Advantages of Using the Mechanics of Continuum to Geometrical Analyse Deformations Obtained from Geodetic Survey

Milan TALICH, Czech Republic

Key words: deformation analysis, displacement vectors, mechanics of continuum

SUMMARY

The paper demonstrates why applying of the theory of continuum is useful to geometrical deformation analyses obtained from repeated positional survey in geodesy. The mechanics of continuum may be applied as basis of communication during the multidisciplinar approach to geodetic and geotechnic monitoring. It serves also as a technological and scientific communication basis between geodesists and specialists of other professions as geotechnics, geophysics, building engineers etc.

The independency of resulting deformation parameters to applied coordinate frame is shown, too. It is not necessary to try to find any conditions of placing the survey network in the coordinate system but the calculation should be adjusted as free network. Then errors originated from erroneous pre-requisites about stability of some selected points that are taken during normal calculation as stable (in the stable part of such location) will be completely eliminated. When compared to the sole listening of displacements this procedure enables to present deformation parameters in much more objective way and serves as a tool to demonstrate the relatively geodynamical trends of the territory in question.

SUMMARY

Příspěvek ukazuje proč je výhodné užití teorie mechaniky kontinua pro analýzy deformací z opakovaných polohových geodetických měření. Mechaniku kontinua lze použít jako základ pro komunikaci v multidisciplinárním přístupu ke geodetickému a geotechnickému monitorování. Slouží tedy jako technologická a vědecká báze pro komunikaci mezi geodety a odborníky jiných profesí jako jsou geotechnika, geofyzika, stavební inženýrství atd.

Poukazuje se také na nezávislost výsledných parametrů deformací na použitém souřadnicovém rámci, kdy se není třeba zabývat tím, jaké volit při vyrovnání podmínky pro umístění geodetické sítě v souřadnicovém systému, je pouze třeba dodržet výpočet vyrovnání sítě jako sítě volné. Eliminují se tak zcela chyby pramenící z mylných (chybných) předpokladů o stabilitě některých vybraných bodů, které při běžných výpočtech posunů z opakovaných měření považujeme za stabilní (ve stabilní části lokality). V porovnání s pouhým výpočtem posunů to znamená, že parametry deformací jsou mnohem objektivnějším nástrojem pro odhalení reálných relativních geodynamických trendů ve vyšetřovaném území.

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1. INTRODUCTION

More and more results of geodetic activity are used in other disciplines. Such cases are, e.g., geodynamic research, where geodesy offers objective and relatively sufficiently exact information about motions and shape and dimension changes of the terrestrial surface or building constructions, generally speaking of some monitored object or locality. Such information is used to further studies, physical interpretations and determining causative factors. That means there is a multidisciplinary lead approach to solving such problems. Such approach requires creating of technological and scientific base for communication between specialists of different professions that will be suitable to all of them. In (Szostak-Chrzanovski et al. 2006) is proposed the mechanic of continuum to this purpose as support of deformation monitoring, their analysis and interpretation. Let us show other reasons and benefits leading to applying this theory in geodetic praxis.

Homogeneous territory or object in question is the required condition, of course. It means that if such condition is not fulfilled, then only division of the whole territory to several homogeneous parts must be tried and calculations has to be done for each one of them separately.

Analysis of deformations of the terrestrial surface or of building constructions does not belong to problems that many geodesists solve in their everyday praxis. The main goal of deformation analysis is determination of deformation mechanism if we look at the object as at the mechanical system under deformations according the laws of mechanics of continuum. After such geometric analysis physical interpretation of results may follow.

Geometrical analysis, describes the change in shape and dimensions of the monitored object, as well as its rigid body movements (translations and rotations). The goal of the geometrical analysis is to determine in the whole deformable object the displacement and strain fields in the space and time domains.

Physical interpretation is based on the relationship between the causative factors (loads) and deformations.

Geodetic monitoring of deformations should follow in two steps. The first one is determination of displacements of selected points on the object in question (standard task). After it determination of displacement field in continuous form by their interpolation (generalized task) can follow. The second step is determination of deformations parameters by geometric analysis of continuum mechanics (strain analysis). Determination of displacements in the first step is usually done by repeated observation of geodetic network. But realisation of the second step is not so much usual among geodesists. Nevertheless strain analysis offers many conveniences.

