Using Adaptive Filtering to Detect Multipath and Cycle Slips in GPS/Acclerometer Bridge Deflection Monitoring Data

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Key words: Adaptive Filtering (AF), Kinematic GPS, Accelerometer, Multipath Mitigation, Cycle Slips, Bridge Deformation Monitoring.

ABSTRACT

The use of accelerometers and GPS, either separately or as a combined approach is becoming a reliable and useful tool for real time bridge deflection monitoring.

RTK GPS for such monitoring, however, is prone to error sources such as multipath and cycle slips. Adaptive Filtering (AF) is being investigated at the University of Nottingham as a tool that can be used to combine the accelerometer and GPS data, as well as to detect and correct the GPS data for multipath and cycle slips.

The following paper details the AF theory used, as well as its incorporation into the GPS/accelerometer bridge deflection data. Trials have been conducted upon a suspension bridge. The results of these trials illustrate that it is indeed possible to detect such error sources using the proposed filtering.

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1 INTRODUCTION

With sophisticated GPS receivers, the deformations of the various structures can be monitored in real-time mode with 3D positioning precision to several millimetres in a low dynamic environment (Dodson et al. 2001). To reach this positioning precision, the various error sources need to be subtly modelled or mitigated (Duff and Hyzak 1997). Of all the GPS error sources, the impacts of multipath and cycle slip together with integer ambiguity resolution to the quality of the measurements is still a recent research emphasis if high positioning precision is pursued (Bisnath et al. 2001; Cross 2000). Many efforts are required to detail the error contents and characteristics. In addition to the applications of modern data processing algorithms, sensor integration is another approach to raise the GPS positioning precision. Whether in a raw data format, such as pseudorange or carrier phase, or in integrated ones, such as coordinates from data processing, the output from GPS receiver can be treated as a time series. Suppression of the inherent GPS noise, the detection of the existing cycle slips and extraction of useful information are the routine in the treatment of these collected time series. Using the cross correlation between two time series, the adaptive filtering (AF) approach can be applied to suppress the noise level and isolate the information of interests (Ge et al. 2000; Roberts et al. 2001a; Meng et al. 2001). The benefit of the AF approach is that relative noisy raw data can be pre-processed and cleaned, and re-entered into the data processing procedure in order to obtain relatively precise resolution. Research reveals the efficiency of the AF approach largely depends upon the degree of cross correlation of the two time series. The higher the correlation is, the better the AF technique is able to strip out the noise component. In this paper, the authors first briefly introduce trials using Leica dual frequency receivers and a trial axial accelerometer to monitor a small footbridge. Then the acceleration data are double integrated, resulting in relative displacements, which are applied to GPS time series to mitigate multipath using AF. The results are analysed and comments on how to use this approach are made. Spectral analyses are applied to the input and output data from the AF procedure to illustrate the efficiency of the AF approach through the comparison of the changes of frequency distribution. The results reveal big improvements both in the GPS and accelerometer data sets.

Cycle slips will cause the correct ambiguity to be lost, which will dramatically reduce the GPS data quality. Starting with the comparison of various cycle slip detection and repair techniques, the authors present two kinds of cycle slip detection methods, which can be used conjunction with the AF approach to isolate residual multipath and ionospherical delay. To get the optimum

AF output and greatly reduce multipath impact to the GPS raw measurements, the exact time match of AF input signals is important to obtain maximum cross correlation index. The time shifts of two continuous days' satellite repeatability are analysed to each satellite, which is further used to align the pseudorange measurements. Also the time shift of total satellite configuration in view is analysed. The positioning differences between the exactly and normally aligned data sets are used to demonstrate the importance of the impact of appropriate data alignment. Through the practical data sets, the authors illustrate how the AF approach can be an efficient tool to output high quality pseudorange multipath template for each individual satellite. This will help detect and determine very small cycle slips and also speed the integer ambiguity resolution.

