The main targets

- A-priori estimation of the measured points’ uncertainty by using the "light" laser scanners.
- Selection of the proper total station and the appropriate scanning distance according to the desired uncertainty result.
- Determination of the scanning parameters such as the scanning steps (horizontally and vertically) and consequently the maximum number of points to be measured, as well as the a-priori standard error of a geometric surface adjustment.

Monte Carlo Technique + Least square Method
"light" laser scanners

- total stations (servo or piezoelectric driven)
- No support of external batteries and individual laptop is needed
- Convenient for geometrical surface documentation
- horizontal and vertical scanning step
- Individual point measurement

<table>
<thead>
<tr>
<th>Total Station</th>
<th>Distance accuracy (RL)</th>
<th>Angle accuracy</th>
<th>Max. Scan speed</th>
<th>Range (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Topcon IS</td>
<td>±5 mm</td>
<td>±0.3mgon</td>
<td>20 points/sec</td>
<td>&lt;250</td>
</tr>
<tr>
<td>Trimble VX</td>
<td>±3 mm ±2 ppm</td>
<td>±0.3mgon</td>
<td>15 points/sec</td>
<td>&lt;150</td>
</tr>
</tbody>
</table>

THE A-PRIORI SURFACE ADJUSTMENT

\[
\delta x_i = \delta y_i = \pm \sqrt{\delta D_i^2 + \frac{D_i^2}{(\bar{D} \pm \bar{D})^2} \cdot (\delta \alpha^{2} + \delta \beta^{2})}
\]

\[
\delta z_i = \pm \sqrt{\delta D_i^2 + \frac{D_i^2}{(\bar{D} \pm \bar{D})^2} \cdot \left( \frac{\delta z_i}{\bar{z}} \right)^2}
\]

Monte Carlo technique

\[
\sigma_{x_i}, \sigma_{y_i}, \sigma_{z_i}
\]

Monte Carlo technique in the surface equation via the least square method

\[
\sigma_{i} (adjustment)
\]

\[
M=10^5
\]
The $\sigma_x$, $\sigma_y$, $\sigma_z$ values for different instruments, for distance measurement with an accuracy of $\pm 3$mm

The a-priori standard error $\sigma_0$ for the adjustment of a plan

\[ 5x + 8y - 2z = 0 \]
how many points have to be scanned on a concrete surface?

\[ \sigma_0 \]

same step, horizontally and vertically

\[ \text{step } s = \sigma_0 \cdot z_{95\%} \]

the approximate number \( n \) of the points

\[ n = \frac{\text{Area}}{s^2} \]

---

**THE A-POSTERIORI SURFACE ADJUSTMENT**

- least square method is used
- The unknown surface is expressed by a linear or no linear function of the calculated \( x, y, z \)
- The unknown parameters are the coefficients \( a_i \)

\[ A \cdot \hat{x} = l + u \]

\[ \hat{x} = \left( A^T \cdot A \right)^{-1} \cdot A^T \cdot l \]

\[ \delta_{a\text{-posteriori}} \]

\[ V_x = \begin{bmatrix} \delta_{x_1}^2 & \delta_{x_2}^2 & \cdots & \delta_{x_n}^2 \end{bmatrix} \]

**Where**

\[ A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix} \]

\[ \hat{x} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}, l = \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_m \end{bmatrix} \]
Statistical Checks

1st

\[ \hat{\sigma}_{ai} \cdot z_{95\%} \leq a_i \]

If equation is valid for all the coefficients then the surface is acceptable.

2nd

\[ \hat{\sigma}_{a posteriori} \leq \hat{\sigma}_{a priori} \cdot z_{95\%} \]

If equation is valid then the measured points belong to the geometric-mathematic model and hence the surface has no manufacture errors.

APPLICATION

satellite antenna

\[
\frac{(x_i - x_0)^2}{a^2} + \frac{(y_i - y_0)^2}{b^2} = \frac{1}{c} \cdot (z_i - z_0)
\]
The determination of the a-priori parameters

![Graph showing distance vs. adjustment with different accuracy levels.]

- Using a total station
- \( \pm 3^{cc} \) angle accuracy
- Distance = 10m
- \( \sigma_0 = 1.6 \text{cm} \)
- \( s = 1.6 \cdot z_{95\%} = 3.2 \text{cm} \)
- \( \sim 1000 \text{ points/m}^2 \)

The determination of the a-posteriori surface

*Trimble VX*

- Scanning step = 4cm
- Horizontally and vertically
- 2600 points

<table>
<thead>
<tr>
<th>a(m)</th>
<th>( \sigma_a ) (mm)</th>
<th>b(m)</th>
<th>( \sigma_b ) (mm)</th>
<th>( x_0 ) (m)</th>
<th>( \sigma_{x_0} ) (mm)</th>
<th>( y_0 ) (m)</th>
<th>( \sigma_{y_0} ) (mm)</th>
<th>( z_0 ) (m)</th>
<th>( \sigma_{z_0} ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.951</td>
<td>( \pm 0.3 )</td>
<td>0.965</td>
<td>( \pm 0.3 )</td>
<td>0.001</td>
<td>( \pm 0.0001 )</td>
<td>-0.019</td>
<td>( \pm 0.0002 )</td>
<td>0.001</td>
<td>( \pm 0.0001 )</td>
</tr>
</tbody>
</table>

- \( \pm 3^{cc} \) for the directions
- \( \pm 3 \text{mm} \pm 3 \text{ppm} \) for distance
- \( \sigma_0 = \pm 12 \text{mm} \)
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<tr>
<th>REMARKS AND CONCLUSIONS</th>
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<tr>
<td>▶ The <strong>a priori standard error</strong> of the points’ adjustment, which belong to a specific surface is strongly influenced by the accuracy that the total station provides and can estimated by using the <strong>Monte Carlo technique</strong>.</td>
</tr>
<tr>
<td>▶ Knowing the number of points, which are necessary to be captured and the desired $\sigma_0$ of the adjustment, the user can have a better understanding of what he needs to collect at the field</td>
</tr>
<tr>
<td>▶ The <strong>comparison</strong> between the a-priori and the a-posteriori $\sigma_0$ of the adjustment can document that the measured surface is <strong>constructed according</strong> to its specifications.</td>
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<td>▶ The <strong>Monte Carlo</strong> technique proved to be a very useful tool for the <strong>a-priori determination</strong> of the measurements’ uncertainty as well as the standard error of the adjustment.</td>
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<td>▶ The development of <strong>total stations</strong>, with laser scanner capability, gives the opportunity for a more <strong>economical</strong> procedure of scanning geometric surfaces compared with the real laser scanner.</td>
</tr>
<tr>
<td>▶ These instruments are more <strong>convenient</strong> when processing data when compared to the laser scanners, as a less bulky computer is needed. They are more <strong>easy to use</strong> as they are lighter and have the same on board software as the conventional surveys.</td>
</tr>
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</table>
Assessing the use of "light" laser scanners and the Monte Carlo technique for the documentation of geometric surfaces

THANK YOU FOR YOUR ATTENTION

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