2. GEOMETRIC ANALYSIS OF DEFORMATIONS BY CONTINUUM MECHANICS

As it was told earlier, the principle of geodetic methods applications is based on repeated measurement and comparison of results of individual stages of measurements. Obtained differences in positions of points represent their displacements. The vector of point displacement is

$$\mathbf{d}_i = (u_1, u_2, u_3)_i^T = \mathbf{x}_i^o - \mathbf{x}_i^t$$

Where $\mathbf{x}_i^o$ (resp. $\mathbf{x}_i^t$) is the vector of P_i point coordinates of fundamental (resp. actual in t-time) stage. This vector may be expressed as a function of coordinates:

$$\mathbf{u} = (u_1, u_2, u_3)^T = \mathbf{u}(\mathbf{x}) = (u_1(\mathbf{x}), u_2(\mathbf{x}), u_3(\mathbf{x}))^T = \mathbf{d}, \quad \mathbf{x} = (x, y, z)^T$$

The strain tensor in P_i is defined as a gradient of the function in this point:

$$\mathbf{E}_i = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{pmatrix}_i = \text{grad}(\mathbf{d}_i) = \begin{pmatrix} \frac{\partial u_1}{\partial x} & \frac{\partial u_1}{\partial y} & \frac{\partial u_1}{\partial z} \\ \frac{\partial u_2}{\partial x} & \frac{\partial u_2}{\partial y} & \frac{\partial u_2}{\partial z} \\ \frac{\partial u_3}{\partial x} & \frac{\partial u_3}{\partial y} & \frac{\partial u_3}{\partial z} \end{pmatrix}_i$$

In the displacement field is valid next relation, see (Welsch 1983):

$$\mathbf{d}_i = \mathbf{E}_i \, \mathbf{x}_i + \mathbf{t}$$

where:
- $\mathbf{d}_i$ is the displacement vector,
- $\mathbf{E}_i$ is the displacement gradient,
- $\mathbf{x}_i$ is the coordinate vector,
- $\mathbf{t}$ is the vector of translation elements.

The strain tensor may be divided into two parts:

$$\mathbf{E}_i = \mathbf{e}_i + \mathbf{\Omega}_i = (e_{jl})_i + (\omega_{jl})_i \quad j,l = 1,2,3$$

where:
- $\mathbf{e}_i$ is the symmetric tensor of deformation,
- $\mathbf{\Omega}_i$ is the antisymmetric tensor of rotation,
- $e_{jl} = (\varepsilon_{jl} + \varepsilon_{lj}) / 2$,
- $\omega_{jl} = (\varepsilon_{jl} - \varepsilon_{lj}) / 2$.

$$\mathbf{e}_i = \begin{pmatrix} e_{11} & e_{12} & e_{13} \\ e_{12} & e_{22} & e_{23} \\ e_{13} & e_{23} & e_{33} \end{pmatrix}_i = \begin{pmatrix} \varepsilon_{11} & \frac{1}{2}(\varepsilon_{12}+\varepsilon_{21}) & \frac{1}{2}(\varepsilon_{13}+\varepsilon_{31}) \\ \frac{1}{2}(\varepsilon_{12}+\varepsilon_{21}) & \varepsilon_{22} & \frac{1}{2}(\varepsilon_{23}+\varepsilon_{32}) \\ \frac{1}{2}(\varepsilon_{13}+\varepsilon_{31}) & \frac{1}{2}(\varepsilon_{23}+\varepsilon_{32}) & \varepsilon_{33} \end{pmatrix}_i$$

$$\mathbf{\Omega}_i = \begin{pmatrix} 0 & \omega_{12} & \omega_{13} \\ -\omega_{12} & 0 & \omega_{23} \\ -\omega_{13} & -\omega_{23} & 0 \end{pmatrix}_i = \begin{pmatrix} 0 & \frac{1}{2}(\varepsilon_{12}-\varepsilon_{21}) & \frac{1}{2}(\varepsilon_{13}-\varepsilon_{31}) \\ -\frac{1}{2}(\varepsilon_{12}-\varepsilon_{21}) & 0 & \frac{1}{2}(\varepsilon_{23}-\varepsilon_{32}) \\ -\frac{1}{2}(\varepsilon_{13}-\varepsilon_{31}) & -\frac{1}{2}(\varepsilon_{23}-\varepsilon_{32}) & 0 \end{pmatrix}_i$$

It could be written:

$$\mathbf{d}_i = (\mathbf{e}_i + \mathbf{\Omega}_i)\,\mathbf{x}_i + \mathbf{t}$$

We could determine the deformation parameters from $\mathbf{e}_i$ and $\mathbf{\Omega}_i$. This may be dome as a 3D solution as well as in plane. Such plane could be e.g., parallel with XY or XZ or YZ or in a more general way any plane of the local coordinate system to which space displacements will be projected.