2 GPS TRIALS

To study the viability of using state-of-the-art dual frequency code/carrier GPS receivers to monitor structural movements, trials were conducted at 5 locations on a footbridge on the 22, 23 and 24 November 2000 by the IESSG staff and a colleague from Brunel University. The data gathering trials were divided into five sessions during the three days. The reference GPS receiver was setup on a survey point on top of an adjacent building. The GPS receivers used were Leica 530 dual frequency geodetic receivers, and the antennas were Leica choke ring antennas, all of which push the capabilities of GPS to the forefront of current technology. In addition, a Kistler triaxial accelerometer was housed within a cage attached to the underneath the GPS antenna on a midspan site. The GPS data rate was 10 Hz and the data rate of the accelerometer was set to 200 Hz on each axis. Due to the large data files, the sessions were split into 5 or 6 sub-sessions for processing purposes. The resulting coordinates were transformed into the bridge coordinates with the developed algorithm (Roberts et al. 2001b).

Existing criteria defining acceptable limits to dynamic motion of footbridge relate to pedestrian comfort. For bridges these have been derived from subjective tests, mainly for vertical movements at frequencies above 1 Hz. The dynamic frequencies less than 1 Hz are of great interests to the bridge engineers to analyse any unusual vibrations of the footbridge. In this paper the focus of data analysis will be on the viability of using GPS and accelerometer integration to detect low frequency vibrations.

3 ADAPTIVE FILTERING APPROACH FOR GPS MULTIPATH MITIGATION

GPS noise and real structural deformation signals tend to fall into the same range of frequencies when GPS is used as a deformation monitoring sensor. The noise sources including multipath are normally changing over time due to changes of GPS satellite configuration and observation environment. An adaptive tuned filter is needed to mitigate or isolate the deformation signal from various noise sources in the real time mode. The principal of the AF approach and application oriented comments, especially for multipath mitigation and deformation isolation in low dynamic environment, are well documented (Ge et al. 2000; Haykin 1996; Dodson et al. 2001).

Through appropriate data ordering or combination, recent research reveals that AF is an efficient tool for the isolation and separation of receiver random noise, multipath, residual ionospherical and tropospherical delay, cycle slips and real structure deformations (Meng et al. 2001). Further research into the statistic characteristics of different error sources can be made based on this research. This is fundamental for various GPS error modelling and data processing techniques.
The key to the AF approach used to isolate different time series is the level of the cross correlation of two time series used as desired and references signals. The simulations reveal that the higher the cross correlation is, the better the results can be obtained from the AF approach. To appropriately align two time series to get optimal results is crucial.

The time series as inputs for the AF can be integrated positions such as coordinates, raw GPS measurements such as pseudorange and carrier phases or the relevant data sets from other sensor such as the relative displacements from acceleration double integral.

To analyse the multipath level and also cycle slips on each satellite in view, the GPS raw measurements are needed. A Matlab M-program was developed and used to separate pseudorange and carrier phase data from RINEX format data file. The outputs are the data files including the raw measurements of each individual satellite.

It is well known that with the current GPS satellite constellation, the whole satellite configuration normally advances about 4 minutes between two continuous days. Using cross correlation and the raw measurements isolated from the RINEX data format output for the data sets collected on 22 November 2000 at one midspan site on the footbridge deck, the time shifts for each satellite between two days are calculated (Table 1). The final column is the total time shift of the satellite constellation in view at this site using the two days' coordinates time series.

Table 1. Time Shift in Two Days of Each Satellite in View

PRN	4	5	6	9	24	25	29	30	Total
Shift	4'06"	3'56"	4'10"	4'08"	4'06"	4'13"	3'50"	4'12"	3'41"

Analysis reveals that the time shifts are affected by actual satellite orbit perturbation caused by the solar pressure and air drag as well as the impact from propagation media. Further research will be carried on in this aspect.

