Estimation of Parameters of Multiplicative Exponential Function Model for Real Estate Market Value Prediction

Anna BARANSKA, Poland

Key words: market value, multiplicative model, exponential function, prediction

SUMMARY

The basis of modelling real estate unit values is information on prices and qualities of real estates as subject of market turnover or on market values of representative real estates and their attributes. Such a data set constitutes a representative basis of real estates for analysing the market, meeting all criteria of multidimensional random variable.

Among different non-linear functions of many variables, in modelling of a real estate unit price or value, multiplicative exponential function is chosen to be examined in relation to the particular attributes. Exponential form of the function assures positive values of real estates and it permits to describe their variability as a monotone function.

The model of real estates unit values in form of multiplicative exponential function was analysed as follow:

$$c = B_0 \cdot B_1^{x_1} \cdot B_2^{x_2} \cdot \cdots \cdot B_m^{x_m}$$

where:

c – real estate unit price or unit value,

$x_1, x_2, ..., x_m$ – real estate attributes, including the attribute of *transaction time*,

B_j – estimated model factors,

B_0 – real estate unit value, for zero of all attributes.

Estimation of model parameters may be done in many ways. In the present paper, the run system for Markow method (in form of least squares weighing method) will be submitted. Verification of estimated model proceeds as follows:

a) investigation of model acceptability regarding the values of factors variability as well as the convergence,

b) analysis of model factors significance,

c) analysis of random components symmetry.

STRESZCZENIE

Podstawą modelowania jednostkowych wartości nieruchomości są informacje o cenach i cechach nieruchomości, będących przedmiotem obrotu rynkowego lub o rynkowych wartościach nieruchomości reprezentatywnych i ich atrybutach. Zbiór takich informacji stanowi reprezentatywną bazę nieruchomości dla analizowanego rynku, która spełnia wszystkie kryteria zmiennej losowej wielowymiarowej.

Spośród różnych nieliniowych funkcji wielu zmiennych, w modelowaniu wartości nieruchomości, będzie rozpatrywana multiplikatywna funkcja wykładnicza względem poszczególnych atrybutów. Wykładnicza postać funkcji zapewnia dodatnią wartość rynkową nieruchomości oraz pozwala opisywać zmienność w formie funkcji monotonicznej.
Rozpatrywany był model w formie następującej multiplikatywnej funkcji wykładniczej:

$$c = B_0 \cdot B_1^{x_1} \cdot B_2^{x_2} \cdot \cdots \cdot B_m^{x_m}$$

gdzie:

c – jednostkowa cena lub wartość nieruchomości,

$x_1, x_2, ..., x_m$ – atrybuty nieruchomości, w tym atrybut *czas transakcji*,

B_j – szacowane współczynniki modelu,

B_0 – jednostkowa wartość nieruchomości, dla zerowych wartości wszystkich atrybutów.

Estymacja parametrów modelu może być wykonana różnymi metodami. W niniejszym opracowaniu przedstawiony został schemat postępowania dla metody Markowa (w postaci ważonej metody najmniejszych kwadratów). Weryfikacja wyestymowanego modelu przebiega następująco:

a) badanie dopuszczalności modelu ze względu na wartości współczynników zmienności oraz zbieżności,

b) badanie istotności współczynników modelu,

c) badanie symetrii składnika losowego.

Estimation of Parameters of Multiplicative Exponential Function Model for Real Estate Market Value Prediction

Anna BARANSKA, Poland

1. INTRODUCTION

The modelling of real estates unit values is based on the information about prices and features of real estates as subject of a market turnover or on market values of representative real estates and their attributes. Such a data set constitutes a basis for constructing a representative database of real estates for the analysed market. This database should meet the following conditions:

- the number of real estates should be greater than 30,
- an admissible difference between the mean price of all gathered data from the analysed market and the mean price of real estates from the representative database should not exceed 30 % of its value,
- an admissible difference between the standard deviation of prices of all gathered real estates from the analysed market and the standard deviation of prices of real estates from the representative database should not exceed 50% of its value,
- the number of real estates from the whole set of data acquired on the analysed market, which prices are contained in the intervals: $\hat{c} \pm \sigma$ and $\hat{c} \pm 2\sigma$ (where $\hat{c}$ is a mean value, and σ is a standard deviation), should be proportional to the number of real estates in the representative database in corresponding intervals.

