Mean Sea Level tidal plane errors

CRC-SI Project 1.14:
“Reconciling Australia Height datum’s: the Vertical component”

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The problem:

Bathymetry and Topography datums don’t match:

- Bathymetry datum is CD/LAT
  - Bathymetry measured from MSL observations & adjusted down to LAT/CD
  - Varying length MSL obs. from days to decades in length
  - Varying length MSL obs. give inconsistent datums between bathymetry datasets

- Topography datum is AHD
  - AHD referenced to 1966 - 1968 MSL

Solution:

- Define a common datum with uncertainties for varying span obs.
Overview:

Method for comparing various obs. spans of MSL:

1. Select long term tide gauges for residual baseline (defining MSL datum).
   • At these gauges uncertainty = 0.000m
2. Get periodogram amplitude estimates (spectrum) of residual via Fourier Transform
3. Interpolate spectrum components to get synthetic spectrum at short span MSL obs. point.
4. Synthetic spectrum modified for short obs. span uncertainty, relative to long term baseline.

Later project/further work:
• Perform method on all varying span MSL obs. with ellipsoidal heights, collate for MSL uncertainty surface.

Data

• Modern 19 yr (1989-2007) data epoch
• Tides removed using TANS¹ analysis
  • Astronomical energy removed
• Spectrum of residuals used for uncertainties

¹ TANS: National Tidal Centre’s in house tidal analysis package
Data

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Tidal residuals

- Tide gauges have unique meteorological noise
- Parseval’s theorem > error from spectrum.
  
  $\sum_j |x_j|^2 - \frac{1}{N} \sum_f |X(f)|^2$

- Spectrums redistributed (and smoothed) into 500+ bins $R/f$ for comparison and interpolation.
Interpolating spectrum

- Interpolated baseline spectrum components to create synthetic spectrum at interpolation point.
- Assumption: Low freq. spectrum has high spatial correlation. (wide bands not individual frequencies)
- Interpolation performed for each redistributed spectrum bin \( R(f) \)
- Inverse distance weighting used (Shepard, 1968)
  - Fits data points \( R(f) \), only 66 points
  - Copes with irregular data points.
  - Very simple, smoother than linear interpolation

\[ S(x) = \sum \frac{w_k(x)}{d(x, x_k)^p} R(f) \]

- Shepard’s inverse distance weighting:
  \( x \) interpolation point
  \( w_k(x) \) weighting function
  \( S(x) \) synthetic spectrum value
  \( R(f) \) spectrum component
  \( d(x, x_k) \) distance
  \( p \) proponent = 2

Frequency based error

- Frequency contribution (1σ) in periods of 1 – 19yrs
- High error in northern regions > 0.06m
- Low error ~0.02m behind great barrier reef.
Frequency based error

- Frequency contribution (1σ) of 3 days – 19.2hrs frequencies
- High error >0.06m in upper reaches of Spencer Gulf
- Low error < 0.024m most of coastline

Moving average filter

Time domain: A moving average (MA) filter applied to tidal residual $z[n]$ is a ‘signal’ of short span MSL’s obs. RMS (standard deviation) of this signal is uncertainty of short span MSL obs. to long term average. RMS (standard deviation) can also be calculated from spectrum by applying filter in frequency domain to calculate uncertainty of ‘synthetic signal’ by modifying synthetic spectrum.
Modified spectrum

- Modify synthetic spectrum with MA filter frequency response function

\[ H(f) = \frac{\sin(Mf)}{M \sin(f)} \]

- Ampl. factor \( M \) hrs obs.
- \( f \) freq (cycles/sample)

Uncertainty (\( \sigma \)) of short span MSL:

\[ \sigma = \frac{1}{N} \sum S[f] \times H[f] \]

- \( S[f] \) Synthetic spectrum
- \( H[f] \) frequency estimate

Short span obs. uncert.

- Modified spectrum is unique to each short span MSL obs.
- Repeated for all short span obs.
- 28 day moving average filter (672 hrs)
- Broadly similar to low freq. error
- Long period tides corrected or added back.

Theoretical uncertainty of short span 28 day MSL obs. around coastline
Summary

- Baseline long term tide gauges used as MSL datum
- Synthetic spectrums are calculated around the coast.
- Synthetic spectrums are modified to produce uncertainty at short span MSL obs. points
- With ellipsoidal heights MSL observations around coastline can be greatly expanded.
- Method adaptable for AHD and older sea levels comparison.

Thank you

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