Figure 1 and 2 are the AF outputs for the normally aligned (4 minutes) and exactly aligned (3 minutes 41 seconds) two days' relative vertical positions at the midspan site. The first rows on both graphs are the relative vertical positions on the second day. The second rows are reference signals from the first day's measurements. The third rows are signals for receiver noise plus small bridge movements relevant to the second day's measurement only. The final rows are the isolated multipath series. The horizontal axes on both graphs are labelled with the number of points, which is 400-second measurement. Figure 3 is the difference of the uncorrelated components of the normally and exactly aligned AF results. Figure 4 shows the difference of residual multipath caused by time series misalignment. It can be concluded that misalignment of a few seconds of the two time series in the AF inputs will introduce centimetre error in multipath template determination and about two-centimetre error in the combined time series of receiver random noise and real deformation. If this multipath template is applied to further data processing to suppress multipath impact, it will certainly cause distortion.

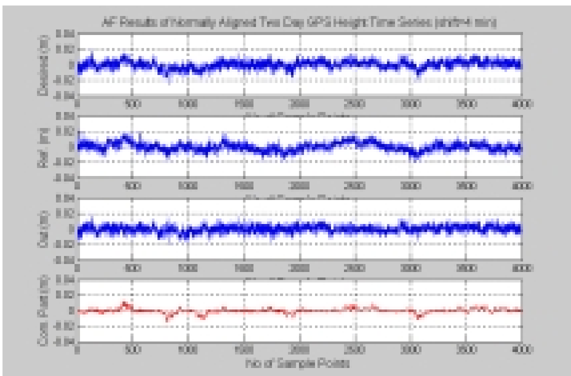

Figure 1. AF results from normally aligned data

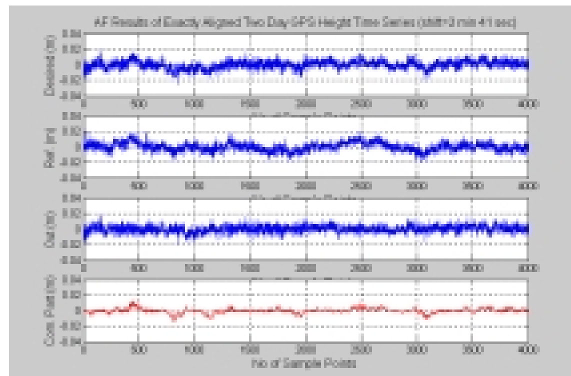

Figure 2. AF results from exactly aligned data

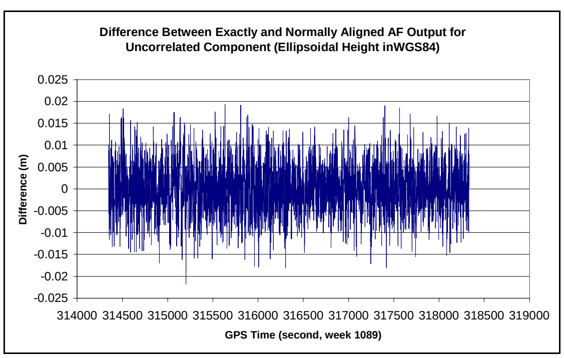

Figure 3. Difference of uncorrelated components

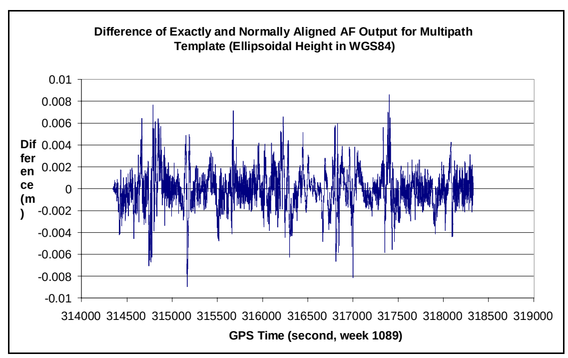

Figure 4. Difference of multipath templates

The raw measurements from the accelerometer are the acceleration of each axis. Even though the two kinds of sensors are physically installed together, the direct comparison of GPS coordinates and accelerations using the AF approach can not be conducted due to different dimensions. The accelerations need to be converted into displacements by double integration. The accelerometer errors such as zero biases accumulate into distance errors according to the equation of motion (Lawrence 1998)

$$S = vt + 1/2at^2 \qquad (1)$$

where v is the initial velocity; a is the acceleration; and s is the distance travelled in time t.