The representative database of real estates for an analysed market satisfies all criteria of multidimensional random variable. For modelling a multidimensional random variable represented by a price or a value of a real estate and, at the same time, by all attributes of a real estate, linear models as well as different non-linear models can be used.

The previous methods of estimating real estates assumed "a priori" that a market value was created as a sum of the attribute shares. These shares could be presented by different elementary functions, also non-linear. This assumption is satisfied especially by multiple linear regression models. Some questions arise: is the additive form of the market value variability model for all real estates markets optimum, i.e. can the multiplicative models give higher accuracy of prediction results? In order to answer this question, we should make a comparative analysis of different models, for different markets of real estates.

Among different non-linear functions of many variables, a multiplicative combination of exponential variability's of the particular attributes was analysed. An exponential form of the function guarantees always-positive market value of real estates for particular attributes and permits to describe their variability as a monotone function (always growing or always decreasing). An additional advantage of this model is the possibility to present the market variability in percentages or to show the cadastral value of a real estate in relation to its value at determined attributes (zero for example).

Farther, we will analyse a model as a multiplicative exponential function:

$$c = B_0 \cdot B_1^{x_1} \cdot B_2^{x_2} \cdot \cdots B_m^{x_m} \tag{1}$$

where:

c – unit price or value of a real estate in database,

$x_1, x_2, \ldots x_m$ – attributes of real estates in database, including the attribute of *transaction time*,

B_j – estimated model coefficients,

B_0 – unit value of a real estate, for zero of all attributes.

Coefficients B_j take on the values close to unity and they have the nature of factors, which multiply the basic value of a real estate with zero attributes. If we subtract 1 from the coefficient B_j, this difference determines the coefficient of change of a real estate value for a unit of j-th attribute.

The estimation of the model parameters (1) can be done by different methods, i.e. using different algorithms or in other words, applying different objective functions. The following estimation methods can be applied:

- least squares method,
- Markow method (in the form of least squares weighing method),
- maximum likelihood estimation,
- moments method.

In the present paper, the run system for the least squares weighing method used for the estimation of model parameters will be submitted (1).

2. ESTIMATION OF MODEL PARAMETERS USING THE LEAST SQUARES WEIGHING METHOD

For the estimation of the coefficients B_j, the function (1) has to be brought to a linear form. For this purpose, we take the logarithms of both sides, using the natural logarithm, and we receive:

$$\ln c = \ln B_0 + x_1 \cdot \ln B_1 + x_2 \cdot \ln B_2 + \ldots\ldots\ldots\ldots + x_m \cdot \ln B_m \tag{2}$$

The system of equations (2) has the features of a probabilistic model, taking on in matrix notation the following form:

$$\{\ln \delta\} + \{A\} \cdot \{\ln B\} = \{\ln c\} \tag{3}$$

where:

$\{A\}$ - rectangular vertical matrix $[n \times (m+1)]$, containing ones and attributes values of real estates in database,

$\{\ln c\}$ - vector $[n \times 1]$, containing natural logarithms of database real estates prices,

$\{\ln B\}$ - vector $[(m+1) \times 1]$, containing estimated values of logarithms of the model natural coefficients B_j of model (1),

$\{\ln\delta\}$ - vector $[n\times 1]$, containing random deviations for natural logarithms of database real estates prices,
where n is the number of considered real estates in the database and m is the number of considered attributes.

The system of equations (3) rewritten as a full matrix form becomes:

$$\begin{Bmatrix} \ln\delta_1 \\ \ln\delta_2 \\ \ln\delta_3 \\ .... \\ \ln\delta_n \end{Bmatrix} + \begin{Bmatrix} 1 & x_{11} & x_{12} & ...... & x_{1m} \\ 1 & x_{21} & x_{22} & ...... & x_{2m} \\ 1 & x_{31} & x_{32} & ...... & x_{3m} \\ ...... & ...... & ...... & ...... & ...... \\ 1 & x_{n1} & x_{n2} & ...... & x_{nm} \end{Bmatrix} \times \begin{Bmatrix} \ln B_0 \\ \ln B_1 \\ \ln B_2 \\ .... \\ \ln B_m \end{Bmatrix} = \begin{Bmatrix} \ln c_1 \\ \ln c_2 \\ \ln c_3 \\ ...... \\ \ln c_n \end{Bmatrix} \tag{4}$$

The confidence weight for every real estate in database can be established for instance on the basis of the attributes creating prices similarity to their mean values in database (e.g. by deviations $x_i - \hat{x}_i$) or on the basis of the value of the attribute "source of information about market real estates":
$p = 1.0$ – own information,
$p = 0.7$ – auction, public sale,
$p = 0.6$ – notarial act, fiscal office,
$p = 0.5$ – offers.

