A Principle for Determining the Optimum Surveying Accuracy

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1. INTRODUCTION

Many countries have national standards for topographic surveying and engineering surveying. These standards usually have specifications for the accuracies to be achieved and even the surveying methods to be used. While these standards provide guidance to the practitioners, the author is not aware of the principles that are followed to develop the standards. In this paper, the author attempts to introduce a principle for determining the optimum accuracy for surveying.

2. OPTIMUM ACCURACY

Is there an optimum accuracy for a surveying project? I will use tunnel surveying as an example for this discussion. Assume that we dig a tunnel from both ends A and B. With a traverse (or triangular network), we can meet at the middle point T as shown in Figure 1.

Due to the nature of the surveying methods, we normally do not end up at T. We assume that the target point surveyed from A is T1 and the target point surveyed from B is T2. We will call the difference in (X, Y) between T1 and T2 the “Delta”.

Figure 1. A tunnel surveying project
Assume that we use a low accuracy of \( \alpha_1 \) and we achieve a Delta of D1. Statistically, D1 will follow a normal distribution. Depending on the magnitude of D1, we will have different costs to address the Delta (which is a consequence of using an accuracy of \( \alpha_1 \)):

<table>
<thead>
<tr>
<th>Magnitude of Delta (cm)</th>
<th>Costs to address the Delta ($)</th>
<th>Probability of the Delta in the range of magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>0cm to 10cm</td>
<td>Cost_1</td>
<td>P_1</td>
</tr>
<tr>
<td>10cm to 100 cm</td>
<td>Cost_2</td>
<td>P_2</td>
</tr>
<tr>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
<tr>
<td>10m to 50m</td>
<td>Cost_N</td>
<td>P_N</td>
</tr>
<tr>
<td>…</td>
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Assume that we use today’s best practice for building tunnels and the best way to address the Delta. We can then determine the “mathematical expectation of the cost to address the Delta for \( \alpha_1 \)” (called E1):

\[
E_1 = \text{Cost}_1 \times P_1 + \text{Cost}_2 \times P_2 + \ldots + \text{Cost}_N \times P_N + \ldots
\]

where \( P_1 + P_2 + \ldots + P_N + \ldots = 1 \)

The probability theory tells us that when we increase the accuracy of surveying (\( \alpha_1 \) becomes smaller in magnitude), E1 should also become smaller. When we increase the surveying accuracy from \( \alpha_1 \) to \( \alpha_2 \), we have higher cost for the surveying effort (called CS2) and a smaller E2. It will make business sense to use \( \alpha_2 \), if the following holds:

\[
\text{CS}_2 - \text{CS}_1 < E_1 - E_2 \quad (1)
\]

\( (\text{CS}_2 - \text{CS}_1) \) represents the increase of cost of surveying. \( (E_1 - E_2) \) represents the savings in the cost of addressing the problems caused by the inaccuracies of surveying.
(1) Can be rearranged as:

\[ CS_2 + E_2 < CS_1 + E_1 \] (2)

It is clear that the optimum surveying accuracy \( \alpha \) is achieved when

\[ (CS + E) \text{ is minimized} \] (3)

(3) says that the best accuracy is the one that leads to the minimum combined cost of surveying and the weighted average of the cost of addressing the problems caused by the inaccuracies of surveying. There may be multiple surveying accuracies that satisfy (3).

According to (3), we need to know the following to determine the optimum accuracy:

- The cost of various applicable surveying methods and their corresponding accuracies;
- The cost of various tunnel construction techniques which can address the various magnitudes of \( \Delta \)

The costs for surveying methods and tunnel construction techniques depend on many factors such as:

- Surveying equipment costs
- Surveying labour costs
- Tunnel construction equipment and material costs
- Tunnel construction labour costs

Therefore, the optimum accuracy for the same tunnel surveying project (identical terrain, geological conditions, height, width, and length of tunnel) will vary from place to place due to the fluctuation of the above-mentioned costs.

3. DISCUSSION

So far we have limited our discussion to the horizontal accuracy of a tunnel surveying project. We can use the same approach to determine the optimum vertical accuracy for a given tunnel surveying project.

For topographic surveying, we also need to address the optimum sampling point density. Principle (3) can still be applied although we now have to compare different combinations of surveying accuracy \( \alpha \) and sampling density \( \beta \). There may be multiple pairs of \((\alpha, \beta)\) that meet principle (3).

(3) is really a generic principle that could be applied in many similar situations. We will use nuclear power station (“NPS”) safety as one example. If we design an NPS to withstand the largest earthquake-tsunami in the last one hundred years, we have a cost of \( C_1 \) for implementing
the NPS safety measure. In case a bigger earthquake-tsunami happens, we have the cost of addressing the Delta (the difference between the actual magnitude of earthquake-tsunami and the targeted earthquake-tsunami in the initial design) $E_1$. When the sum of $C$ and $E$ is minimized, we will have the most appropriate level of safety in our NPS safety design.

After the BP oil spill in the Gulf of Mexico, many people wanted the governments to regulate the oil companies more. Some people are concerned that too much regulation will increase the cost of off-shore oil exploration and production which may lead to less investment of oil companies. The “optimum level of regulation” can be achieved if (3) is satisfied. In this case, the $C$ will be the cost of regulation compliance and the $E$ will be the cost of addressing the problems that are not covered by the regulations.

4. CONCLUSION

This paper presents a principle for determining the optimum surveying accuracy. The same principle can also be applied to topographic surveying to determine the optimum surveying accuracy and point sampling density.

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