Tightly Coupled Integration of GPS-PPP and MEMS-Based Inertial System Using EKF and UKF

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INTRODUCTION

- The most common navigation systems are the Global Navigation Satellite System (GNSS) and the Inertial Navigation Systems (INS).
- Currently, GPS system is the most widespread GNSS system and the only fully operational system.
- GPS mainly provides worldwide positioning, velocity and time synchronization.
- Unfortunately, accurate GPS solution may not always be available as a result of multi-path, GPS partial or complete outages.
- To overcome these limitations, GPS can be integrated with a relatively environment-independent system, the inertial navigation system (INS).

INTRODUCTION

- Unlike GPS system, the INS performance is not affected in environments as urban canyons; it is independent of external electro-magnetic signals.
- In contrast with GPS, which need complicated system to give the attitude solution, INS system gives the attitude solution directly.
- However, the main drawback of an INS is the degradation of its performance with time. In order to control the errors to an acceptable level continues updates from, for example, GPS are necessary.

Problem Statement

- Currently, most of the integrated GPS/INS systems for precise positioning applications are based on *high-end INS grades* and *differential GPS (DGPS).*
  - *This involves the deployment of a base station, which controls the range of navigation area.*
  - *The need of the surplus equipment (minimum two receivers) increases the complexity and cost.*
  - *The high grades INS are very expensive which add other limitation.*
  - *Most of the integrated system research employed Extended Kalman filter (EKF) as an estimation filter may cause divergence in positioning estimation during GPS outages.*

Research Objectives

- To overcome the limitation mentioned before this research aims to develop a new integrated navigation system for precise positioning and attitude applications based on;
  - *Precise Point Positioning (PPP) technique is used instead Differential mode to reduce the complexity and the cost.*
  - *Both pseudorange and carrier phase GPS measurements are considered.*
  - *Low-cost micro-electro-mechanical sensors (MEMS) accelerometers is used reducing the cost of the system with fiber optic gyros for precise attitude detecting.*
  - *Both EKF and UKF estimation filters are employed to develop the optimal filter for the proposed integrated system.*

MATHEMATICAL MODELS *(GPS-PPP)*

- Both pseudorange and carrier phase GPS measurements are processed.

$$P_1 = \rho(t, t-\tau) + c[dt_r(t) - dt^s(t-\tau)] + T + I_1 + c[d_r(t) - d^s(t-\tau)]^1 + m_{p1} + e_{p1}$$

$$P_2 = \rho(t, t-\tau) + c[dt_r(t) - dt^s(t-\tau)] + T + I_2 + c[d_r(t) - d^s(t-\tau)]^2 + m_{p2} + e_{p2}$$

$$\phi_1 = \rho(t, t-\tau) + c[dt_r(t) - dt^s(t-\tau)] + T - I_1 + c[\delta_r(t) - \delta^s(t-\tau)]^1 + \delta m_1 + e_1 + \lambda_1[N_1 + \phi_{r1}(t_0) - \phi_1^s(t_0)]$$

$$\phi_2 = \rho(t, t-\tau) + c[dt_r(t) - dt^s(t-\tau)] + T - I_2 + c[\delta_r(t) - \delta^s(t-\tau)]^2 + \delta m_2 + e_2 + \lambda_2[N_2 + \phi_{r2}(t_0) - \phi_2^s(t_0)]$$

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MATHEMATICAL MODELS *(GPS-PPP)*

- Un-differenced ionosphere-free linear combinations of pseudorange and carrier phase measurements is employed.

$$P_3 = \frac{f_1^2 P_1 - f_2^2 P_2}{f_1^2 - f_2^2} = \rho + c\,dt_r - c\,dt^s + T + c(2.546 d_{r1} - 1.546 d_{r2}) - c(2.546 d^{s1} - 1.546 d^{s2}) + e$$

$$\phi_3 = \frac{f_1^2 \phi_1 - f_2^2 \phi_2}{f_1^2 - f_2^2} = \rho + c\,dt_r - c\,dt^s + T + c(2.546 \delta_{r1} - 1.546 \delta_{r2}) - c(2.546 \delta^{s1} - 1.546 \delta^{s2}) + (\overline{\lambda N}) + \varepsilon$$

$$P_3 = \rho + c[dt_r - dt^s] + T + B^r_{p3} - B^s_{p3} + e_{p3}$$

$$\phi_3 = \rho + c[dt_r - dt^s] + T + B^r_{\phi 3} - B^s_{\phi 3} + \overline{\lambda N} + e_{\phi 3}$$

