THE TOISE-METRE PROBLEM IN THE STRUVE ARC

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1. INTRODUCTION

Geodetic results of the 1816-1852 meridian arc measurements have been presented in 1857 and 1860 by head of that famous international enterprise F. G. W. Struve (Struve 1860). The quantities were expressed in old French units "toises", as it is described in the table (Fig.1). The Toise-Metre Problem (TMP) in the Struve arc was similar to that in other European triangulations based on various national pre-metric or "old-metric" length standards. It occurred much later than the arc measurement had been completed owing to the new International Metre (IM) standard brought into service since 1889. Standards supporting former results had to be subjected to special certification procedures in terms of the IM. Particular attention had to be given to the possibility of geometrical change during the elapsed time. But there seems to have been no "problem" so far regarding.

PRACTICAL USE of the Struve arc. A large number of geometricians from Bessel, 1841 to Zhongolovich, 1956 have made successful use of the results presented by Struve. That was because the arc was not used as a single geodetic or separately but always as a composition of its 12 segments and together with similar arcs throughout the world. The most probable average values levelled some UNCERTAINTY in converting toises into metres. Turning to those great measurements today one can feel free of practical limitations. For us those arcs are merely masterly works of technological art, pieces of our cultural heritage (Fig.2). Both their merits and dismerits are just interesting, and the mentioned uncertainty is too.

2. THE ARC LENGTH STANDARDS

The principal standard of the Struve arc was the double-toise bar of iron denoted as "N". Being made in 1827 it was certificated in 1828 in relation to the Toise of Peru (TP) by Struve himself. That was made indirectly, through the mediation of an identical Fortin's copy of the TP. As Struve put it, "The length N= 1728.01249 lines of the Toise of Peru has been invariably taken as the starting figure through all of our computations of linear quantities" (Struve 1860, translation).

Divided by 864 the cited quantity is equivalent to:
N= 2.0000145 TP. (1) [56] or 2.8 ppm

The intermediate probable errors estimated by Struve accumulate in this value in a possible maximum error below 3 ppm. But was there any independent evidence of the accuracy obtained by Struve in his certification ? Yes, there were five.

In 1865-1871 Clarke re-determined five different bars of those which had been investigated by Struve in relation to his standard N. Three of those bars were found almost of the same (within 1 ppm) length, the other two exceeded Struve's values by only 2 ppm (in 40 years!).

So it should firmly be stated that it is the length of the TP which is the unit of the Struve results. The bar N presented the standard measure embodying the unit, just a PRECISE SCALED COPY of the TP.

The transported standards used in the total of 10 base measurements were: N (1827, 1844, 1845), its two copies R (1848) and P (1850, 1851, 1852), and K. Tenner's standard "sazhen" T (Russian fathom) used on three southern base-lines (1820, 1827, 1838). The "family tree" of the bar N included a number of interconnected "end" and "line" copies extensively used in 1847-1928. It is of importance that the five "senior" members of the "family", N and its four primary copies have been compared with European standards during the period of 1847 - 1902. Some of those relations can be traced up to the IM standards. This material presents the multitude of INDIRECT links between the Struve standard and that of the IM. Besides, in 1893 the bar N was DIRECTLY certificated in terms of the IM by Benoit at the B.I.P.M. with the result (Sokoloff 1894):

N= 3.897760 IM at 7.20 Celsius. (2) [10] or 2.6 ppm

A possible maximum error was estimated by 10 microns. The equation (2) still required an adjustment to the legal temperature of every toise standard, i.e. to 16.25 Celsius. For the purpose the thermic coefficient n = ~ 11.394 ppm (determined in 1852 by Struve, Lindhagen et al) was used.