It holds e.g., for displacements projected to XY of the local coordinate system:

$\Delta = e_{11} + e_{22}$	- total dilatation
$\gamma_1 = e_{11} - e_{22}$	- shear strains
$\gamma_2 = 2e_{12}$	- shear strains
$\gamma = \sqrt{\gamma_1^2 + \gamma_2^2}$	- total shear
$\varepsilon_1 = \frac{1}{2}(\Delta + \gamma)$	- axis of maximum strain
$\varepsilon_2 = \frac{1}{2}(\Delta - \gamma)$	- axis of minimum strain
$\varphi = \frac{1}{2}\mathrm{arctg}\left(\frac{\gamma_2}{\gamma_1}\right)$	- direction of axis of maximum strain
$\psi = \varphi + \frac{1}{4}\pi$ for $\omega_{12} > 0$	- direction of shear strain
$\psi = \varphi - \frac{1}{4}\pi$ for $\omega_{12} < 0$	- direction of shear strain

Theoretical solution and derivation of these formulas in question may be found in many publications — e.g., (Szostak-Chrzanovski 2006), (Altiner 1999), (Talich 1994) and (Talich, Kostelecký, Vyskočil 1993).

3. DEFORMATION PARAMETERS CHARACTERS AND BENEFITS RESULTING FROM THEM

It is worth noting that all displacements depend on selected coordinate frame. On contrary, all **deformation parameters** of last equations except the ψ and φ directions **are on used coordinate frame independent**, and insensitive to translation and rotation. And this is the reason why deformation parameters and their applications are important in practice.

In the case of point displacements calculation the resulting character of displacements is quite unambiguously given by the applied coordinate frame or by conditions of geodetic network placing in the coordinate frame.

The first condition that has to be fulfilled is net adjustment as free network to prevent its scale changes or even its deforming. Usually selected points that are expected to be in the stable part of location are chosen like fixed. More exactly said: in the case of free networks, these points are chosen like points included at the condition of being placed in the coordinate frame (e.g. by selecting among identical points during the Helmert transformation). Selecting of such points is usually based on expected physical properties of the locality. Nevertheless we are never quite sure that our expectations about points stability are good and well fulfilled.

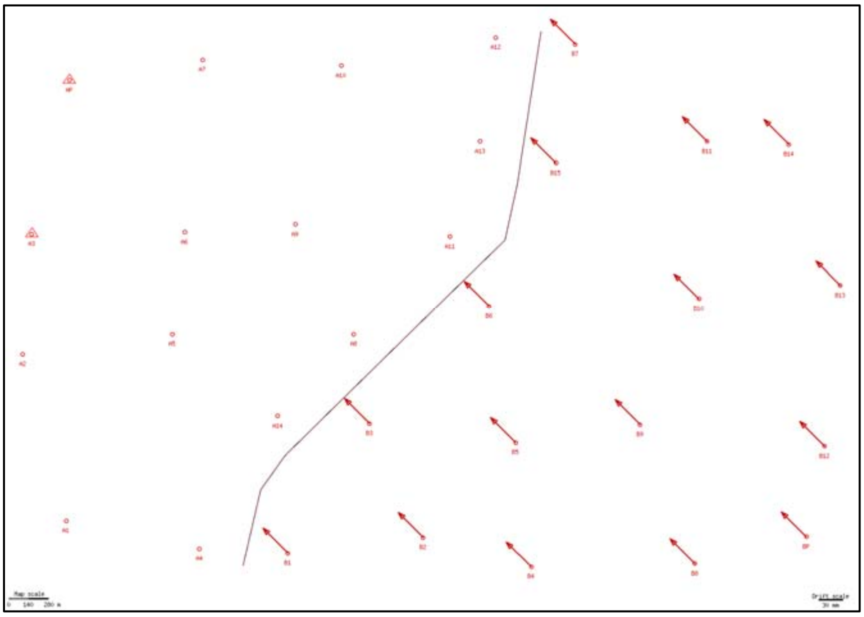

Figure 1: Fixed point AP, A3

The situation may be shown on models. Let us expect a fault in the locality in question and moving of two quite rigid plates again each other (this may happen e.g. by plunging of one plate under the second plate). The following figures 1 to 3 show such model situation. There are three different ways of fixed point chosen leading to quite different character of calculated displacements of determined points of the same network. Points that are selected as fixed points and that are supposed to be in the stable part of location, are always marked by red triangle. To accentuate character of determined displacements there are on following figures 4 to 6 also interpolated displacements fields represented. As seen from pictures, following physical interpretation of such calculated displacements could lead to quite erroneous or contradictory results. And this all is nothing else than influence of calculation and data selecting.