Simulation calculation reveals that an uncompensated bias of 1mg in a journey of 1 hour (starting at rest, v=0) will give a distance error of 65 km with Equation 1. Corresponding to the accelerations sensed by the instrument, the accelerometer outputs voltages, which are proportional to the inputs. As most systems are controlled by digital computers, the analogue outputs should be digitised by an A/D converter. The scale factor is the ratio between the change in the output signal and the change in input signal. Accelerometer scale factor error will cause distance error when the sensor is accelerating. For bridge monitoring, if the accelerations experienced are less than 0.1g, a 1% scale factor error will give an acceleration uncertainty of 1mg, which is undistinguished from the above bias and will cause 65km distance error in 1 hour measurement period.

Equation 2 is used to calculate velocity from acceleration, an approach to approximate acceleration integral,

$$v(t) = v(t-1) + \frac{\Delta t}{2}(a(t) + a(t-1)) \qquad (2)$$

where $v(t)$, $v(t-1)$, $a(t)$, $a(t-1)$ are the velocity and acceleration at time t and $t-1$, respectively. Δt is the time interval of data sampling.

Since the research is interested in the relative displacements, the initial velocity can be set to zero. A moving averaging (MA) filter can be applied to cope with the drift problems of the velocities and displacements (Equation 3). In principal, the MA approach is a low-pass filter. It can be used to isolate longer period movements from the time series of integrated velocity and displacement, which are contaminated by the systematic errors of the accelerometer. The residual between original time series and the smoothed output from MA constitutes the local higher frequency variation (high-pass filter), which mainly represents real structural vibration and the impact from high frequency noise. Simulation reveals that the selection of sample number used for averaging is crucial. This requires considerable experience plus knowledge of the frequency aspects of time series analysis. Spectral analysis is used for this frequency identification purpose. In the spectral analysis of the bridge data gathered from GPS and accelerometer, the frequency of this research interest is below 2 Hz. To use relative displacements calculated from accelerometer data as an input of the AF approach, the accelerations are re-sampled from 200 Hz down to 10 Hz. Simulation results show that noise with frequency larger than 2 Hz is filtered out in the MA procedure via 10 sample averaging.

$$MA(a_t) = \frac{1}{|s|+q+1} \sum_{r=-q}^{+s} a_r \qquad (3)$$

$MA(a_t)$ is the smoothed output for time series a at time t. a could be acceleration or filtered velocity.

Figure 5 is the comparison of original acceleration, calculated velocities and relative displacements using the above proposed algorithm. The original data are ten-minute long vertical accelerations, which are chopped from one-hour continuous acceleration data set. It is obvious that the drifts caused by systematic errors have been successfully removed. The obvious vibration pattern can be identified through acceleration double integral. Figure 6 illustrates the spectrum comparison of the accelerations, velocities and displacements. Except for the reduction in spectral power during the double integral and MA procedure, the main frequencies of the bridge movements are left unchanged.

