Establishing an Accurate Continuous Nationwide Cadastre Based on the Cadastral Triangulation Method

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INTRODUCTION

• Cadastre - method of land property registration and land parcels information

• Principles of Israeli cadastre (Torrens principles) - parcel boundaries are determined based on ground surveying

• Currently, the Israeli cadastre is based on block maps, mutation plans and parcel boundaries - and ground marks having legal validity
CADASTRAL CHARACTERISTICS

• Based on various geodetic control networks in preparing cadastral works (in the past - Cassini-Soldner, Israeli TM, local systems; presently - IG05 based on implementation of satellites)

• Low accuracy of parcel points coordinates due to systematic errors of geodetic control networks in the past

• Great difficulty in integrating adjoining blocks into a spatial cadastral continuity

• Not too many existing cadastral control points due to urban development activity and construction

POSSIBLE SOLUTION

• Transition from:
  the existing graphical based cadastre

• To
  a coordinate based cadastre
  optimal turning point positions
  improved accuracy

→→ Mathematical analytical / analogical adjustment procedures and processes
PROPOSED METHOD

Cadastral Triangulation (CT) Method:

- Global transformation of separate cadastral projects aiming to create a homogeneous seamless space by applying the Block Adjustment method by Independent Models

- The method is based on Chained Transformation by applying the Generalized Least Squares Adjustment

PROPOSED METHOD

CT Method Principles:

- Refers to separate cadastral projects (blocks and mutation plans) optimally pre-processed, determined in various coordinate systems (origin grids)

- Defines **tie points** - peripheral common turning points belonging to adjoining cadastral projects

- Defines **control ("authentic") points** - cadastral project points remained in the field and re-measured (in the target grid)
PROPOSED METHOD

MATHEMATICAL MODEL

Planar Similarity Transformation

\[
\begin{bmatrix}
Y_t \\
X_t
\end{bmatrix} = \begin{bmatrix}
a & -b \\
b & a
\end{bmatrix} \begin{bmatrix}
Y_o \\
X_o
\end{bmatrix} + \begin{bmatrix}
c \\
d
\end{bmatrix}
\]

\[y_t = F(\beta)\]

where

- \(Y_t, X_t\) - point coordinates in target grid \(\Rightarrow Y_t\)
- \(Y_o, X_o\) - point coordinates in origin grid
- \(a, b\) - parameters of scale and rotation
- \(c, d\) - shift parameters \(\Rightarrow \beta\)
### MATHEMATICAL MODEL

#### Tie Point

<table>
<thead>
<tr>
<th>Non-authentic:</th>
<th>Authentic Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{ti}^j - y_{ti}^k = 0$</td>
<td>$y_{ti}^j - y_{ti}^s = 0$</td>
</tr>
<tr>
<td>$y_{ti}^j - y_{ti}^s = 0$</td>
<td>$y_{ti}^k - y_{ti}^s = 0$</td>
</tr>
</tbody>
</table>

Where

- $y_{ti}^j$, $y_{ti}^k$ - adjusted coordinates of point “i” in parcellations “j” & “k”
- $y_{ti}^s$ - known (authentic) coordinates of point “i”

#### Authentic Point

- Non-tie: $y_{ti}^j - y_{ti}^s = 0$

### MATHEMATICAL MODEL

**Generalized LS Adjustment**

$$X\beta + Z\varepsilon + w = 0$$

Where

- $X$, $Z$ - partial derivatives
- $\varepsilon$ - point position residuals
- $w = y_{ti}^j - y_{ti}^k$ - for tie points
- $w = y_{ti}^j - y_{ti}^s$ - for authentic points
SCOPE ESTIMATION

- Approximate number of valid parcellations (blocks and mutation plans) in Israel ~ 75,000
- Number of unknown parameters per parcellation – 4/6
- Number of tie points and authentic points ~ millions
- Y & X unknown coordinates of tie points ~ millions

Estimated size of adjustment matrix:

- Number of Rows ~ several Millions
- Number of Columns ~ 300,000/450,000

REAL DATA PROCESSING
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- Group of 11 adjacent cadastral blocks (digitized from maps at scale of 1:2,500)
- Origin grid – local grid of digitization; target grid – the accurate mutation plans' grid (point coordinates serving as authentic points)
- The estimated accuracy of the digitization – 0.8 mm on the map (2 meters on the ground)
- The estimated accuracy of mutation plan coordinates - 0.1 meter

REAL DATA PROCESSING

- Two methods have been compared:
  - **Existing**: separate transformation and joining of adjacent cadastral blocks by computing the average position of peripheral common points (the current method used by the Survey of Israel)
  - **Proposed**: simultaneous transformation and joining of adjacent cadastral blocks by the CT adjustment process

- Two kinds of transformation:
  - Similarity - 4 parameters
  - Affine - 6 parameters
## REAL DATA PROCESSING

<table>
<thead>
<tr>
<th>Type of transformation</th>
<th>Existing method (meters)</th>
<th>Proposed method (meters)</th>
<th>Improvement Ratio (existing vs. proposed)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSE</td>
<td>Max residuals</td>
<td>MSE</td>
</tr>
<tr>
<td>Similarity (4 parameters)</td>
<td>1.71</td>
<td>5.15</td>
<td>0.90</td>
</tr>
<tr>
<td>Affine (6 parameters)</td>
<td>3.48</td>
<td>13.04</td>
<td>0.61</td>
</tr>
</tbody>
</table>

**Notes:**

1. Residuals of the existing method - differences between positions of transformed peripheral block points and their average positions; MSE - sum of squared differences divided by number of points
2. Residuals and MSE of the proposed method – from adjustment process

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**Graph:**

- MSE and max residuals for Similarity (4 parameters) and Affine (6 parameters) transformations are shown.
- The proposed method shows improved residuals compared to the existing method.
CONCLUSION

Applying CT method enabled to:

- Convert separate cadastral blocks digitized in origin (local) grid into cadastral continuity in a uniform geodetic target grid
- Reduce considerably the position discrepancies between adjoining cadastral blocks
- Increase the position accuracy of parcel boundary turning points compared to the existing boundaries matching methods

FUTURE WORK

An additional study is planned to analyze:

- The optimal number of transformation parameters referring to separate blocks during the global transformation and the adjustment of the adjoining boundaries
- The optimal scattering scheme and number of authentic points in the adjusted area
- How to improve the computational algorithm in order to deal with tens and hundreds of thousands of unknowns on a nationwide implementation process
Thank You