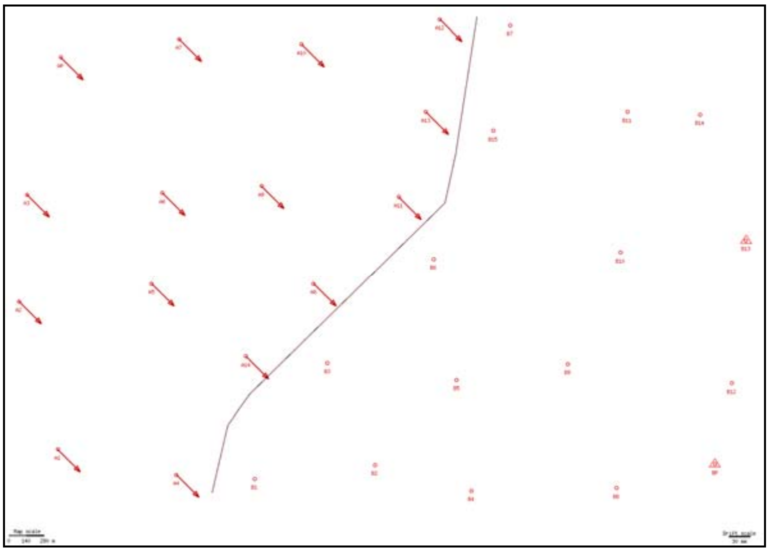

Figure 2: Fixed point BP, B13

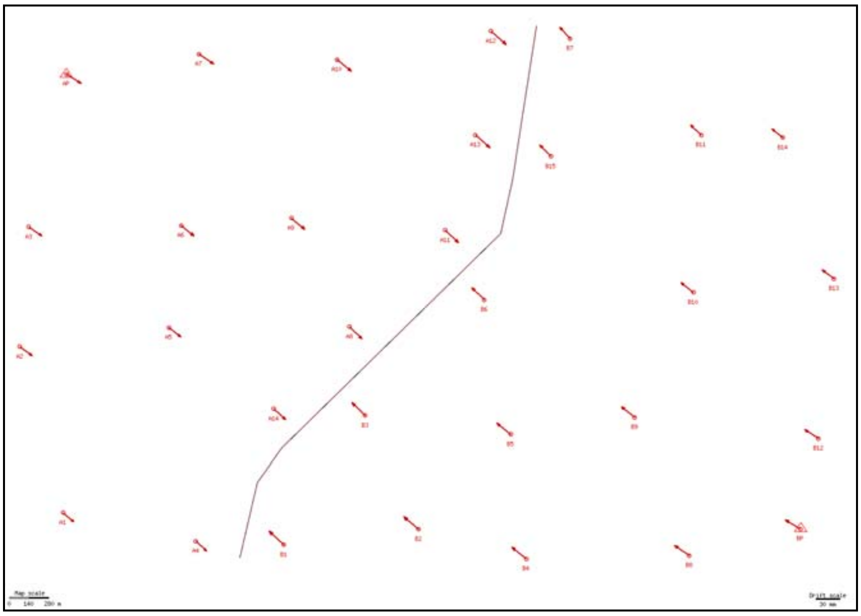

Figure 3: Fixed point AP, BP

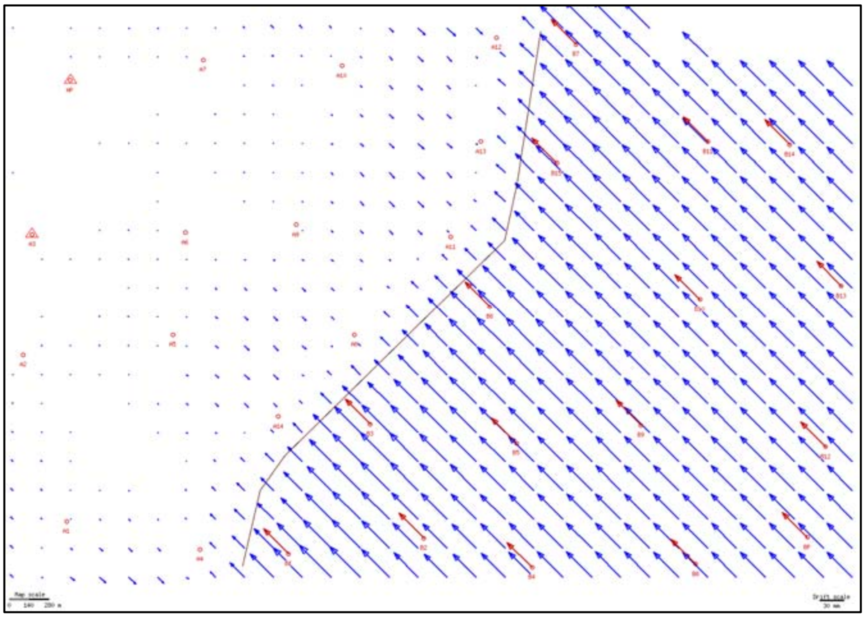

Figure 4: Fixed point AP, A3

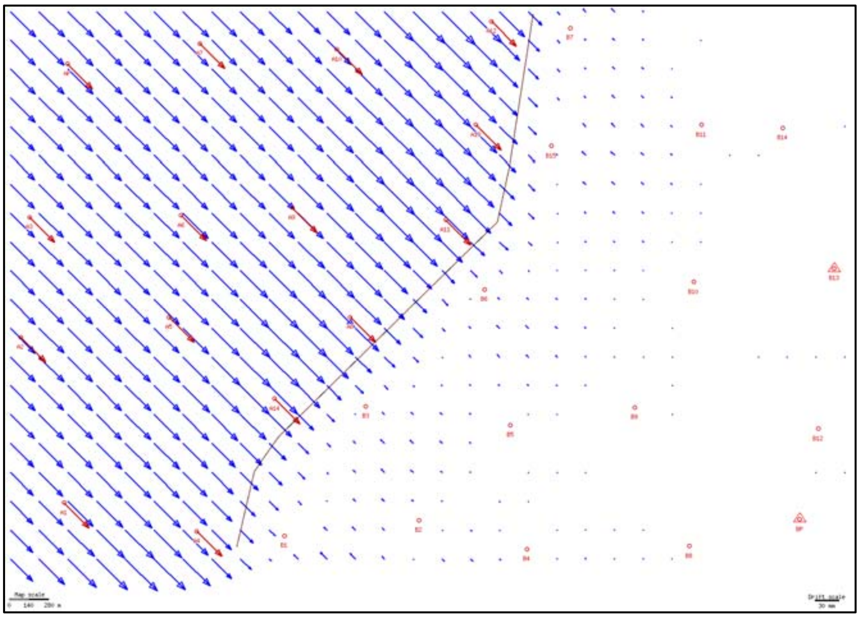

Figure 5: Fixed point BP, B13

At this moment it will be quite convenient to go to the second step, determination of deformation parameters according the mechanics of continuum. With regard to their independency to the selected coordinate frame and insensitivity to translation or rotation we will obtain from all possible variants of displacements calculation always the same deformation values. It means that it is not necessary to deal with conditions of placing the geodetic network in the coordinate frame (fixed points declaration). In other words, which points should be declared as fixed or not. The only remaining thing is calculation of network adjustment as free network, i.e. to choose only necessary number of conditions to their placement in the coordinate frame and thus not “deform” the adjusted network. Such conditions are in the case of horizontal network with given dimension (at least one measured length) three and in the case of horizontal network without given dimension four. Different condition selections do not mean anything else in the case of free network than application of rotation and translation and calculation of deformations does not depend on it, as was already said. The above mentioned is given on figure 7 which is common to all variants of calculated displacements.