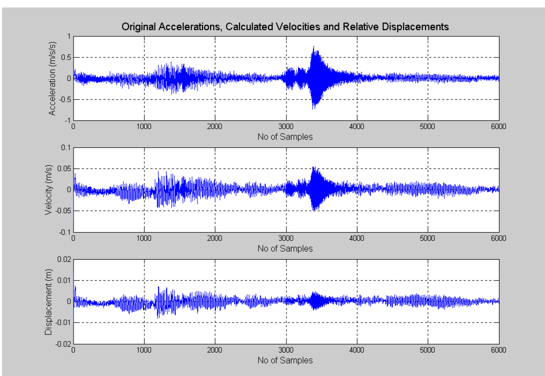

Figure 5. Acceleration, velocity and displacements

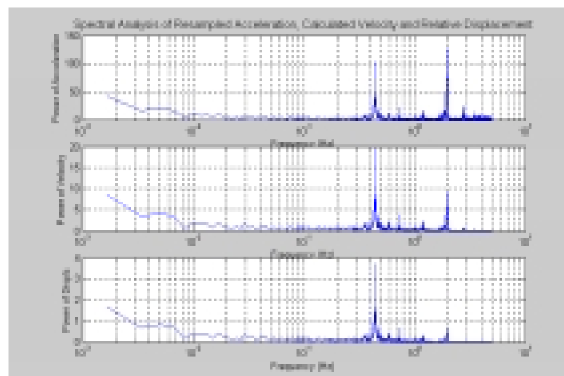

Figure 6. Spectrum of the time series on the left

Figure 7 is the output of the accelerometer-aided AF approach. The desired signal is the relative movement of the bridge calculated from GPS coordinates in Bridge Coordinate Systems. The relative displacements from the accelerometer act as a reference signal. The third row is the noise output relevant only with GPS. The fourth row of this graph is the correlated component of the two sensors, which is the bridge movements sensed by both instruments at the same time. The time series synchronisation of the two sensors was realised through a software package at this stage. The relative displacements calculated by the AF approach are about 1.0 cm, resulting only from the near natural movements caused by the wind blowing because there were no pedestrians on the bridge during the data collection. The graphs in Figure 5 show obvious vertical movement of the bridge deck, which can not be identified from the GPS data. Figure 8 is the spectrum of the desired signal (GPS data only) and Figure 9 is that for the filtered output from the AF algorithm (the common part of GPS and accelerometer data). In Figure 8, the multipath shows itself as a very strong signature with a frequency signature of 0.05 Hz, which stands for a 3 minute and 20 second multipath period. The natural frequency of the bridge movement is buried within other frequencies and can hardly be identified from them. Through the AF procedure, both GPS multipath and the accelerometer noise have been successfully filtered out (bear in mind that the fourth row of Figure 7 shows only correlated part is the output). The natural frequency with relative high power spectrum can now be very easily identified from Figure 9.