The reliance weights assigned to all real estates in the database, the weight matrix $\{P\}$ can be set up as a diagonal matrix:

$$\{P\} = \begin{Bmatrix} p_1 & 0 & .... & 0 & 0 \\ 0 & p_2 & .... & 0 & 0 \\ 0 & 0 & .... & 0 & 0 \\ 0 & 0 & .... & p_{n-1} & 0 \\ 0 & 0 & .... & 0 & p_n \end{Bmatrix} \tag{5}$$

By applying the least squares weighing method and taking into account Gauss-Markow principle, we receive the following formulas for estimated parameters of a non-linear model:

$$\{\ln\hat{B}\} = \left(\{A^T\}\{P\}\{A\}\right)^{-1} \times \{A\}^T\{P\}\{\ln c\} \tag{6}$$

To determine a covariance matrix for estimated regression coefficients, we use the formula:

$$Cov\{\ln\hat{B}\} = \hat{\sigma}_0^2\left(\{A\}^T\{P\}\{A\}\right)^{-1} \tag{7}$$

where σ^2_0 is the variance of the multiplicative non-linear model estimation. Its estimator is given by the following expression:

$$\hat{\sigma}_0^2 = \frac{\{\ln c\}^T\{P\}\{\ln c\} - \{\ln\hat{B}\}^T\{A\}^T\{P\}\{\ln c\}}{n-m-1} \tag{8}$$

The elements on the main matrix diagonal (7) are the squares of natural logarithms standard deviations for particular coefficients of a non-linear model (1), i.e.:
$\sigma^2(\ln B_0)$, $\sigma^2(\ln B_1)$, $\sigma^2(\ln B_2)$,......,$\sigma^2(\ln B_m)$.

3. PROCEDURE OF VERIFICATION OF AN ESTIMATED MODEL

To confirm the usability of an estimated model, we have to verify it by suitable statistical methods:

a) investigation of model acceptability regarding the values of the factors variability and the convergence:

- coefficient of variation V:

$$V = \frac{\sigma_\delta}{\hat{c}} \tag{9}$$

where: σ_δ - standard error of remainders,
$\hat{c}$ - mean value of real estates prices in database,

- coefficient of convergence φ^2:

$$\varphi^2 = \frac{\sum_{i=1}^{n} \delta_i^2}{\sum_{i=1}^{n} (c_i - \hat{c})^2} \tag{10}$$

where: $\delta_i = c_i - c_{i(p)}$ - remainder for i^{th} real estate (difference between a price observed and a price forecast according to the model),
$\hat{c}$ - mean value of real estates prices in the database.

Boundary values for V and φ^2 are determined arbitrarily.
Mostly they are: V_0 = 0,10 and φ_0^2 =0,40.

b) investigating the random components symmetry:

Suppose k is the number of positive (or negative) remainders δ_i. We verify the hypothesis concerning the remainder structure indicator with established sign (so-called fraction p) in the following form: H_0: p=0.5, against the alternative hypothesis H_1: $p \neq 0.5$. The test statistics takes on the form:

$$Z = \frac{\left| \frac{k}{n} - 0.5 \right|}{\sqrt{\frac{\frac{k}{n}\left(1 - \frac{k}{n}\right)}{n-1}}} \tag{11}$$

which, for small random sample (n<30) has a T-Student distribution (marked T), while for numerous tests – a normal one.
If the test indicates the necessity to reject the null hypothesis, the analytic form of the model has to be modified.

c) examining the significance of model coefficients:
- verification of a parameters system significance on the basis of Fisher-Snedecor's statistics, using the following hypothesis H_0: $\sum_{j=0}^{m} B_j^2 = 0$ against the alternative hypothesis H_1: $\sum_{j=0}^{m} B_j^2 \neq 0$; statistic form:

$$F = \frac{1-\varphi^2}{\varphi^2} \cdot \frac{n-m-1}{m} \tag{12}$$

which, if the null hypothesis is true, has a F-Snedecor's distribution with (m, n-m-1) degrees of freedom,
- verification of particular regression coefficients significance on the basis of T-Student statistics, under the null hypothesis H_0: $B_j = 0$ against the alternative hypothesis H_1: $B_j \neq 0$ statistic form:

$$T = \frac{\ln \hat{B}_j}{\sigma\left(\ln \hat{B}_j\right)} \tag{13}$$

which, if the null hypothesis is true, has a T-Student distribution with (n-m-1) degrees of freedom.