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MATHEMATICAL MODELS *(INS)*

- A system of non-linear first-order differential equations (mechanization equations) is used
- The inertial measurements (specific force and angular rate) are used to solve it to provide positions, velocities and attitudes

$$\begin{bmatrix} \dot{r}^n \\ \dot{V}^n \\ \dot{R}_b^n \end{bmatrix} = \begin{bmatrix} D \cdot V^n \\ R_b^n f^b - (2\Omega_{ie}^n + \Omega_{en}^n) \cdot V^n + g^n \\ R_b^n(\Omega_{ib}^n - \Omega_{in}^b) \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & \frac{1}{M+h} & 0 \\ \frac{1}{(N+h)\cos\varphi} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\omega_{ie}^{\ n} = \begin{bmatrix} 0 & \omega\cos\varphi & \omega\sin\varphi \end{bmatrix}^T \qquad \omega_{in}^{\ b} = R_n^b \cdot (\omega_{ie}^{\ n} + \omega_{en}^{\ n})$$

$$\omega_{en}^{\ n} = \begin{bmatrix} \frac{-V_n}{M+h} & \frac{V_E}{N+h} & \frac{V_E \tan\varphi}{N+h} \end{bmatrix}^T$$

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MATHEMATICAL MODELS *(Integration mechanism)*

- Loosely coupled
  - Both the GPS and the INS mathematical models are processed separately and the integration is applied in the level of the navigation outputs (positioning, velocities and attitudes)
- Tightly coupled
  - The GPS raw measurements are not processed in a separate filter as in the Loosely coupled case, but are directly combined in a unique filter.
  - The tight integration strategy is preferred to use during the periods of partial GPS availability.
  - As a result, in this research Tightly coupled is employed

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MATHEMATICAL MODELS (Integrated system processing)

1. Process model

$$\delta \dot{x} = F(t)\delta x + G(t)\,w(t)$$

$$\begin{vmatrix} \delta \dot{r}^n \\ \delta \dot{v}^n \\ \dot{\varepsilon}^n \\ \delta \dot{b}_a \\ \delta \dot{b}_g \\ \delta \dot{S}_a \\ \delta \dot{S}_g \\ \dot{\delta} t_{off} \\ \dot{\delta} t_{dri} \end{vmatrix} = \begin{vmatrix} F_{rr} & F_{rv} & F_{r\varepsilon} & 0_3 & 0_3 & 0_3 & 0_3 & 0 & 0 \\ F_{vr} & F_{vv} & F_{v\varepsilon} & R_b^n & 0_3 & R_b^n F^b & 0_3 & 0 & 0 \\ F_{\varepsilon r} & F_{\varepsilon v} & F_{\varepsilon\varepsilon} & 0_3 & R_b^n & 0_3 & R_b^n W^b & 0 & 0 \\ 0_3 & 0_3 & 0_3 & \beta_{ba} & 0_3 & 0_3 & 0_3 & 0 & 0 \\ 0_3 & 0_3 & 0_3 & 0_3 & \beta_{bg} & 0_3 & 0_3 & 0 & 0 \\ 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & \beta_{sa} & 0_3 & 0 & 0 \\ 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & \beta_{sg} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} \delta r^n \\ \delta v^n \\ \varepsilon^n \\ \delta b_a \\ \delta b_g \\ \delta S_a \\ \delta S_g \\ \delta t_{off} \\ \delta t_{dri} \end{vmatrix} + \begin{vmatrix} 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & 0 & 0 \\ R_b^n & 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & 0 & 0 \\ 0_3 & R_b^n & 0_3 & 0_3 & 0_3 & 0_3 & 0 & 0 \\ 0_3 & 0_3 & I_3 & 0_3 & 0_3 & 0_3 & 0 & 0 \\ 0_3 & 0_3 & 0_3 & I_3 & 0_3 & 0_3 & 0 & 0 \\ 0_3 & 0_3 & 0_3 & 0_3 & I_3 & 0_3 & 0 & 0 \\ 0_3 & 0_3 & 0_3 & 0_3 & 0_3 & I_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{vmatrix} \begin{vmatrix} w_a \\ w_g \\ w_{ba} \\ w_{bg} \\ w_{sa} \\ w_{sg} \\ w_{off} \\ w_{dri} \end{vmatrix}$$

