

# A New Method for 3D Cadastral Parcel Merging Based on Conformal Geometry Algebra

Jiyi ZHANG, Pengcheng YIN, Gang LI, Zhaoyuan YU, Di HU and Yongyi ZANG, China

**Key words:** Cadastre, Data Model, Conformal Geometry Algebra, 3D Parcel Merging

## SUMMARY

With the development of urbanization, conflicts between the demand for land due to urban expansion and the limitation of land resources contribute to the appearance of complex buildings below and above land surface. Traditional two dimension (2D) cadastre has encountered great challenge in registering overlapping and interlocking constructions in the urban area. There is no doubt that developing three dimension (3D) cadastre could resolve the problems in registering and managing objects with complex structures more efficiently. Much research has been carried out on the development of 3D cadaster in recent years, which is conducive to the implementation of 3D cadastral management. However, since Euclidean geometry lacks consistency in unified representation form for cadastral objects with different dimensions, traditional cadastral data models based on Euclidean geometry represent dimensional cadastral objects in a rather different way. Due to the dimensional isolation characteristics of Euclidean geometry, representation and management of 3D cadastral objects are more complex than 2D cadastral objects, thus hindering implementation of 3D cadastral objects management and updating. In order to represent cadastral objects with different dimensions in a unified multidimensional manner, Conformal Geometry Algebra (CGA) is introduced in this paper. As the hierarchical Grassmann structure corresponding to the hierarchical structure of dimensions in CGA, cadastral objects in different dimensions can be expressed in a unified form with outer product. Different dimensional objects can be organized and stored by the multivector structure in a multidimensional unified way in CGA. The advantages of CGA in multidimensional expression are used to construct a new cadastral data model which is introduced briefly in this paper to represent multidimensional objects in a unified form. Since 3D cadastral objects spatial updating is important in 3D cadastral management, operators based on CGA are developed to realize 3D cadastral objects' spatial merging in a symbolic way. The method for 3D cadastral spatial parcel merging based on CGA operators is analyzed in detailed in this paper. Case studies have been carried out to elevate the new method for 3D cadastral objects' updating. Results show that our method can merge spatial objects in a more symbolic and geometry-oriented way compared with the traditional way based on Euclidean geometry.

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## 1. INTRODUCTION

Since the two dimension (2D) cadastre cannot clearly express and manage the complex buildings and cadastral rights in urban area, the three dimension (3D) cadastre has been developed to address those limitations in traditional 2D cadastral management. Numerous developments for 3D cadastre have been achieved in the past few years. Several types of 3D cadastral data models have been developed (Henssen 1995, Meyer 2001, ePlan 2010, ISO19152 2912).

Since the land ownership boundaries and the rights should be strictly and accurately defined in cadastral registration, 3D cadastral data models use the topological relations among dimensional components to represent the construction structures of the 3D cadastral parcels (Tse and Gold 2003, Zlatanova et al 2004, Ying et al 2014). Therefore, the most typical data representation models for the 3D cadastre are the topology-based. They focus on the description of topological relations among different dimensional construction elements of 3D objects. The essence of those topological data models is actually a 3D extension of topological relations among point, line and plane used in 2D dimensional space. And the combining and splitting of the 3D cadastral parcels are based on the complex topological relation-based intersection and splitting.

Almost all the current existing 3D cadastral data models are constructed in Euclidean geometry space. However, different dimensional objects in Euclidean space cannot be represented in a unified expression form. Expression methods for 3D objects in Euclidean space are distinct from 2D objects. Spatial representations for 3D parcels in a cadastral database are more complicated than 2D parcels. The complexities of topological relationships among components of 3D objects increase the difficulty of development of 3D cadastre. The hierarchical Grassmann structure is not consistent with the hierarchical structure of dimensions in Euclidean space, which is the main reasons for the division of multidimensionality in Euclidean space (Yuan et al 2013). Limitations of Euclidean geometry in multidimensional expression cause problems in the extension and development from 2D cadastre to 3D cadastre. There are still no universally accepted 3D cadastral data models for complex geometries representation efficiently. How to store and update geometry and topological relations for 3D cadastral objects in database efficiently is still a challenge in the 3D cadaster development (van Oosterom 2013).

To solve problems encountered in 3D cadastre development, we have introduced Conformal Geometric Algebra (CGA) in 3D cadastral data model construction (Zhang et al 2016). In this paper we mainly focus on the algorithm for 3D objects updating which is represented by 3D cadastral spatial data model based on CGA.