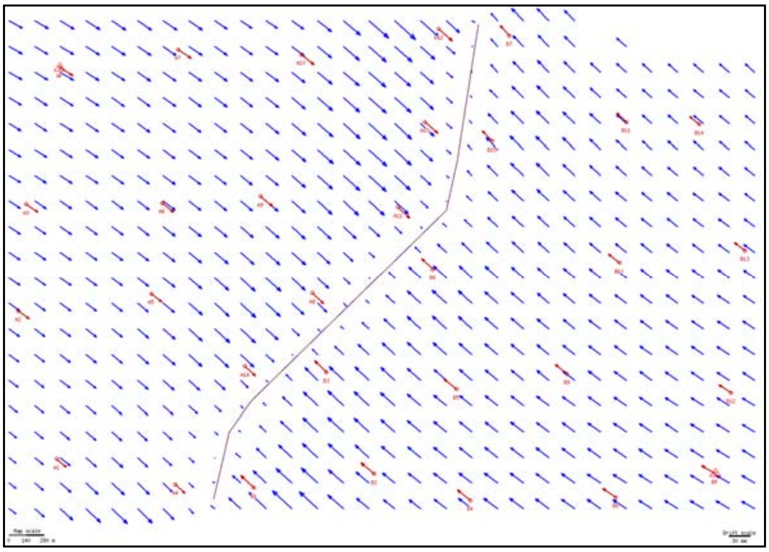

Figure 6: Fixed point AP, BP

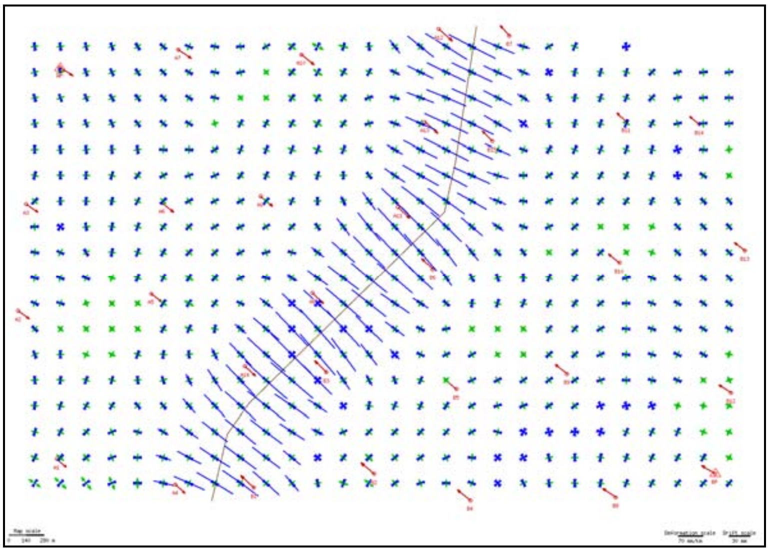

Figure 7: Deformations parameters are the same for all variants of fixed points

This means in practice that errors from bad (erroneous) expectations about stability of some selected points, that we consider at common calculations of displacements from repeated measurement as stable (in the stable part of locality), are totally eliminated.

As such a mistake we could consider for instance GPS antenna exchange (change of phase centre of a new antenna against the old one) of a permanent station at access point of the GPS net. Even this mistake will be during calculation of deformation parameters totally eliminated provided that it leads to the shift of whole network and this point is not included into calculation of the field of displacements and deformation.

The other advantage is the fact that it is not necessary (regarding calculation of deformation parameters) to discuss the problem of transforming displacements given e.g. in coordinates frame ITRF into ETRF, or to reduce displacements in ITRF by movements of tectonic plate according to some of geodynamic models as e.g. APKIM2000 (Drewes and Angermann 2001) or NNR-NUVEL (Shuanggen and Wenyao 2004).

In our model case it is possible to deduce the real geodynamic activities based on given deformation parameters. Above all, the real situation is disclosed, i.e. movement of two relatively consistent plates one against the other with unambiguously defined process of fault in the territory, as described in figure 7.

That means that, contrasting to displacements, deformations represent objective tool for disclosure of real relative geodynamic trends in the researched territory.

4. PRACTICAL EXAMPLE

Deformation network GEOSUD in the area of Polish Sudetes and the Fore-Sudetic Block discussed in (Cacon et al. 2005) is chosen as a practical example. It is geodynamic network of repeated GPS measurements. Deformation parameters calculations are made by support of web application (Talich and Havrlant 2008) accessible at http://www.vugtk.cz/~deformace/ .

In figure 8, resulting displacements are stated in coordinate frame ITRF 2000. In figure 9 “residual” displacements after their reduction using model APKIM2000 are displayed. All displacements in ITRF are approximately of the same size, 24 to 27 mm /year, and approximately of the same direction. On the other hand, “residual” displacements after reduction to local system are of different character with size from 0,3 to 3,7 mm / year and different directions. These trends are further emphasized by displacements field, where displacements on measured points are displayed in red colour and displacements stated by their interpolation in grid in blue colour.