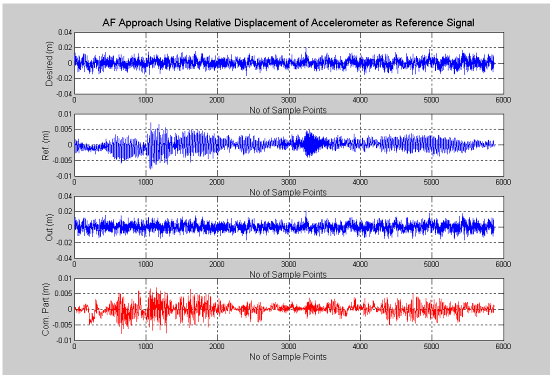

Figure 7. Accelerometer Aided AF approach

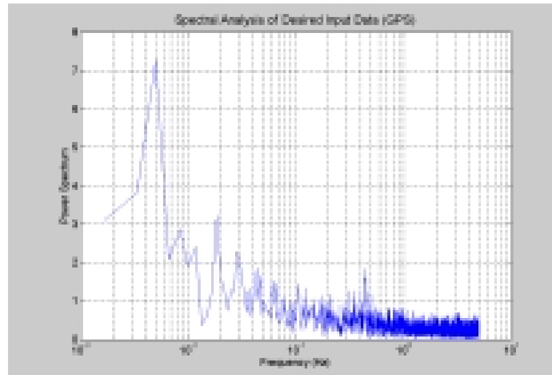

Figure 8. Spectrum of GPS data

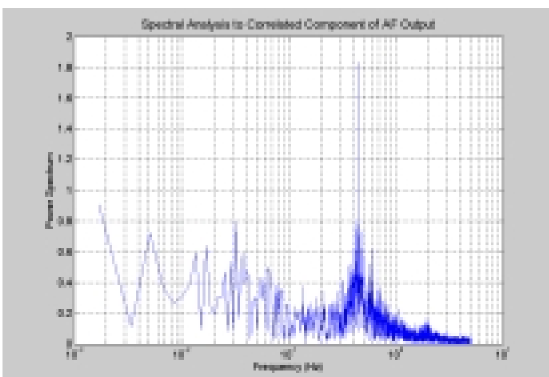

Figure 9. Spectrum of correlated AF output

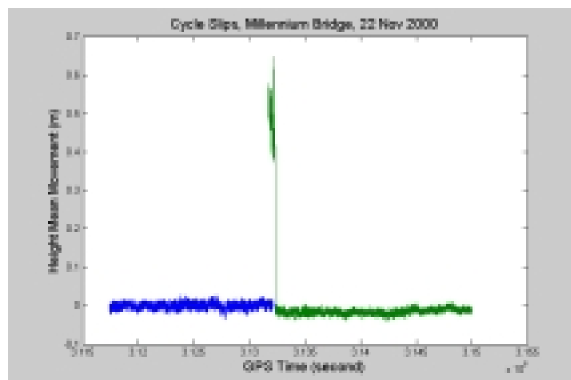

Figure 10. Cycle slips and their impact to ambiguity resolutions

4 APPROACHES FOR THE DETECTION OF VERY SMALL CYCLE SLIP AND THE CALCULATION OF RESIDUAL IONOSPHERICAL DELAY AND MULTIPATH

Cycle slips are discontinuities in the measured carrier beat phase resulting from loss-of-lock in the tracking loop of a GPS receiver (Strang and Borre 1997). The procedure of cycle slip treatment includes detection, determination and repair. Cycle slip detection and repair requires the location of the jump (i.e., cycle slip) and the determination of its size. Detection is accomplished by a testing quantity. The determination of the cycle slip size and the correction of the phase data is often denoted as cycle slip "fixing" (Hofmann-Wellenhof et al. 1997). Then, repairs are made by correcting all the subsequent phase observations for certain satellites and their carrier by the fixed amount. The formulation of testing quantities is based on measured carrier phases and pseudoranges. For a single site, the testing quantities are phase, phase combinations or combinations of phases and code ranges. Single-, double-, and triple-differences provide testing quantities for two receiver sites. With dual frequency receivers, the test quantities can be formed with the linear combination of carrier phases and possibly pseudorange observations (Bisnath 2000). Each of the described testing quantities enables the location of cycle slips to be found by checking the difference of two consecutive epoch values. This also yields an approximate size of the cycle slip.

Different approaches are in use to identify the discontinuities of the testing quantities (Seeber 1993). They include:

- A low degree polynomial is fitted to the time series.
- A dynamic model is set up to predict subsequent observations by Kalman filtering. A comparison between predicted and observed data can lead to indication of possible cycle slips (Bastos and Landau 1988; Collin and Warnant 1995).
- A scheme of first, second, third and even fourth differences can be set up. Discontinuities show rather strong signals in the higher order differences.
- Wavelet approach to detect the changes of cycle slips over time (Collin and Warnant 1995).

As aforementioned, in order to obtain the positioning results from 10 Hz GPS measurements, the whole session data need to be sub-divided into 45-minute subsections to cope with the deficiency of the current available data processing software. Figure 10 shows the relative displacements in the vertical direction at the same midspan site. Detailed analysis of the satellite configuration shows that satellite PRN 6 was obstructed by an adjacent building and this caused temporary full loss-of-lock of signal on L2 both on carrier phase and pseudorange. Ambiguity resolution re-fixing caused obvious position drift (green time series in Figure 10). Excluding PRN 6 from the data processing introduced a poor satellite geometry and further worsens the position solutions. Also detailed carrier phase and pseudorange analysis reveals that there are small cycle slips on the L1 carrier as well.