This lead to the final result of the metric certification of the Struve standard:

N= 3.898162 IM at 16.25 Celsius in 1893. (3a)

3. THE PROBLEM AND APPROACHES

In the same year, just before the above certification Helmert (1893) had justified the most probable metric valueof Struve's standard:

N= 3.8981525 IM at 16.25 Celsius. (3b)

The author was basing on the most reliable INDIRECT evidence originating from the Struve, Bayer and Clarke comparisons involving Russian, German and British standards. Helmert used his result (3b) for processing the Russian part of the 52th parallel arc measurements. The former author has also furnished a variety of indirect results:

N from 3.898141 to 3.898165 IM. (3c)

He made no critique about the range and concluded with the following words: "The 1893 B.I.P.M. result might probably exceed by a few microns the true length of the standard but, should it be the case, this incorrectness is of no practical importance... It is difficult to say how many obstacles would withstand the wish to obtain a higher accuracy, taking into consideration that the terminating surfaces of the bar bear evidence of rust..." (loose translation).

Since 1894 the value (3a) has been regarded as the most reliable. The corresponding value of the toise-to-metre Conversion Coefficient (CC) was calculated as follows:

(3a) / (1) = 1.949067 IM/TP. (CCa)

Evidently, the tacit assumption of the INVARIABILITY of the Struve standard in between 1828 and 1893 was adopted at this calculation. The coefficient (CCa) was made geodetic use of independently by Wassiliew (1905), Helmert (1906), Zhongolovich (1956) et al. There were only small differences between the authors in the higher decimals of that value.

A typical example of practical approach to the TMP is presented by the mentioned Wassiliew's paper. In 1901-1902 he performed a special research which argued the above mentioned traditional value of the thermic coefficient of the bar N and found:

n~ = 11.609 ppm + 13 ppb, against 11.394 ppm
as previously. This lead Wassiliew to revise the 1893 result of the metric certification of the bar N and derive a larger one (Wassiliew 1905):

N = 3.898174 IM for 1893. (3d)

However, he took into consideration the lower value (very close to 3b) supported by a reliable German source and chose the medium value (3a). In his next work (Wassiliew 1907) he changed his mind and maintained his own result (3d). However, he seems to have not been followed in that choice by anyone else since then. Strangely enough, no attention was paid to a similar incident with the thermic coefficient of the famous Bessel's iron toise made in 1823 . The preceding Bessel's value 11.40 ppm was in 50 years changed for 11.60 ppm (the same quantity for iron) by Benoit at the B.I.P.M. Therefore Wassiliew's result (3d) cannot be considered less probable than (3a) for it has its own reason.

Thus there is a rather large uncertainty of the required metric value of the length of the bar N which may be supposed valid for the times of Struve:

N from 3.898141 IM to 3.898174 IM (3e)

It corresponds to the following (CC) range:

CC from 1.949057 to 1.949073 IM/TP (CCe)

which covers values supported by either direct or indirect evidence. One can notice that INDIRECT evidence comes presumably to LOWER values, within the left third of the latter range. It should not be forgotten that the more popular value (CCa) was based on the assumption of INVARIABILITY of the bar N between 1828 and 1893.

In literature regarding the TMP in the Struve arc one can find three more values staying outside the range (CCe). Some authors use the value 1.949081 IM/TP which proceeds, to my mind, from mere confusion in definitions. Basing on the fact that the bar N was a DOUBLE-toise copied from an identical Fortin's copy of the TP those authors came to recognize the "toise" of Struve's results as a "SINGLE toise" of the Struve standard N. This speculation leads to the equation:

1 "single Struve toise" = (3a) / 2 , or:
CC = 1.949081 IM/TP.

Of course, such an approach should be considered definitely wrong as the unit "Struve toise" was identical to the unit presented by the Toise of Peru as showed in paragraph 2.

The second outsider is the Toise of Peru itself, as far as it was in 1887-1891 when its metric certification was made. The result can be presented as:

CC = 1.949090 IM/TP.

Available sources give reasons to assume that the TP has probably suffered lengthening after 1823. A reliable (not single) evidence of that comes from the well-known copy of the TP made in 1823 for Bessel. In that year the copy was certificated as nearly identical but in 1891 it was found by 29 microns (14 ppm) shorter than its original. At the same time the length of Bessel's toise remained the same if referred to its own copy "T9" (Helmert 1893).