If for any of explaining variables, the statistical test does not demonstrate reasons for rejecting the null hypothesis, we eliminate this variable from the model and we reestimate the parameters.

4. FORECASTING A MARKET UNIT VALUE OF A REAL ESTATE

The prediction of an arbitrary real estate unit price c_p (market value) is determined by substituting to the evaluated model the form (2) of the values of the attributes a_j. By converting this, we receive the forecast value for the analysed real estate:

$$c_p = \exp\left(\ln B_0 + a_1 \cdot \ln B_1 + a_2 \cdot \ln B_2 + \ldots + a_m \cdot \ln B_m\right) \tag{14}$$

The standard deviation of the forecast price natural logarithm is determined using the covariance matrix (7) for the coefficients of the linearizated model using the expression:

$$\sigma^2(\ln c_p) = \left(\begin{Bmatrix} 1 \\ a_1 \\ a_2 \\ \dots \\ a_m \end{Bmatrix}^T Cov\{\ln \hat{B}\} \begin{Bmatrix} 1 \\ a_1 \\ a_2 \\ \dots \\ a_m \end{Bmatrix} \right) \tag{15}$$

The standard deviation, accordingly to the propagation of errors law is calculated by formula:

$$\sigma(c_p) = c_p \cdot \sigma(\ln c_p) \tag{16}$$

Analysing the variance, we can state that the forecast price (value) of the analysed real estate, at confidence level (1-α)=0.95, should be contained in the following interval:

$$p_p \pm t(0{,}975; n-m-1) \cdot \sigma(c_p) \tag{17}$$

with $t(0.975; n\text{-}m\text{-}1)$ denoting an appropriate quantile of T-Student distribution.

To make the forecast reliable and usable for the real estate estimation, two following principles must be observed:
- the values of a_j attributes of an estimated real estate must be contained in variability intervals corresponding to the attributes of real estates in database,
- the confidence interval width should not exceed half of the market value of an estimated real estate.

The estimation reliability indicator, including inaccuracy of all parameters appearing in the real estate price modelling, is expressed by an index of dispersion as:

$$\lambda = \frac{t\left(1 - \frac{\alpha}{2}; n-m-1\right) \cdot \sigma(c_p)}{c_p} \tag{18}$$

On the basis of the proposed scale for the index of dispersion, the expert can evaluate the reliability of the estimated real estate, i.e.:

$\lambda \leq 0{,}05$ – very high reliability,
$0{,}05 < \lambda \leq 0{,}10$ – high reliability,
$0{,}10 < \lambda \leq 0{,}15$ – high enough reliability,
$0{,}15 < \lambda \leq 0{,}20$ – sufficient reliability,
$0.20 < \lambda \leq 0{,}25$ – acceptable reliability.

If $\lambda > 0{,}25$, it must be stated that the estimation of the real estate on the basis of estimated coefficients of non-linear regression is inadmissible. In such a case, the database ought to be revised and the parameters of the model ought to be re-estimated.

5. EXAMPLE

Results of non-linear multiplicative models estimation for two different real estate data basis are presented below.

Model 1
multiplicative model with 13+1 variables:

$$c = B_0 \cdot B_1^{x_1} \cdot B_2^{x_2} \cdot B_3^{x_3} \cdot \ldots \cdot B_{13}^{x_{13}} \tag{19}$$

coefficient of convergence: $\varphi^2 = 0.32$
variance of the multiplicative non-linear model estimation: $\sigma_0^2 = 0.02$ [-]
standard error of estimation: $\sigma_0 = 0.14$ [-]

In table 1 there are parameters of model 1 with the result of verification of particular parameter significance. Only marked values are significant.

