of 10m displayed about 9\( \mu \)m so that it can be adequately bisected by the crosshair’s lines.
2.3 The number of targets

To calculate the minimum required number of targets, for the control field establishment, the used equation (10) was simulated $10^6$ times by using the Monte Carlo method according to the JCGM guidelines (BIMP, JCGM 101:2008, 2008, James, 1980).

**Monte Carlo methods** are a class of computational algorithms that rely on repeated random sampling to compute their results. Monte Carlo methods are often used in simulating physical and mathematical systems. Because of their dependence on repeated computation of random or pseudo-random numbers, these methods are most suited to calculation by a computer and tend to be used when it is unfeasible or impossible to compute an exact result with a deterministic algorithm.

Monte Carlo simulation methods are especially useful in studying systems with a large number of coupled degrees of freedom, such as fluids, disordered materials, strongly coupled solids, and cellular structures. More broadly, Monte Carlo methods are useful for modeling phenomena with significant uncertainty in inputs, such as the calculation of risk in business. When Monte Carlo simulations have been applied in space exploration and oil exploration, actual observations of failures, cost overruns and schedule overruns are routinely better predicted by the simulations than by human intuition or alternative "soft" methods.

The term "Monte Carlo method" was coined in the 1940s by physicists working on nuclear weapon projects in the Los Alamos National Laboratory. (Princeton, 2014).

The Monte Carlo technique is applied by following the next steps (Pantazis & Nikolitsas, 2011):

Using a set of generated samples the Probability Density Function (PDF, 2014) for the value of the output quantity $Y$ in equation (6) will be numerically approximated (Alkhatib et al, 2009).

$$ Y = f(Z_1, Z_2, ..., Z_n) = f(Z) $$ (6)

**Step 1**: A set of random samples $z_1, z_2, ..., z_n$, which have $n$ parameters, is generated from the PDF for each random input quantity $Z_1, Z_2, ..., Z_n$. The sampling procedure is repeated $M$ times for every input quantity.

**Step 2**: The output quantities $y$ will be then calculated by:

$$ y^{(i)} = f(z_1^{(i)}, z_2^{(i)}, ..., z_n^{(i)}) = f(z^{(i)}) $$ (7)

Where $i = 1, ..., M$ are the generated samples of the random output quantity $Y$.

**Step 3**: Particularly relevant estimates of any statistical quantities can be calculated.

- the expectation of the output quantity

$$ \bar{E}(f(z)) = \bar{E}(y) = \frac{1}{M} \sum_{i=1}^{M} f(z^{(i)}) $$ (8)

- the estimation of the variance of the output quantity
Concerning as import simulation data, x which are the standard values of directions and y are the modified standard values, according to the expected error of each measurement. The Monte Carlo simulation was run. The expected error of each measurement is defined as the sum of the nominal (manufacturer) uncertainty of the TS and the pointing error. The results of the simulation illustrated in Figure 3. On the Y axis the uncertainty σ in the standard deviation σο and on the X axis the number of the targets, for TSs of nominal uncertainty ±3", ±5" and ±7" are presented.

![Figure 3. The number of targets.](image)

It is obvious that, the first critical curvature change is shown at twenty targets while the next change which occurs at forty targets provides a little improvement in the error about ±0.2". So, the best option is to select the twenty targets, which also ensures the minimum measurement time and satisfies the degrees of freedom for the adjustment.

3. THE UNCERTAINTY EQUATION

A first degree equation (10) could be used in order to certify the measurement accordance between the TS under examination and the reference TS.

\[ y = a \cdot x + b \tag{10} \]

where:
- y: are the measurements from TS under test.
- x: are the corresponding standard (true) measurements from the reference TS.
- b: is the systematic error of TS under test.
- a: is the scale of TS (mathematically expresses the slope of the adjustment line).

The number of the formed equations is equal to the number of the targets used at the control field. Thus both unknowns a and b are determined by using a least squares adjustment (Math works, 2014). Moreover their uncertainties σa and σb are calculated. The standard deviation of the measurements fitting is expressed by the standard error σο of the adjustment.

It is considered that random errors follow the same distribution at the entire range of the control field.