However, both displacements values lead to the same field of deformation displayed in figure 10. The independence of deformation parameters on translations and rotations was thus proved by practical calculation, in this case represented by reduction in accordance with model APKIM2000. In other words, displacements reduction from ITRF2000 into local system for disclosure of real relative geodynamic trends was not necessary.

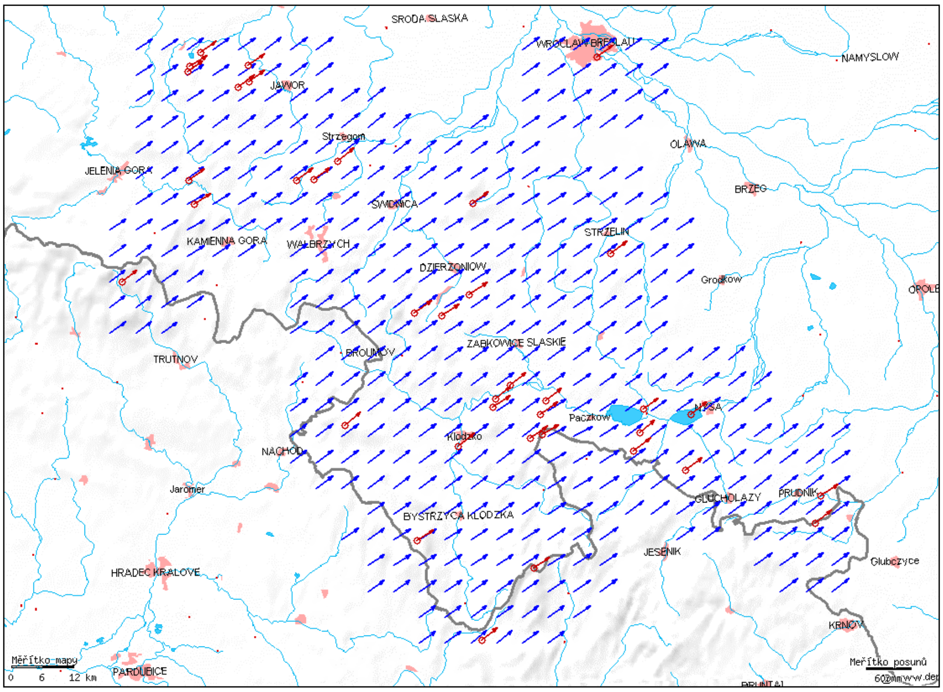

Figure 8: GEOSUD network, displacement field in ITRF 2000

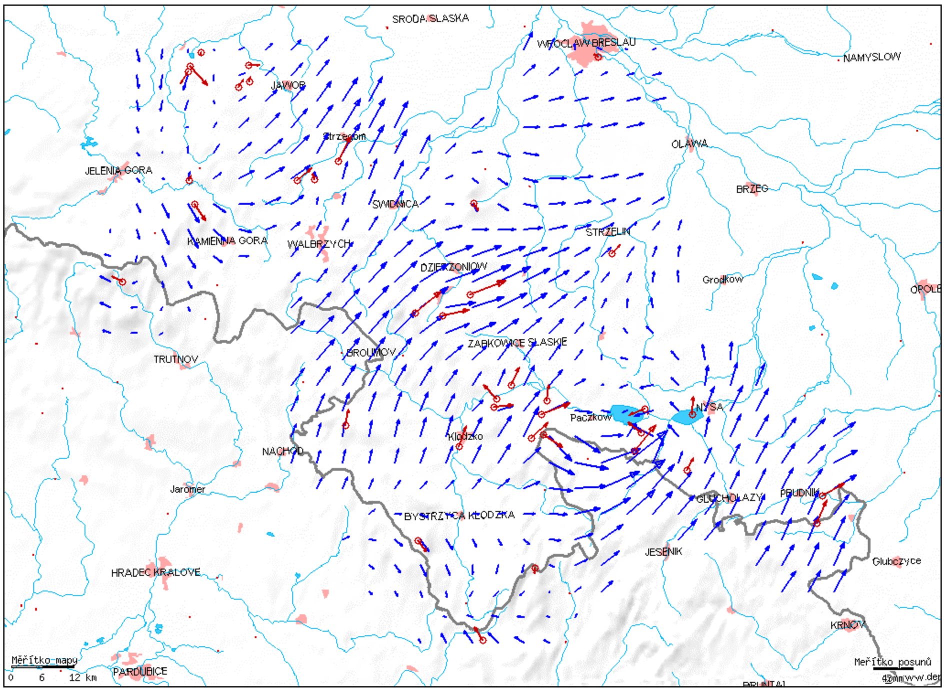

Figure 9: GEOSUD network, displacement field in “local” system

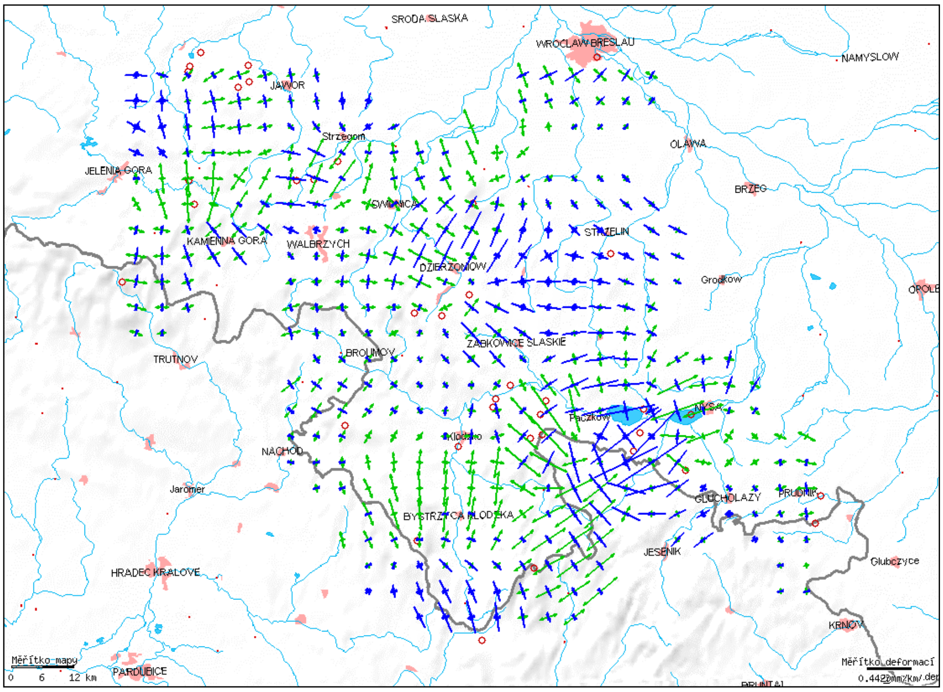

Figure 10: GEOSUD network, deformation parameters

5. CONCLUSION

The aim of this article was to show the practical advantage of the usage of theory of mechanics of continuum for disclosure of real relative geodynamic effects in researched territories, eventually of dynamics researched entities or construction structures. Mechanics of continuum give results, that are better understandable even for experts from different fields than geodesy. Therefore it can serve as technological and scientific basis for communication among experts from different professional fields. Besides that, its usage also eliminates some impacts of measurement and calculations during geodetic networks processing, which enables access to more credible and objective results, especially in comparison with pure displacements determination of points in the monitored object.

