On the Use of Crustal Deformation Models in the Management of ETRS89 Realizations in Fennoscandia

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Outline

- The Fennoscandia land uplift process
- What is ETRS89?
- Applying models of crustal deformations!
- The new model NKG2016LU
- Details on transformations

→ next talk by Pasi Häkli
Motivation – users asking for high performance in real time!
The Fennoscandia Post Glacial Rebound, or GIA process!
The Glacial Isostatic Adjustment (GIA) phenomenon
What is ETRS89?

- The European Terrestrial Reference System
- Coincident with ITRS at epoch 1989.0, and fixed to the stable part of the Eurasian tectonic plate.

- From Boucher & Altamimi (2011) Memo v8, chapter 3:

$$X^E(t_c) = X^I_{YY}(t_c) + T_{YY} + \begin{pmatrix}
0 & -\hat{R}3_{YY} & \hat{R}2_{YY} \\
\hat{R}3_{YY} & 0 & -\hat{R}1_{YY} \\
-\hat{R}2_{YY} & \hat{R}1_{YY} & 0
\end{pmatrix} \times X^I_{YY}(t_c) \cdot (t_c - 1989.0)$$

the plate motion model
Realizations of ETRS89 in Fennoscandia (the Nordic /Baltic countries)

<table>
<thead>
<tr>
<th>Country</th>
<th>Name of realization</th>
<th>ETRF version</th>
<th>Realization epoch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denmark</td>
<td>EUREF-DK94</td>
<td>ETRF92</td>
<td>1994.704</td>
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<td>Estonia</td>
<td>EUREF-EST97</td>
<td>ETRF96</td>
<td>1997.56</td>
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<td>Faroe Islands</td>
<td></td>
<td>ETRF2000</td>
<td>2008.75</td>
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<tr>
<td>Finland</td>
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<td>ETRF96</td>
<td>1997.0</td>
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<td>Latvia</td>
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<td>ETRF89</td>
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<td>Lithuania</td>
<td>EUREF-NKG-2003</td>
<td>ETRF2000</td>
<td>203.75</td>
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<td>Norway</td>
<td>EUREF89</td>
<td>ETRF93</td>
<td>1995.0</td>
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<tr>
<td>Sweden</td>
<td>SWEREF 99</td>
<td>ETRF98</td>
<td>1999.5</td>
</tr>
</tbody>
</table>
To note:

In presence of crustal deformations, the epoch is crucial.

Therefore:

Time tag everything!
The NKG_RF2003_vel velocity model

Horizontal (0 to 2 mm/yr): The GIA model in Milne 2001 transformed to the GPS-velocities (in Lidberg 2007).

Vertical (-1 to 10 mm/yr): The NKG2005LU(ABS) model Based on: TG, repeated levelling, and GPS. (Ågren & Svensson 2006)
Some formulas for the use of the model of crustal (intraplate) deformation

From velocities to coordinate differences

\[
\begin{pmatrix}
  dX \\
  dY \\
  dZ
\end{pmatrix} = \left( t_{\text{target\_epoch}} - t_{\text{observation\_epoch}} \right) \begin{pmatrix}
  V_{X_{\text{int\_ra}}} \\
  V_{Y_{\text{int\_ra}}} \\
  V_{Z_{\text{int\_ra}}}
\end{pmatrix}
\]

From velocities in (n,e,u) to (X,Y,Z) frame

\[
\begin{align*}
\dot{X} &= -\frac{Z}{R} \frac{X}{P} \dot{n} + \frac{-Y}{P} \dot{e} + \frac{X}{R} \dot{u} \\
\dot{Y} &= -\frac{Z}{R} \frac{Y}{P} \dot{n} + \frac{X}{P} \dot{e} + \frac{Y}{R} \dot{u} \\
\dot{Z} &= \frac{P}{R} \dot{n} + \frac{Z}{R} \dot{u}
\end{align*}
\]

Where: \( R = \sqrt{X^2 + Y^2 + Z^2} \)

And: \( P = \sqrt{X^2 + Y^2} \)

(assuming a spherical earth)
Principle transformation scheme from ITRFs to national realization of ETRS89

- ITRS $t_{\text{obs}}$
- ETRS89 $t_{\text{obs}}$
- ETRS89 $t_{\text{campaign}}$

Standard transformation provided by EUREF (the MEMO)

Model of intraplate deformations, e.g. NKG_RF03vel
Practical transformation scheme while connecting to known permanent GNSS stations – example Sweden

Standard transformation provided by EUREF (the MEMO)

3D Helmert

ITRS
Internal epoch 1999.5

ETRS89
$t_{\text{campaign}}$

ETRS89
$t_{\text{obs.}}$

ITRS
$t_{\text{obs.}}$

NKG_RF03vel

Precise analysis done in ITRF current epoch

SWEREF 99 ep 1999.5
CRD of reference stations

NKG_RF03vel
Comparing the national realizations of ETRS89 in Fennoscandia

The NKG2008 campaign in ETRF2000 compared to national realizations.

**Left**, @ epoch 2008.75.

**Right**, @ epoch 2000.0, using a model for intraplate velocities *(NKG_RF03vel)*

Statistics: \((n,e,u)\) in mm

<table>
<thead>
<tr>
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<th>RMS</th>
<th>Mean</th>
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<tbody>
<tr>
<td>Left</td>
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<td>-4</td>
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<tr>
<td></td>
<td>12</td>
<td>5</td>
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<td>69</td>
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<table>
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<tr>
<th></th>
<th>RMS</th>
<th>Mean</th>
</tr>
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<tbody>
<tr>
<td>Right</td>
<td>8</td>
<td>-3</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>19</td>
</tr>
</tbody>
</table>

NKG2003 based on ITRF2000,
NKG2008 based on ITRF2005.

Left, NKG2008 @2008.75;
NKG2003 @ 2003.75

Right, booth @ epoch 2003.75, using the model NKG_RF03vel
(No fit – just coordinate differences!)

Statistics: (n,e,u) in mm
RMS   4  5  24
Mean  -5 -4 16

Statistics: (n,e,u) in mm
RMS   4  4  8
Mean  0 -3 -3
Creating a new model for the Fennoscandia crustal deformations

**Input to the modelling**

- Observations from repeated levelling
- 3D GPS velocities from the BIFROST project (10-15 year time series)
- Also a GIA-model
- But tide-gauge data is not used for the modelling!
The semi-empirical land uplift model NKG2016LU_abs/lev
difference between BIFROST GNSS and NKG2016LU_ABS

- Statistics (mm/year):
  - # 179
  - Min -2.00
  - Max 1.32
  - Mean 0.02
  - StdDev 0.42
The horizontal velocity model

GPS velocities minus GIA model "best sites":
(0.4, 0.2, 0.4) (n,e,u) mm/yr std.
(after 6-par fit, applying rotation and translation rates)

Note the systematic differences in the North direction
Developments in GIA modelling: New Thermo-mechanical ice model examples at LGM

The "old" ice history model (Lambeck 1998)

The "new" ice history model from Lev Tarasov.

The ice history governed by models for climate and glaciology.
New GIA model rotated to the GPS velocities (in ETRS89)

Using 66 well defined GPS sites
RMS: 0.2 mm/year in north and east
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