In case that TS under examination fulfils its specifications, then σο should be less than the given nominal accuracy. If the systematic error b or the random error σο of TS is greater than the nominal TS’s accuracy, then this instrument is improper for use.
In the case that the uncertainty $\sigma_b$ of the systematic error is close or greater than the value of $b$, then $b$ value is negligible so there is no systematic error. Also TS may be used in applications when the demanded accuracy $\sigma_e$ is greater than the total error, which consists of the systematic and the random part, namely

$$\sigma_e \geq \pm \sqrt{\sigma_a^2 + b^2}$$ (11)

The scale $a$ of TS, represents the uniformity of the error’s distribution all over the measurements and should be 1 (by approximation $10^{-6}$) and its standard deviation $\sigma_a$ should be also of the same order.

In case that there is a coarse error or failure in the total station’s proper operation then the value of the scale is different of 1, while the standard deviation $\sigma_a$ has values in the range of $\pm 10^{-3}$ to $\pm 10^{-4}$. Accordingly, the $\sigma_a$ of the adjustment is greater than the total station’s nominal uncertainty.

4. EXPERIMENTAL APPLICATION

By using the above mentioned procedure four total stations were checked. The check was carried out by a skilled observer in a laboratory hall where a control field with the aforementioned presuppositions has been established. The environmental conditions were stable. As the targeting distances fluctuate from 5m to 13m, therefore the special targets of fig.2 are used to eliminate the targeting error. The check field consists of 20 targets all around per 18° in order to cover the entire horizontal measuring circle. Correspondingly 20 targets are put in the vertical direction all around per 15° for the vertical direction check, except the area under the TS where sighting is not possible.

For the acquisition of the standard values of the directions, each target is measured nine times, according to the diagram of Fig.1, by using the standard TS, Leica TM30, which provides uncertainty $\pm 0.5''$ for the directions measurements (Leica, 2009).

Four others TSs were checked, which have nominal accuracy ranging from $\pm 3''$ to $\pm 6''$. The measurements were carried out in two periods within 30 min for each TS. The result of the adjustment namely the scale $a$ for each TS, the systematic error $b$, the random error $\sigma_a$ and their uncertainties $\sigma_a$ and $\sigma_b$ are presented in the table 2.

<table>
<thead>
<tr>
<th>Total Station</th>
<th>Nominal Uncertainty</th>
<th>$a$</th>
<th>$\sigma_a$ (arcsec)</th>
<th>$b$</th>
<th>$\sigma_b$ (arcsec)</th>
<th>$\sigma_e$ (arcsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal directions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>±5''</td>
<td>1.0000008</td>
<td>±1.5·10^{-6}</td>
<td>0.4''</td>
<td>±1.3''</td>
<td>±2.5''</td>
</tr>
<tr>
<td>B</td>
<td>±5''</td>
<td>0.9999823</td>
<td>±3.6·10^{-3}</td>
<td>64.5''</td>
<td>±3.0''</td>
<td>±6.3''</td>
</tr>
<tr>
<td>C</td>
<td>±3''</td>
<td>1.0000251</td>
<td>±8.2·10^{-5}</td>
<td>-29.3''</td>
<td>±7.1''</td>
<td>±15.2''</td>
</tr>
<tr>
<td>D</td>
<td>±6''</td>
<td>0.9999993</td>
<td>±2.3·10^{-6}</td>
<td>2.4''</td>
<td>±2.0''</td>
<td>±4.2''</td>
</tr>
<tr>
<td>Vertical directions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>±5''</td>
<td>1.0000001</td>
<td>±6.1·10^{-6}</td>
<td>1.5''</td>
<td>±2.4''</td>
<td>±3.9''</td>
</tr>
<tr>
<td>B</td>
<td>±5''</td>
<td>1.0000298</td>
<td>±3.8·10^{-3}</td>
<td>18.3''</td>
<td>±4.2''</td>
<td>±7.2''</td>
</tr>
<tr>
<td>C</td>
<td>±3''</td>
<td>1.0000373</td>
<td>±5.0·10^{-5}</td>
<td>4.9''</td>
<td>±4.1''</td>
<td>±8.9''</td>
</tr>
<tr>
<td>D</td>
<td>±6''</td>
<td>1.0000009</td>
<td>±4.2·10^{-6}</td>
<td>2.1''</td>
<td>±1.3''</td>
<td>±3.6''</td>
</tr>
</tbody>
</table>

**Table 2:** The results of the TSs check

Figures 4, 5, 6 and 7 illustrate the residuals of the measurements’ adjustment for the horizontal and vertical directions of TSs A and C.
For TSs A and D is concluded that the systematic as well as the random errors of the direction measurement are within the nominal accuracies given by the manufacturer. Moreover the residuals have a smooth distribution.