Table 1. Parameters of model 1

Number of parameter	$\ln \hat{B}_i$	$\sigma(\ln \hat{B}_i)$	T_{cal}	T_{tab}
0	0,5335	0,1705		1,99
1	0,0165	0,0041	**4,05**	
2	0,0068	0,0018	**3,78**	
3	-0,0148	0,0530	-0,28	
4	0,0308	0,0285	1,08	
5	-0,0908	0,0232	**-3,92**	
6	-0,0186	0,0247	-0,76	
7	0,0427	0,0129	**3,32**	
8	-0,0885	0,0302	**-2,93**	
9	-0,0223	0,0067	**-3,31**	
10	0,0520	0,0276	1,89	
11	0,1070	0,0217	**4,93**	
12	-0,0221	0,0333	-0,66	
13	-0,0485	0,0391	-1,24	

Model 2
multiplicative model with 14+1 variables:

$$c = B_0 \cdot B_1^{x_1} \cdot B_2^{x_2} \cdot B_3^{x_3} \cdot \ldots \cdot B_{14}^{x_{14}} \tag{20}$$

coefficient of convergence: $\varphi^2 = 0.16$

variance of the multiplicative non-linear model estimation: $\sigma_0^2 = 0.01$ [-]
standard error of estimation: $\sigma_0 = 0.07$ [-]

In table 2 there are parameters of model 2 with the result of verification of particular parameter significance. Only marked values are significant.

Table 2. Parameters of model 2

number of parameters	$\ln \hat{B}_i$	$\sigma(\ln \hat{B}_i)$	T_{cal}	T_{tab}
0	1,2896	0,1223		
1	-0,0012	0,0033	-0,36	
2	-0,0026	0,0010	**-2,63**	
3	0,0736	0,0229	**3,22**	
4	-0,0357	0,0189	-1,89	
5	-0,0263	0,0136	-1,94	
6	-0,0123	0,0164	-0,75	
7	-0,0110	0,0091	-1,21	2,00
8	0,0515	0,0183	**2,81**	
9	-0,0038	0,0054	-0,72	
10	0,0129	0,0235	0,55	
11	0,1510	0,0182	**8,28**	
12	-0,1033	0,0168	-0,79	
13	-0,0168	0,0155	-1,09	
14	-0,1614	0,0347	**-4,65**	

Hereafter, in the tables form, the results of models verification are presented. Suitable statistical tests were executed for real estate data bases containing $n_1 = 91$ and $n_2 = 77$ number of real estate. In table 3 there are values of coefficients of variation V and convergence φ^2 for particular models with calculated statistic form and critical value from statistical board for test investigating the random components symmetry, with number of positive remainders k. Last two lines there is the tests result of verification of e parameters system significance.

Table 3. Results of models verification

Model	1	2
V	0,13	0,08
φ^2	0,32	0,16
K	50	42
Z_{cal}	0,55	0,80
Z_{tab}	1,96	1,96

F_{cal}	12,49	23,63
F_{tab}	1,85	1,86

One of the models has a coefficients of variation value lower then 0.10. Both models have a symmetrical distribution of random components (calculated statistics values are lower than critical value, that means there are not reason to reject the null hypothesis which assumes that the number of positive and negative remainders are equal). The result of verification of the coefficients system significance is positive for both models.

After elimination, from the model, variables which are not statistically significant, the new estimation of parameters was made with the verification of model.

Model 1a:

$$c = B_0 \cdot B_1^{x_1} \cdot B_2^{x_2} \cdot B_3^{x_3} \cdot \ldots \cdot B_7^{x_7} \tag{21}$$

coefficient of convergence: $\varphi^2 = 0.36$
variance of the multiplicative non-linear model estimation: $\sigma_0^2 = 0.02$ [-]
standard error of estimation: $\sigma_0 = 0.15$ [-]

In table 4 there are parameters of model 1a with the result of verification of particular parameter significance. All of parameters are significant.

Table 4. Parameters of model 1a

Number of parameter	$\ln \hat{B}_i$	$\sigma(\ln \hat{B}_i)$	T_{cal}	T_{tab}
0	0,4223	0,0936		1,99
1	0,0199	0,0036	**5,56**	
2	0,0083	0,0016	**5,32**	
3	-0,0907	0,0213	**-4,26**	
4	0,0395	0,0128	**3,09**	
5	-0,0918	0,0284	**-3,23**	
6	-0,0215	0,0063	**-3,40**	
7	0,1078	0,0190	**5,67**	

Model 2a:

$$c = B_0 \cdot B_1^{x_1} \cdot B_2^{x_2} \cdot B_3^{x_3} \cdot B_4^{x_4} \cdot B_5^{x_5} \tag{22}$$

coefficient of convergence: $\varphi^2 = 0.18$
variance of the multiplicative non-linear model estimation: $\sigma_0^2 = 0.01$ [-]
standard error of estimation: $\sigma_0 = 0.07$ [-]

In table 5 there are parameters of model 2a with the result of verification of particular parameter significance. All of parameters are significant.