For further study of these phenomena and in the context of bridge applications, single difference of L1 or L2 phase/code range combination is used to detect small cycle slips up to one cycle accuracy.

Neglecting the relative small quantities of multipath impacts (quarter of a cycle in carrier phase), the measured phase $\Phi_i^j(t)$ can be simply modelled by

$$\lambda\Phi_i^j(t) = \rho_i^j(t) + \lambda N_i^j + c\Delta\delta_i^j(t) - \Delta^{iono}(t) + \Delta^{trop} \quad (4)$$

and code pseudorange by

$$R_i^j(t) = \rho_i^j(t) + c\Delta\delta_i^j(t) + \Delta^{iono}(t) + \Delta^{trop} \tag{5}$$

The testing quantity at epoch t from a phase/code range combination is as follows

$$\lambda\Phi_i^j(t) - R_i^j(t) = \lambda N_i^j - 2\Delta^{iono}(t) \tag{6}$$

The testing quantity at $t+1$ is as follows

$$\lambda\Phi_i^j(t+1) - R_i^j(t+1) = \lambda N_i^j - 2\Delta^{iono}(t+1) \tag{7}$$

Single difference of a phase/code range combination for consecutive two epochs is as follows

$$\Delta(\lambda\Phi_i^j(t+1) - R_i^j(t+1)) = 2\Delta^{iono}(t+1) - 2\Delta^{iono}(t) \tag{8}$$

Because the ionospheric fluctuation is a relative slow changing procedure, the sudden jump caused by cycle slips can be easily identified through the above procedure. Figure 11 illustrates the two to twenty cycle slips identified from L1 carrier phase/code combination when there were cycle slips on L2. This means when there are big cycle slips on L2 there might be small cycle slips on L1 as well. The above approach can be used to detect and determine the location and size of very small cycle slips on both carriers.

The high precision pseudorange can help the integer ambiguity resolution. Based on the exactly aligned pseudorange observations for two days from one receiver site to the same satellite, a new algorithm, which can be used to extract residual ionospherical and multipath as well as to detect cycle slips is developed using pseudoranges from both carriers.

The pseudorange for each individual satellite on L1/L2 on two days can be expressed as

$$P_{L_i/day_j} = \rho_{day_j} + I_{L_i/day_j} + T_{day_j} + c*(\delta^S_{day_j} - \delta^R_{day_j}) + M_{L_i/day_j} + \varepsilon_{L_i/day_j}$$

The difference of the pseudorange observations on the two carriers on the same day is

$$\Delta P_{day1} = \Delta I_{day1} + \Delta M_{day1} + \Delta\varepsilon_{day1}$$
$$\Delta P_{day2} = \Delta I_{day2} + \Delta M_{day2} + \Delta\varepsilon_{day2}$$

When AF approach is used to analyse these two time series, the correlated part is the residual multipath and the uncorrelated part is residual ionospherical delay relevant to desired input. Any cycle slip will appear on the time series of the uncorrelated AF output. One cycle slip can be detected using this approach because the residual ionospherical delay is very small (less than 1 cycle in this case, refer to Figure 12 for details).

Figure 12 and 13 are the time series of the extracted residual ionospherical delay and residual multipath respectively for satellite PRN 9. These time series can be applied back to the original pseudorange time series to get greatly improved raw pseudorange data. It can be further used to identify small cycle slip through the quality improvements to the test quantity of the combination of carrier phase and pseudorange. With this higher precision pseudorange, a fast integer ambiguity resolution can be realised. The extracted residual multipath can also be used as a template to mitigate the multipath on the following days.