2. Measurement model

$$\delta z = \begin{vmatrix} P_i^{GPS} - P^{INS} \\ \phi_i^{GPS} - \phi^{INS} \\ \dot{P}_i^{GPS} - \dot{P}^{INS} \\ \vdots \end{vmatrix} \qquad H = \begin{vmatrix} A_i & 0 & 0\,0\,0\,1\,0\,0 & 0 & \cdots\cdots \\ & \vdots\vdots\vdots & & & \\ B_i & 0 & 0\,0\,0\,1\,0\,\lambda & 0 & \cdots\cdots \\ & \vdots\vdots\vdots & & \ddots & \\ 0 & C_i & 0\,0\,0\,0\,1\,0 & 0 & \cdots\cdots \\ & \vdots\vdots\vdots & & & \end{vmatrix}$$

The complete state vector consists of 23+n states describing the basic state vector (the nine navigation parameter errors), the inertial sensors errors (bias drift and scale factor), errors unique to the GPS measurements such as clock offset and drift and finally the ambiguity parameters (n)

Where $A_i = \frac{\partial P}{\partial x}$, $B_i = \frac{\partial \Phi}{\partial x}$, $C_i = \frac{\partial \dot{P}}{\partial V}$ are the partial derivatives of the pseudorange, phase and Doppler, respectively, with respect to the receiver position X and velocity V;

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MATHEMATICAL MODELS (Estimation filter)

1. Extended Kalman filter (EKF)
   - ❑*Apply linearization to the nonlinear models using first order Tylor expansion and neglecting higher order terms.*
   - ❑ *Restrict the probability distribution of the motion and measurement models to Gaussian distribution.*

Prediction step

$$\delta \hat{x}_{k,k-1} = \Phi_{k,k-1}\delta \hat{x}_{k-1}$$

$$P_{k,k-1} = \Phi_{k,k-1}P_{k-1}\Phi^T_{k,k-1}$$

Update step

$$K_k = P_{k,k-1}H_k^T(H_kP_{k,k-1}H_k^T + R_k)^{-1}$$

$$\delta \hat{x}_k = \delta \hat{x}_{k,k-1} + K_k(\delta Z_k - H_k\delta \hat{x}_{k,k-1})$$

$$P_k = (I - K_kH_k)P_{k,k-1}$$

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MATHEMATICAL MODELS (Estimation filter)

- Unscented Kalman filter
  - *Nonlinear models are used no Tylor expansion used.*
  - *Restrict the probability distribution of the motion and measurement models to Gaussian distribution.*

1. Predict step

$$\begin{bmatrix} \dot{r}^n \\ \dot{V}^n \\ \dot{R}_b^n \end{bmatrix} = \begin{bmatrix} \\ R_b^n f^b - (2\Omega \ldots \\ R_b^n(\ldots \end{bmatrix}$$

2. Sampling step

$$X_0 = \bar{x}, W_0 = \frac{\lambda}{(n+\lambda)}$$

$$X_i = \bar{x} + \sqrt{(n+\lambda)P_x}, W_i = \frac{1}{2(n+ \ldots}$$

$$X_{i+n} = \bar{x} - \sqrt{(n+\lambda)P_x}, W_{i+n} = \frac{}{2(\ldots}$$

$$z_i = h(X_i), z = \sum_{i=0}^{2n} W_i z_i, x = \sum_{i=0}^{2n} W \ldots$$

$$P_z = \sum_{i=0}^{2n} W_i (z - z_i)(z - z_i)^T$$

$$P_x = \sum_{i=0}^{2n} W_i (x - x_i)(x - x_i)^T$$

3. Update step

$$K_k = P_{k,k-1} H_k^T (H_k P_{k,k-1} H_k^T + R_k)^{-1}$$

$$\delta\hat{x}_k = \delta\hat{x}_{k,k-1} + K_k(\delta Z_k - H_k \delta\hat{x}_{k,k-1})$$