Table 5. Parameters of model 2a

Number of parameters	$\ln \hat{B}_i$	$\sigma(\ln \hat{B}_i)$	T_{cal}	T_{tab}
0	1,0533	0,0706		1,99
1	-0,0019	0,0007	**-2,59**	
2	0,0617	0,0182	**3,39**	
3	0,0404	0,0160	**2,52**	
4	0,1628	0,0145	**11,25**	
5	-0,1202	0,0280	**-4,30**	

Comparing coefficients of determination R^2=1-φ^2 obtained for models 1 and 1a (0,68 and 0,64) as well 2 and 2a (0,84 and 0,82), we can notice that numbers of variables reduction has a small influence on models quality.

Hereafter, in the tables form, the results of models verification are presented.

Table 6. Results of models verification

Model	1a	2a
V	0,14	0,09
φ^2	0,36	0,18
k	48	37
Z_{cal}	0,53	0,34
Z_{tab}	1,96	1,96
F_{cal}	21,53	23,63
F_{tab}	2,13	2,35

One of the models has a coefficients of variation value lower then 0.10. Both models have a symmetrical distribution of random components (calculated statistics values are lower than critical value, that means there are not reason to reject the null hypothesis which assumes that the number of positive and negative remainders are equal). Also the result of verification of the coefficients system significance is positive for both models.

6. SUMMARY

An analogical analysis can be done for other methods of model parameters estimation. The equality of parameters, estimated by different methods, may be verified using suitable statistical tests. Within the framework of the comparison of estimation methods, also the covariance matrixes for parameters achieved by different estimation methods should be compared.

REFERENCES

[1] Baranska A.: 2004 "Criteria of database quality appraisement and choice stochastic models in prediction of real estate market value", **FIG Working Week 2004 and 27th General Assembly in Athens, Greece, 22 – 27 May 2004**

[2] Baranska A.: 2005 "Appraisement of Equalization of Geodetic Observations Quality Applying Values of Defined Invariants Parameters", **FIG Working Week 2005 and the 8th International Conference on the Global Spatial Data Infrastructure (GSDI-8) "From Pharaohs to Geoinformatics" in Cairo, Egypt, 16-21 April 2005.**

[3] Barańska A.: *Kryteria stosowania modeli stochastycznych w predykcji rynkowej wartości nieruchomości*. Rozprawa doktorska, Akademia Górniczo – Hutnicza, Wydział Geodezji górniczej i Inżynierii Środowiska. Kraków 2003

[4] Czaja J.: *Metody szacowania wartości rynkowej i katastralnej nieruchomości*. Kraków, 2001

[5] Czaja J., Preweda E.: *Analiza statystyczna zmiennej losowej wielowymiarowej w aspekcie korelacji i predykcji*. Kraków, UWND AGH, Geodezja, t.2, 2000

[6] Krysicki W. i in.: *Rachunek prawdopodobieństwa i statystyka matematyczna w zadaniach*. Warszawa, PWN, 1986

BIOGRAPHICAL NOTES

Name: **Anna Baranska**
Employment: AGH University of Science and Technology in Krakow Poland (from 01.11.2003)
Degree: Philosophy Doctor of Geodetic Sciences (AGH Krakow, 12.06.2003)
Study: Doctor Study at the Faculty of Mining Surveying and Environment Engineering (AGH Krakow, 10.1998 – 06.2003)
Degree: Master of Geodetic Science, Specialization: Real Estate Administration (AGH Krakow, 23.06.2000)
Study: Master Study at the Faculty of Mining Surveying and Environment Engineering (AGH Krakow, 10.1995 – 06.2000)
Degree: Master of Mathematics, Specialization: Applied Mathematics (Jagiellonian University Krakow, 24.06.1996)
Study: Master Study at the Faculty of Mathematics and Physics Faculty (Jagiellonian University Krakow, 10.1991 – 06.1996)

CONTACTS

PhD Anna Barańska
University of Science and Technology
Faculty of Mining Surveying and Environmental Engineering
Terrain Information Department
al. A. Mickiewicza 30
30 – 059 Krakow
POLAND
Tel. + 48 12 617 23 00
Fax + 48 12 617 22 77
Email: abaran@agh.edu.pl