On the contrary TSs B and C, represent systematic errors of 64.5" and -29.3" for the horizontal directions as the random errors are ±6.3" and ±15.2" respectively. Also at the vertical directions represent systematic errors of 18.3" and 4.9" as the random errors are ±7.2" and ±8.9" respectively, greater than the nominal accuracy limit. Additionally the residuals have a non-uniform distribution. The above results were confirmed by the official distributor of these TSs in Greece where they are delivered the next days for maintenance (Tree Company Co, 2011), in order to confirm the results.

Figure 4. The residual of the adjustment for horizontal directions of TS A

Figure 5. The residual of the adjustment for horizontal directions of TS C

Figure 6: The residual of the adjustment for vertical directions of TS A

Figure 7: The residual of the adjustment for vertical directions of TS C
5. CONCLUSIONS

By using the proposed method the achieved precision of the measured directions by the TS is calculated. Moreover by this method any systematic error, which occurs due to a damage of electronic measuring configuration, is detected by the uncertainty equation. Also the scale of the direction measurements is calculated so that the normal operation of TS is ensured. The control field installation is convenient, cost effective and quick organized.

The standard ("true") values which are indispensable for the procedure could be easily acquired by several periodical measurements using a reference TS of ±0.5” or a Laser Tracker. The proposed method is convenient and time effective as it provides immediate and reliable results.

Additionally the same simple first degree equation is used for the measurements’ elaboration of both horizontal and vertical directions. This is very practical and manageable.

As a comparison between the measurements acquired and the "true" values is applied, the accreditation of TS is carried out.

The application indicates that TSs, A and D work within their nominal accuracies. On the contrary TSs B and C are found out of their proper operation. This fact is confirmed by the official distributor where they were delivered for maintenance (Tree Company Co, 2011)

A comparison of the suggested procedure with the recommended tests for TSs by ISO (17123-3 and 17123-5) shows the following (ISO 17123, 2001).

The main advantage of the proposed method is that the ISO 17123-3 prescribes separate measurements at different control fields and different separate mathematical procedures for the horizontal and the vertical directions test respectively. The proposed method requires simultaneous measurements at the same control field and the same method of measurements’ analysis by using the least squares method adjustment of the equation y=ax+b.

Both the present process and ISO 17123-3 use the original measurements of directions to make the calculation as well as the statistical checks. On the contrary ISO 17123-5 uses the sequence result, namely the coordinates. (Nikolitsas & Lambrou, 2010)

Moreover, both process use as norm the nominal accuracy which is given by the manufacturer of TS, for all the statistical checks and the conclusions.

ISO 17123 tests underline that "these tests are intended to be field verifications of the suitability of a particular instrument for the immediate task at hand and to satisfy the requirements of other standards. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature." (ISO 17123, 2001).

Thus ISO 17123-3 only certifies the internal precision of TS by using repeatable measurements to targets without any comparison to standard values.

The field work needs about 1 hour for the measurements with the standard TS and about half an hour for any TS under check, while the ISO procedure demands at least double or triple this time. (Lambrou & Pantazis, 2004)

Consequently, it is proved that the proposed method could represent a new indoors checking methodology for the metrology test of TSs by using appropriate targets and skilful observers.

To ensure the quality of every geodetic application the appropriate choice of instrumentation is critical. For this reason the accuracy provided by TS must be reliable. The evolution of the technology in TS electronic machinery manufacturing has constrained the majority of the apparent errors. Most of them are detected and corrected automatically. Nevertheless TSs, due to their complicacy, are still sensitive to the influence of the external
parameters as the maintenance, the user treatment and the environmental conditions. Thus, the imperative need of the detection, of gross systematic or random errors remains.

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