THEORETICAL AND EMPIRICAL MINIMUM DETECTABLE DISPLACEMENTS FOR DEFORMATION NETWORKS

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Introduction

Objective

Theoretical Minimum Detectable Displacement

Empirical Minimum Detectable Displacement

Simulated Networks

Results

Conclusions
What is Minimum Detectable Displacement (MDD)?

- **sensitivity**,  
  - Detectability of expected displacements and deformations at the designated network (Even-Tzur, 2006)

- **reliability**,  
  - Displaced points that are detected by Conventional deformation analysis can be validated whether these points are actually displaced or not by means of simulation.

- **accuracy assessment**,  
  - Difference between estimation value and simulated value
Objective

- Theoretical Minimum Detectable Displacement (TMDD) depends on the power of test; such as %80, %70.
  - Which one is more realistic?

- To reach an optimal Minimum Detectable Displacement (MDD), Empirical Minimum Detectable Displacement (EMDD) can be obtained as an alternative.
  - Empirical Technique: Using Displacement Ellipse
  - Empirical Technique 2: Step-by-Step Approach
\[
\lambda = \frac{d_k^T Q_{dd}^+ d_k}{\sigma_0^2} ; \quad \lambda > \lambda_0
\]

If \( \lambda > \lambda_0 \), it is inferred that the deformation network is sensitive. This comparison process called as sensitivity analysis (Caspary et al., 1983; Niemeier, 1985; Cooper, 1987; Even-Tzur, 2006; Aydin et al., 2004).

- \( d_k \): the expected deformations vector
- \( \sigma_0^2 \): a priori variance of unit weight
- \( Q_{dd}^+ \): Pseudoinverse of the cofactor matrix
\[ g = [\cos t_1 \sin t_1 \cos t_2 \sin t_2 \ldots \cos t_p \sin t_p]^T \]

p: The number of points

In vector \( g \) the components of the undisplaced points are assumed as “0”

\[ d_k = a g \]

\( a \): The computable scale factor

\[ \lambda = \frac{a^2}{\sigma_0^2} g^T Q_{dd} g ; \lambda > \lambda_0 \]

\[ \frac{a^2}{\sigma_0^2} g^T Q_{dd} g > \lambda_0 \quad \rightarrow \quad a_{min} = \sigma_0 \sqrt{\frac{\lambda_0}{g^T Q_{dd} g}} \]
Empirical MDD: Using Displacement Ellipse

The circle whose area is equal to the area of the expected displacement ellipse could be chosen as given in Fig. 1, so that the total area of the positive part is equal to the total area of the negative part.

Expected displacement ellipse and corresponding displacement circle of Point P (a and b are the semi-major and semi-minor axes of the expected displacement ellipse) (Hekimoglu et al., 2010).
The semi axes of error ellipse

$$\lambda_1 = \frac{Q_{dxixi} + Q_{dyiyi} + w}{2}, \quad \lambda_2 = \frac{Q_{dxixi} + Q_{dyiyi} - w}{2}$$

- $Q_{dxixi}$ and $Q_{dyiyi}$ = elements of the respective submatrix of the cofactor matrix $Q_{dd}$, which belongs to the $i^{th}$ point
- $\lambda_1$ and $\lambda_2$ = Semi-major and semi-minor axes of the standardized expected displacement ellipse

$$w = \sqrt{(Q_{dxixi} - Q_{dyiyi})^2 + 4Q_{dxixi}^2}$$

The formulas for the elements of the rescaled expected displacement ellipse
EMDD - Using Displacement Ellipse

\[ a = \sigma_0 \sqrt{\lambda_1 \chi^2_{2,1-\alpha}} \quad b = \sigma_0 \sqrt{\lambda_2 \chi^2_{2,1-\alpha}} \]

\[ \sigma_0^2 = \text{“True” variance component} \]