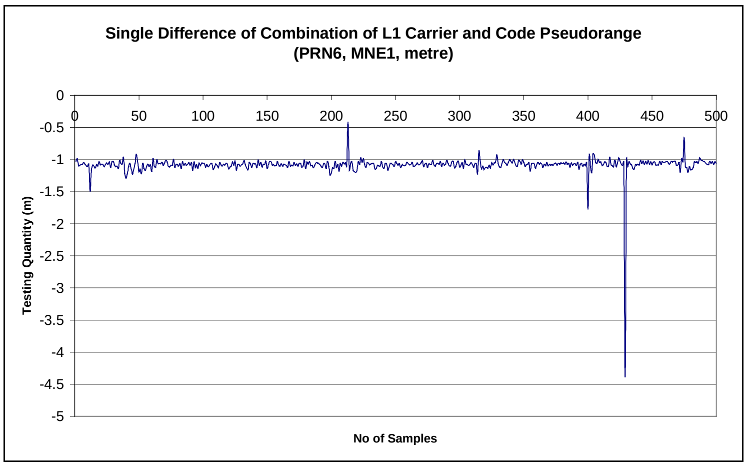

Figure 11. Single difference of L1 carrier and pseudorange

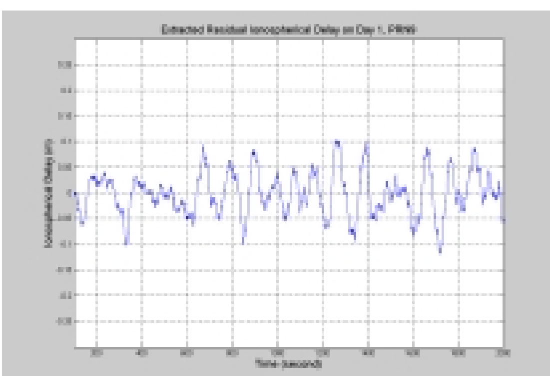

Figure 12. AF residual ionospherical delay output

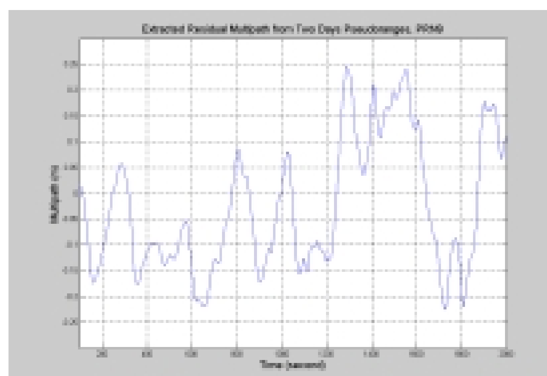

Figure 13. AF residual multipath output

CONCLUSIONS

In this paper, the authors present three kinds of multipath mitigation techniques, which can be used to treat the multipath in raw measurements and integrated GPS positions. The algorithm for double integration of acceleration to calculate relative displacements is illustrated. Spectral analysis is used as a validation tool to confirm the correctness of this algorithm. With the AF approach and using the relative displacements calculated from the accelerometer, the position precision can reach mm level for vertical displacements. For detecting very small cycle slips on L1 carrier phase, a formula using single difference of the combination of carrier phase and pseudorange is derived and applied to the footbridge data. The results show that the proposed method can be used to detect very small cycle slips. Another cycle slip algorithm based on the AF approach is demonstrated in this paper. Using appropriately aligned raw pseudoranges on two days, the residual multipath and ionospherical delay relevant to desired signal are successfully isolated, due to the improvement in pseudorange measurement

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BIOGRAPHICAL NOTES

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Professor Alan Dodson is Director of the Institute of Engineering Surveying and Space Geodesy (IESSG), and Dean of the Faculty of Engineering. He has a BSc in Civil Engineering and a PhD in Engineering Geodesy, both from Nottingham. He has extensive research experience in a range of subject areas including physical and space geodesy, and engineering surveying. His main current research interests are the application of the Global Positioning System (GPS) to a range of environmental, engineering and navigation applications. He is also collaborating with IESSG and European colleagues in the development phase of the proposed

European satellite navigation system, Galileo. He is president of Section 1 of the International Association of Geodesy and a Fellow of the Royal Institute of Navigation.