$$P_k = (I - K_k H_k) P_{k,k-1}$$

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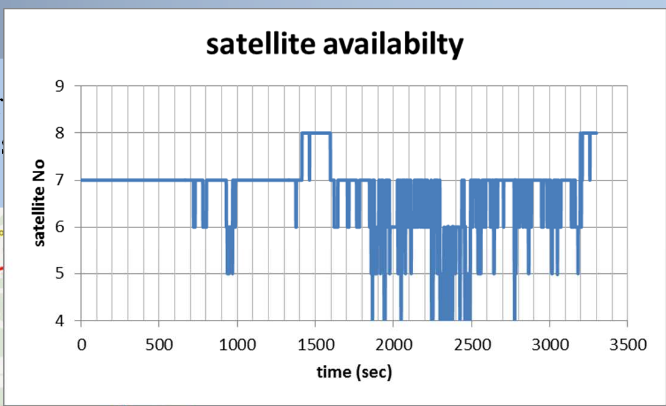
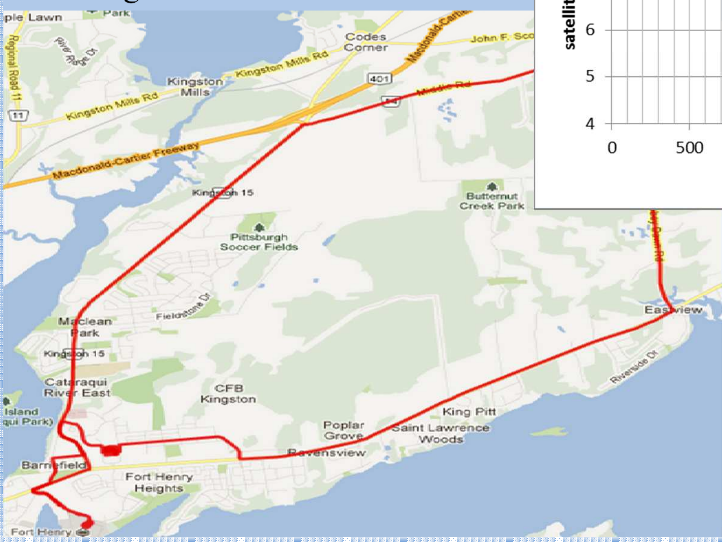
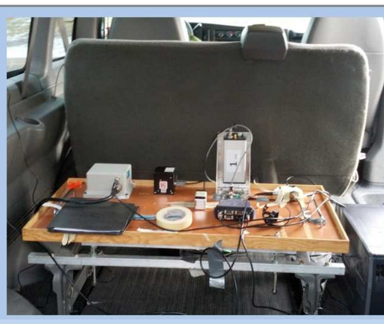
VEHICLE TEST

- A vehicular test was carried out thr... and shows difficult scenarios for ... outages of several seconds.







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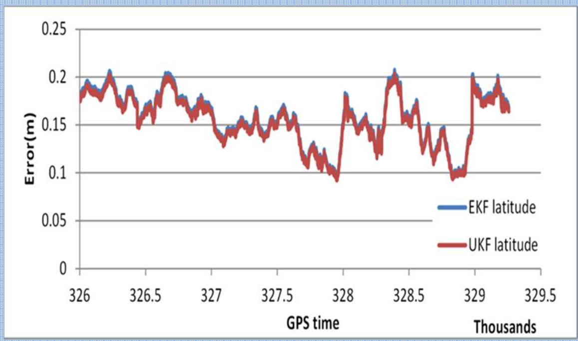
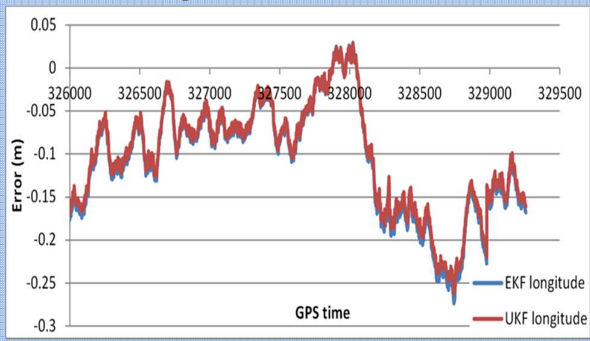
RESULTS (GPS AVAILABILTY)