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BIOGRAPHICAL NOTES

Milan Talich was graduated from the Czech Technical University in Prague, Faculty of Civil Engineering, Department of Geodesy and Cartography. Then he was engaged in the Research Office of Geodesy, Topography and Cartography (VÚGTK), working since 1987 in the International Centre on Recent Crustal Movement (ICRCM) at geodetic networks processing and geodynamic problems. In 1992 - 1993 he was at one-year stage in the Institute of Applied Geodesy in Frankfurt/Main (IfAG), where he compiled the Czech, Slovak and Hungarian parts of the European Reference System (EUREF) GPS network. Then he was engaged in solutions of GPS processing oriented to geodynamic applications and to questions related to deformation analysis of those data. Since 1995 he solved some problems of informatics in the Branch Information Centre (ODIS) of the VÚGTK. At present he works at information systems oriented to web information as a leader of ODIS. In 2002, Milan Talich presented his doctorial thesis "Information Systems to On-Line Providing of Geodetic Information and Their Creation".

CONTACTS

Ing. Milan TALICH, Ph.D.
Research Institute of Geodesy, Topography and Cartography
Ustecka 98,
CZ-250 66, Zdiby
CZECH REPUBLIC
Tel. +420 284 890 515
Fax +420 284 890 056
Email: Milan.Talich@vugtk.cz
Web site: www.vugtk.cz