**Definition of the Geoid**

Geoid is the equipotential surface of the gravity field of the earth.

**Geoid**

Level surface of global undisturbed oceans

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**Earth Surfaces**

- Topographic Surface
- Ellipsoid Surface
- Geoid Surface

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**Geoid - Ellipsoid Separation**

\[ N = \frac{R}{4} \int \lambda \phi^2 (\psi \cos^2 \phi) \]
Why to combine h, H and N??

- Modernize regional vertical datums.
- Uniform connect national regional datums between neighboring countries.
- Transform between different types of height data (GPS-leveling).
- Refine and test existing geodetic geoid models.
- Better understanding of data error sources.
  - Calibrate geoid model.
  - Assess noise in GPS heights, test a-priori error measures.
  - Evaluate leveling precision, test a-priori error values.
- Other applications: see level change monitoring, post-glacial rebound studies, etc.

Problems of combining heterogeneous heights

- Random errors in the desired heights h, H, and N.
- Datum inconsistencies inherent among the height types.
- Systematic effects and distortions (long wavelength geoid errors, poorly modelled GPS errors and over-constrained leveling network adjustments).
- Assumptions/theoretical approximations made in processing observed data (neglecting tax surface topography or river discharge corrections at tidal gauges).
- Approximate or inexact normal/orthometric height corrections.
- Instability of reference station monuments over time (isostatic effects, land uplift/subsidence).

EGM96 gravity data (20 mgal contour interval).

Hatta is obviously on the edge of a major tectonic gravity trend.

EGM96 – GEOID MODEL

Gravity data observation Scheme

Area -1: observed at every 1 X 1 km interval.
Area -2: observed at every 2 X 2 km interval.

Statistics of Gravity Data Reduction

Area covered by detailed DSM data – mainland Dubai

Area covered with 100 m DEM data – Hatta

Comparisons of gravimetric geoid to Dubai Municipality GPS leveling

<table>
<thead>
<tr>
<th>Unit: m</th>
<th>Mean</th>
<th>Std.Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dubai area (3157 GPS pts)</td>
<td>-1.40</td>
<td>0.09</td>
</tr>
<tr>
<td>EGM96 only</td>
<td>-0.83</td>
<td>0.16</td>
</tr>
<tr>
<td>Hatta area (110 pts)</td>
<td>-1.39</td>
<td>0.12</td>
</tr>
<tr>
<td>EGM96</td>
<td>-1.32</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Gravimetric geoid of the Dubai region, from EGM96, gravimetry and DSM’s

Bouguer anomalies of Dubai main area (5 mgal contour interval)
Progress Toward a cm Geoid for Dubai Emirate

The gravimetric geoid computation

The Dubai precise geoid has been computed in two steps:

- A gravimetric geoid model, computed by spherical FFT in a global datum
- A GPS-tailored local geoid, which fits the GPS observations and the Dubai vertical datum to a few cm. This step has involved an iterative editing of GPS-leveling outliers.

Why to use FFT?

- FFT provides very fast evaluation of convolution sums/integrals with gridded data
- In planar approximation, the Stokes and terrain correction integrals are convolutions
- In spherical approximation, these integrals are convolutions along the parallels, and so are the summations for the GM-contributions
- Gravity and topography data are usually provided on regular grids
- Computations for very large areas can be performed on a PC

Computational Procedure:

- Subtract effect of GM from $\Delta g$ (long wavelength)
- Subtract effect of terrain from $\Delta g$ (short wavelength)
- Use the reduced $\Delta g$ in the FFT formulas
- Add to the results (reduced co-geoid) the GM effect
- Add to the results (reduced co-geoid) the indirect terrain effects

Gravimetric Geoid Computation

The geoid signal $N$ is constructed by subdividing it into three parts:

$N = N_1 + N_2 + N_3$

Where

- $N_1$ = Spherical harmonic expansion complete to degree and order 360
- $N_2$ = From the topography
- $N_3$ = From contributions of residual gravity

Long wavelength geoid contribution

$\Delta g_{\text{long}} = \sum_{l=2}^{n} (a-1) \sum_{m=-l}^{l} [C_{lm} \cos m \lambda + S_{lm} \sin m \lambda] P_{lm}(\sin \phi)$

$N_3 = \sum_{l=2}^{n} \sum_{m=-l}^{l} [C_{lm} \cos m \lambda + S_{lm} \sin m \lambda] P_{lm}(\sin \phi)$
Medium wave length Geoid contribution

\[ \delta h_{(q,\lambda)} \approx \frac{1}{2\sigma^2} \int (\Omega_{(q,\lambda)} + \Omega_{0}) F_{(q,\lambda)} F_{0} \text{d}x \text{d}y \]

Short wave length Geoid contribution

\[ \delta N_p \approx \frac{1}{2\sigma^2} \int \frac{k_p}{\gamma} H_f^2 - \frac{k_p}{6\gamma} \int \frac{H_f^3}{P^3} \text{d}x \text{d}y \]

Fitting the geoid to GPS/levelling data

\[ \delta = N_{grav} - N_{GPS} = f(\phi, \lambda) + \varepsilon' \]

The basic principle used here is to model the gravimetric and GPS geoid difference by a smooth function consisting of a trend function \( f \) (a polynomial) and a residual \( \varepsilon' \) to be modeled by least/squares collocation.

Improving the GPS Heighting

\[ H_i = h_i - N_i \quad \text{or} \quad h_i - H_i - N_i = 0 \]

\[ \delta h = c^T x + v_i \]

\[ \phi_i = \cos \phi_i \cos \lambda_i \lambda_1 + \cos \phi_i \sin \lambda_i \lambda_2 + \sin \phi_i \lambda_3 + \lambda_4 \]
R.M.S. fit of GPS leveling data to the geoid

<table>
<thead>
<tr>
<th>Unit m</th>
<th>Mean</th>
<th>Std-dev</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>All data (3640 pts)</td>
<td>0.041</td>
<td>0.209</td>
<td>0.245</td>
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<tr>
<td>DMA GPS data, Dubai (1950 pts)</td>
<td>0.036</td>
<td>0.115</td>
<td>0.21</td>
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</tr>
<tr>
<td>Data area (330 pts)</td>
<td>-0.11</td>
<td>0.11</td>
<td>0.423</td>
<td>0.403</td>
</tr>
</tbody>
</table>

Dubai Geoid computed from gravity, GPS and leveling. Contour interval 20 cm.

Dubai Geoid from gravity, GPS and levelling. Contour interval 20 cm.
CONCLUSION

- The derived Geoid model is precise enough to meet the Third order levelling Standards.
- In the main land Dubai the accuracy could be of the order of 1-3 cm on average.
- In Hatta region the accuracy could be of 5-10 cm. Still more information and data is necessary to improve the model in this region.
- We believe this Geoid model could meet the requirement of many potential users who would intend to convert GPS heights into their corresponding Mean Sea Level heights.