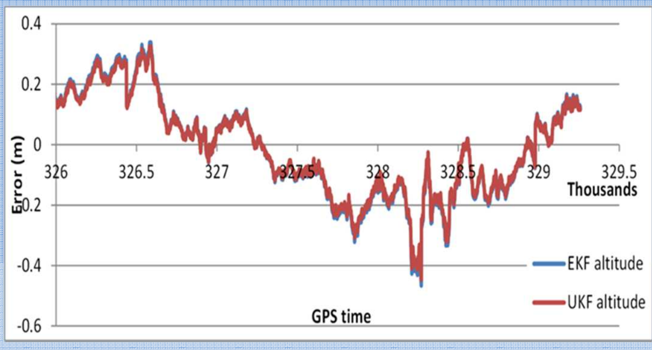
latitude



Longitude



Altitude



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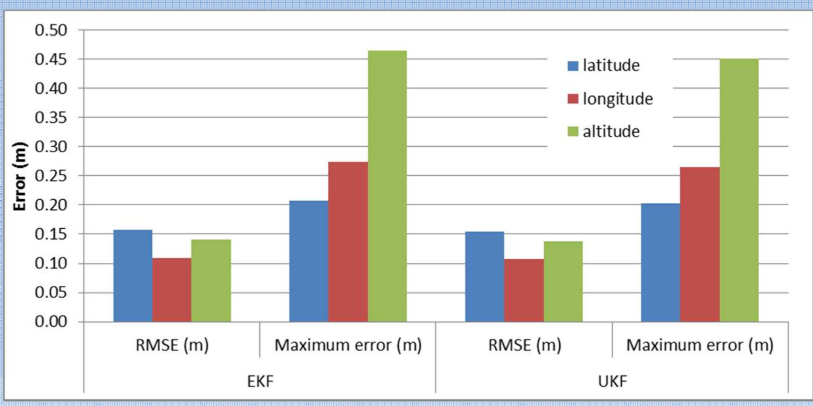
RESULTS (GPS AVAILABILTY)

Estimation filter	EKF		UKF	
Positioning	RMSE (m)	Maximum error (m)	RMSE (m)	Maximum error (m)
latitude	0.15756	0.20749	0.15441	0.20334
longitude	0.10866	0.27348	0.10740	0.26527
altitude	0.14151	0.46528	0.13727	0.45133



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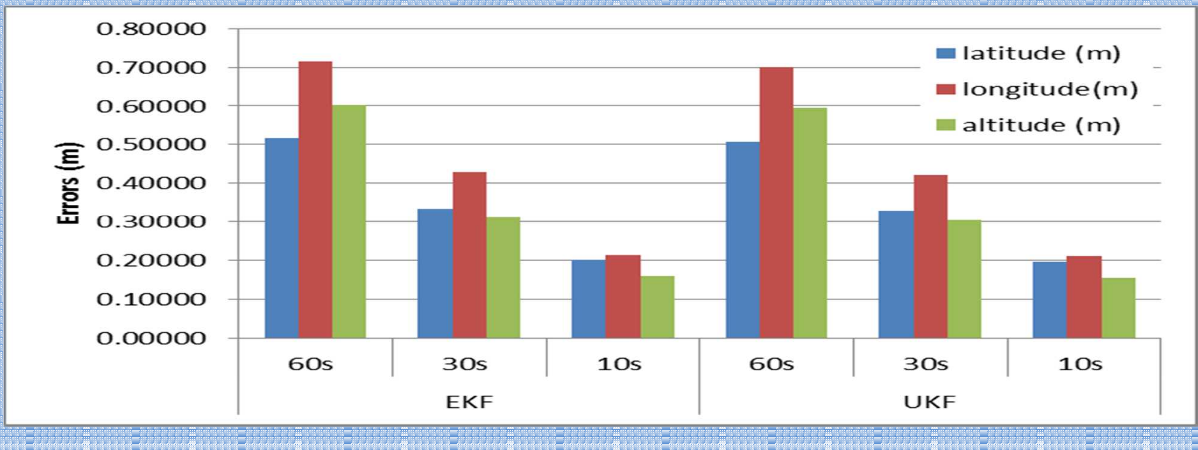
RESULTS (GPS OUTAGES)

- To simulate the challenging conditions of the trajectory trip including high and slow speeds, twelve simulated GPS outages of 60s, 30s and 10s are introduced.
- Both EKF and UKF have similar accuracy-level during the outages.



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CONCLUSION

- This research develops new algorithms for the integration of GPS PPP and MEMS-based IMU.
- un-differenced ionosphere-free linear combinations of pseudorange and carrier phase measurements are processed.
- Both EKF and UKF are used as estimation filters for the integrated system.
- The positioning results of integrated system show decimeter level accuracy.
- Both EKF and UKF results show the same level of accuracy for positioning .
- As a results, considering the computation cost of UKF compared with EKF, EKF is sufficient as an estimated filter for the proposed GPS-PPP/MEMS based IMU integrated system.

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Thanks

Questions

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