- \[ \chi^2_{2,\alpha} = \alpha \text{- fractile of the } \chi^2_{2} \text{-distribution for 2 degrees of freedom} \]
- \[ \alpha = \text{Error probability} \quad ; \quad a, b = \text{semi-axes for each points} \]
- \[ r = \sqrt{ab} \]
- \[ r = \text{Radius of the corresponding displacement circle} \]
- \[ \alpha \text{ is chosen here as 0.001 so the stochastic effect is reflected almost entirely in the simulated displacement magnitude.} \]
EMDD – Step-by-step Approach

- Second EMDD magnitude obtained by step approach testing.
- This technique estimates the minimum magnitude of displacement for different directions depending on the global congruency test.
- To reach the final step quicker, the first value has chosen randomly which can both detectable and reflect the stochastic model of network.
- Then first value was increased or decreased 0.1 mm to get the minimum displacement magnitude.
Chosen random first displacement magnitude

- $H_0$ is accepted
  - 0.1 mm increased
    - Global Test
      - $H_A$ is accepted
      - $H_0$ is accepted

- $H_A$ is accepted
  - 0.1 mm decreased
    - Global Test
      - $H_0$ is accepted
      - $H_A$ is accepted

MDD
The horizontal control network consists of 16 point observations with 9. DoF = 48 (by taking 9 orientation unknowns into account).

The lengths of the distance measurements are varied approximately between 3 and 7 km.

\[ \sigma_{\text{direction}} = \pm 0.3 \quad \text{mgon} \]

\[ \sigma_{\text{distance}} = \pm 3 + 2 \quad \text{ppm} \]
\( l_1 = \bar{l} + e_1 \)  \hspace{1cm} A: Coefficient matrix
\( l_2 = \bar{l} + e_2 + A \ast z \)  \hspace{1cm} z: deformation vector
\( \bar{l} \): uncontaminated measurements
\( e_1 \) and \( e_2 \): normally distributed random error vectors
random errors differently for each epoch \( N(\mu=0,\sigma^2_d) \),
\( z = [z_{1x} \ z_{1y} \ z_{2x} \ z_{2y} \ ... \ z_{ux} \ z_{uy}] \); displacements of
the points
\( z_x \): The projection of deformation vector \( z \) to \( x \) axes of the
deforation,
\( z_y \): The projection of deformation vector \( z \) to \( y \) axes of the
deformation
\( A \ast z \): The deformation vector for the corresponding
measurements
Results for Horizontal Control Network

- The components of displacement circle were obtained for points A, B and C which are assumed as displaced and radius was calculated as $r = 29.9$ mm.

- MDD were computed at 40 different directions for 3 MDD techniques

- with $\alpha = 0.05$ and $\beta = 0.20$. The displacements magnitudes were obtained for $\alpha = 0.05$ and $\beta = 0.30$ as well
Two circles correspond to radius \((r, 2r)\) interval that were obtained from 1\(^\text{st}\) Empirical Technique: Using Displacement Ellipse.

“+” refers to 2\(^\text{nd}\) Empirical Technique: Step-by-Step Approach.

“∗” \(1-\beta=0.80\),

“○” \(1-\beta=0.70\) shows the TMDD.
TMDD and EMDD magnitudes for point B
TMDD and EMDD magnitudes for point C
Simulated Networks: GPS Network

- 3 points at the Davutpasa Campus of Yildiz Technical University
- 4 points TUSAGA-AKTIF (SARY, KABR, SLEE, ISTN)
- 2 points from IGS network (ISTA, TUBI)
- \( \sigma_0^2 = 3.59^2 mm^2 \) (Eckl et al., 2001)
- 32 baselines were observed
- 7 hours of GPS measurements were carried out
the radius was computed as \( r = 7.1 \text{ mm} \)
TMDD and EMDD magnitudes for point OBC2
TMDD and EMDD magnitudes for point OBC3
Conclusions

• The magnitudes of the TMDD techniques are greater than the ones of the EMDD techniques.

• Accordingly, the simulation of the displacement should be based on EMDD techniques for the performance of the deformation analysis method.

• The obtained result values from Empirical MDD more realistic than the Theoretical MDD.

• Two Empirical techniques converges to each other.
  – Step-by-step approach is the slowest amongst these techniques.
Thank you for your attention!