William Dawes’ Gravity Measurement in Sydney Cove, 1788

Case BOSLOPER, Australia
Clock Rate and setting of nut
Oct 1, 1788

Clock has last for the rate of 37.25 on 3rd of Oct, 1788, on one sides of the day, the mean was 15, and on the 2nd, but without 15 before the 17th. A day ago, it was 17. This goes by the Sunday to the

William Dawes
1762 - 1836
Dawes’ List of Observations

Gravity Observation by Cook

The pendulum vibrated from 2° 1° 4 to 1° 3° 9 on each side (o), which is 1° more than before; this seems owing to the weather being much warmer.
Philosophical Transactions of the Royal Society
Vol. 58 Dec 1768 Page 329

Astronomical Observations [made at ...] for determining the going of the Clock
sent thither by the Royal Society
in order to find the Difference of Gravity between the Royal Observatory at Greenwich and the place where the Clock was set up [.....].

The Shelton Astronomical Regulator Clock
Error Budget

Linearity issue
Pendulum equation

\[ T = 2\pi \sqrt{ \frac{L}{g} \left( 1 + \frac{1}{2^2} \sin^2 \frac{\theta_M}{2} + \frac{1}{2^2} \frac{3^2}{4^2} \sin^4 \frac{\theta_M}{2} + \cdots \right) } \]

\[ g = \left( \frac{4\pi^2 L}{T^2} \right) \times (1 + \Delta)^2 \]

Setting the Pendulum length
A Regulator Nut

Solve for Pendulum Lengths with EGM 2008
Mean 39.111\,\text{inches}, \text{ Std dev of mean 0.002\,\text{inch} (51\,\text{microns})}

<table>
<thead>
<tr>
<th>Places</th>
<th>Date</th>
<th>Clock gains on differential time</th>
<th>Latitude</th>
<th>I.D.</th>
<th>latitude line normal gravity</th>
<th>Addo Gravity Anomaly</th>
<th>Observed setting</th>
<th>Dev.</th>
<th>WCW observed</th>
<th>From vet</th>
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</thead>
<tbody>
<tr>
<td>Greenwich</td>
<td>March 1722</td>
<td>0.02</td>
<td>35.26.07N</td>
<td>67.98</td>
<td>0</td>
<td>0</td>
<td>931.77/27.7</td>
<td>0.00000</td>
<td>3.00</td>
<td>1.84</td>
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<tr>
<td>Akula</td>
<td>July 1722</td>
<td>0.06</td>
<td>33.25.54N</td>
<td>67.56</td>
<td>17.18.23W</td>
<td>97.705/74.10</td>
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<td>39.95.25S</td>
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<td>18.28.23E</td>
<td>97.056/31.16</td>
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<tr>
<td>Sydney</td>
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<td>166.182</td>
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<td>40.49</td>
<td>42.11</td>
<td>271.46.26</td>
<td>9.620/6305</td>
<td>0.00020</td>
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<td>39.113</td>
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<tr>
<td>Palos Verdes</td>
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<td>33.50.55S</td>
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<td>205.06.16</td>
<td>9.784/6506</td>
<td>0.00000</td>
<td>6.46</td>
<td>1.74</td>
<td>39.111</td>
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<tr>
<td>Tenerife del Hoyo</td>
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<td>0.0121.1</td>
<td>36.52</td>
<td>55.22</td>
<td>90.37</td>
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</tbody>
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FIG Congress 2010
Facing the Challenges – Building the Capacity
Sydney, Australia, 11-16 April 2010
Length change for one rev of nut

From length ratio to length difference:

\[ \frac{L_0}{L_1} = \frac{T_0^2}{T_1^2} \]

\[ L_1 - L_0 = L_1 \left(1 - \frac{T_0^2}{T_1^2}\right) \]

Estimate \( L \) with \( g/\pi^2 \)

Result: One revolution of nut is 0.0258 inch of length change for the pendulum.

London pendulum length or not?

From normal gravity ratio to pendulum period ratio:

\[ \frac{T_L^2}{T_S^2} = \frac{g_S}{g_L} \]

\[ T_L = T_S \sqrt{\frac{g_S}{g_L}} \]

Conclusion: William Dawes pendulum is too short by one revolution of the regulator nut.

Result:

Sydney Cove gravity value in 1788

979.705 gal
The Appropriate Pendulum Lengths

<table>
<thead>
<tr>
<th>Places</th>
<th>Date</th>
<th>Adopted pendulum length</th>
<th>1775 Gravity value</th>
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<tbody>
<tr>
<td>Greenwich</td>
<td>March 1772</td>
<td>39.137</td>
<td>9.81364</td>
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<td>July 1772</td>
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<td>9.79739</td>
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<td>Cape of Good Hope</td>
<td>Nov 1772</td>
<td>39.137</td>
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<td>Dusky Bay</td>
<td>April 1773</td>
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<td>Point Venus</td>
<td>August 1773</td>
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William Dawes’ Gravity Measurement in Sydney Cove, 1788

Case BOSLOPER, Australia

THE END
Supporting background information
A Six-figure relative gravity value in 1768

Page 405, Philosophical Transactions of the Royal Society, 1771

July 18, 1768. Therefore the force of gravity at Greenwich is to that at King George's Island, as 1000000 to 997075. N. M.

EGM 2008 Anomalies
Facing the Challenges – Building the Capacity  
Sydney, Australia, 11-16 April 2010

\[ y(t) = x(t) \left( 1 - e^{-at} \right) \]

\[ u(t) = \frac{1}{a} \left[ 1 - e^{-at} \right] \]

\[ y_a(t) = 9.794218 \times 10^{-7} \left( 0.081 \times 10^{-7} \right) \times 9.72 \times 6.5 \times 10^{-6} \]

\[ \beta = -\frac{1}{2} + \frac{1}{2} \gamma - \frac{1}{2} \beta_m \left( \gamma + \frac{1}{2} \beta_m \right) \]

\[ \beta_m = \frac{1}{\gamma} \left( \frac{\gamma - \beta}{\gamma} \right) \]

The re-normalized second degree natural quadratic coefficient is given by

\[ \frac{1}{\gamma} \left( 1 + \beta_m \left( \gamma + \frac{1}{2} \beta_m \right) \right) \]

\[ \beta_m = \frac{1}{\gamma} \left( \frac{\gamma - \beta}{\gamma} \right) \]

\[ \gamma = \frac{1}{\beta_m} \left( 1 + \beta_m \left( \gamma + \frac{1}{2} \beta_m \right) \right) \]

\[ b = a \left( \gamma - 1 \right) \]

\[ J = \frac{1}{\beta_m} \left( 1 + \beta_m \left( \gamma + \frac{1}{2} \beta_m \right) \right) \]

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Philosophical Transactions of the Royal Society
Vol. 58 Dec 1768 Page 329

XLIII. Astronomical Observations, made in the Forks of the River Brandwine in Pennsylvania, for determining the going of a Clock sent thither by the Royal Society, in order to find the Differences of Gravitation between the Royal Observatory at Greenwich, and the Place where the Clock was set up in Pennsylvania, to which are added, an Observation of the Rises and Eclipses of the Moon, and some Inversions of Jupiter's First Satellite observed at the same Place in Pennsylvania: By Charles Mason and Jeremiah Dixon.

Read December 9, 1768.