The third well-known value:
1 toise = 1.9490363 (or 1.9490366) "metres"
has no relation to the IM, but refers to the so-called "legal" metre defined after the 1799 French Royal statute.

The difference between the two "metres" ("unfortunate duplication" after Bomford) has much been highlighted previously and therefore it is omitted here. Thus traditional source material related to the "Russian version" of the TMP leaves the 8 ppm uncertainty (CCe). It makes it NOT INTERESTING to compare between the Struve results and possible GPS re-measures of remaining arc fragments. It can be supposed, however, that lack of information is responsible for the problem.

4. ANOTHER EVIDENCE.

Two Swedish sources (Lindhagen 1863, Jäderin 1915) have been studied which have never been considered with respect to the TMP. They evidence that the 1893 B.I.P.M. certificate of the bar N (2) is unlikely to be valid for the Struve times. The basic Struve standard has probably LENGTHENED some time in between 1862 and 1899. This supposition originates from the material of comparisons performed in those years between the bar N and its last primary copy made in 1861 for Swedish Academy of sciences. The detected change of the preceding length difference between the bars was 22 - 26 microns. At the same time the Swedish bar remained practically unchanged if referred to its own subsequent copy.

This new evidence presents the most serious reason so far to question the validity of previous practical solutions of the TMP. Neither the value (3a) nor its corrected version (3d) can undoubtedly match the starting expression (1). Too many suppositions are needed to support the use of those DIRECT results.

INDIRECT links of the Struve bar mentioned in paragraph 2 referred to the three European standards: the British yard (Y), the toise of Bessel (TB, a copy of the TP) and the klafter of Vienna (K). Each one has been certificated afterwards in terms of the IM. The lacking metric value of the bar N can be obtained from the most reliable final metric figures. Using the necessary source quantities, respectively, from (Struve 1860, Clarke 1866 and Bomford 1862), (Struve 1860 and Helmert 1893), (Struve 1860 and Allmer 1990) one can derive:

N<Y> = 0.9144025 (1728.01249 / 405.34622) = 3.898146 IM,
N<TB> = 2 TB + 31 microns = 3.898153 IM, cf. (3b),
N<K> = 1.8965092 (1728.01249 / 840.70370) = 3.898153 IM.

Three completely different standards agree to 2 ppm in reproducing the required metric "identity" of the Struve standard:

N from 3.898146 to 3.898153 IM. (3f)

It looks like the upper two thirds in the questioned range (3e) cover the least supported values in question. The latter range corresponds to apparently the most reasonable CC value:

CC from 1.949059 to 1.949062 IM/TP, (CCf)

where the uncertainty leaves nothing to be desired.

5. CONCLUSION

The possibility of reduction of the inherited dispersion of the toise-to-metre Conversion Coefficient related to the Struve arc has serious reasons. Adoption of the most reasonable value (CCf) would SHIFT to some extent the evaluations of the actual accuracy of the two Struve arc segments derived previously (Kaptüg et al 1996). In the northern part of the arc triangulation (Hogland-Fuglenaes, see Fig.1) the discrepancy between the 1994 GPS results and those by Struve would approach 3-sigma level or roughly 20 ppm per 1189 km. In the southern part (Hogland-Staro-Nekrassowka) the discrepancy would not exceed one sigma or 4 ppm per 1641 km. Thus a significant DIFFERENCE of the TWO PARTS of the Struve arc seems to be true.

It is interesting that due to inverse signs of those discrepancies, the general Struve's result:

1447787 toises, see Fig.1, is in a VERY GOOD AGREEMENT (within few metres per 2822 km) with the observed GPS value on the WGS-84 ellipsoid.

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Appendices

Figure 1. Geodetic results of the 1816-1852 arc measurements presented by F.G.W.Struve.
Figure 2. A measurement with the Struve base apparatus (by permission of the Library of Pulkovo astronomical observatory).

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