## 2. CONFORMAL GEOMETRIC ALGEBRA

### 2.1 Background of CGA

Lots of mathematicians try to integrate different geometric system into a unified algebraic framework in middle of 19th century. The concept of Quaternions is first developed by Irish mathematician Hamilton in 1843, which successfully extended complex number to higher dimension (compare to 2D). In the next year 1844, Germany mathematician Grassmann published his masterpiece, *The Theory of Linear Extension, a New Branch of Mathematics*, in which he developed the concept of outer product. Based on outer product, hypercomplex number can be extended to n dimensions. And after thirty five years, professor Clifford proposed the concept of geometric product based on which he developed the theory of geometric algebra that integrated Grassmann extensive algebra and Hamilton Quaternions in 1878. Geometric algebra is a kind of algebraic language which is used to describe and solve geometric problems (Clifford 1878). It was developed based on Hamilton Quaternions and Grassmann extensive algebra by professor Clifford. As a result, geometric algebra also called Clifford algebra.

The ability of geometric algebra in unified multidimensional expression and geometric computation can be used to solve geometric problems in algebraic form. Inner product, outer product and geometric product are three basic arithmetic operators for geometric space construction and geometric objects representation in geometric algebra (Perwass 2009). Outer product can be used to extend dimensions while inner product can realize the opposite effect. For instance, the outer products between two vectors produce bivector with two dimensions and the inner products of two vectors produce scalars. Outer product and inner product are integrated in geometric product as following definition.

Definition 1: If  $A$  and  $B$  are two objects with arbitrary dimensions, the geometric product of  $A$  and  $B$  can be define as follow:

$$AB = A \cdot B + A \wedge B$$

in which  $A \cdot B$  denotes the inner product of  $A$  and  $B$  and  $A \wedge B$  means outer product. If both  $A$  and  $B$  are vectors, we can obtain a scalar from inner product and a bivector from outer product. This mean that in the result of geometric product different dimensional objects can be integrated which is similar to real and imaginary parts in complex number.

Euclidean geometry, homogeneous geometry and conformal geometry are three types of geometric algebraic systems. Euclidean geometric space is the most familiar one to people because it is in line with people's understanding of the world. However, the structure of outer product (also called hierarchical Grassmann structure) of different dimensions in Euclidean geometric space are inconsistent with the construction structure of geometric objects. For example, points in Euclidean geometric space are represented by vectors (from original point to current point). From theory of geometric algebra we know that the outer product of two vectors produce bivector with two dimensions which can be denote by a plane containing the two vectors. This mean that the outer product of two Euclidean points produce a plane instead of the line containing them. Homogeneous

geometric space is developed on the basis of Euclidean space by introducing an additional projection dimensions  $e_0$ . The hierarchical Grassmann structures in Homogeneous geometric space are consistent with the construction structure of geometric objects. For example, outer products of two homogeneous points represent the line that determined by the two points. And outer products of three homogeneous points represent the corresponding plane. Although homogeneous geometric space realizes the consistency between Grassmann structure and construction structure of geometric objects, the geometric meaning in inner products of homogeneous points is still unclear. Conformal geometric algebraic space is developed on basis of homogeneous geometric space by introducing another additional dimension – the infinite point  $e_\infty$ . The introduction of infinite point means that the inner product in conformal geometric algebraic space has the same meaning with Minkowski inner product. The inner product of two CGA points represents the Euclidean distance between the two points. Meanwhile, the geometric meaning of circle and line, sphere and plane have been unified in CGA space. Compared with other geometric systems, such as Euclidean geometry and homogeneous geometry, CGA has the unique advantages of simple and intuitive structure, distinct geometric meaning and unified characteristics with regard to the expression of the geometry (Yuan et al 2014, Rosenhahn and Sommer 2005, Hildenbrand 2011).

## 2.2 Relative Studies on CGA's Applications

As a branch of Clifford algebra, CGA was developed by David Hestenes, Hongbo Li and Alyn Rockwood (Hestenes 2001, Li et al 2001 and Li 2008). Advantages of CGA in unified multidimensional representation and hybrid dimensional computations (Hestenes 2002) have lots of important applications in many research fields such as geometry, engineering, computer vision and so on (Perwass 2009, Cameron et al 2005, Wareham et al 2005). Theory of CGA was applied in area of Geographic information system (GIS) in recent years. Based on geometric algebra, Yuan et al (Yuan et al 2010) developed a prototype software system in which they provides a useful tool for investigating and modeling the distribution characteristics and dynamic process of complex geographical phenomena under the unified spatio-temporal structure. Yu et al (Yu et al 2015) constructed a Delaunay – Triangulated Irregular Network intersection for change detection with 3D vector data based on advantages of CGA in topological relationship computations. Yuan et al (Yuan et al 2014) studied the network topology expression and algorithm construction based on Clifford algebra. Hu et al (Hu et al 2015) introduced geometric algebra to design methods for data modeling, spatio-temporal queries and dynamic visualization. Yuan et al (Yuan et al 2013) developed data models, data indexes, and data analysis algorithms for multidimensional vector data, raster and vector field data based on advantages of geometric algebra in multidimension-unified and coordinate-free. These applications of geometric algebra in GIS research indicate that the advantages of geometric algebra in multidimensional representation and coordinate-free in spatial topological relations computations provides a new mathematical tool for development of GIS.

## 2.3 A Cadastral Data Model Based on CGA

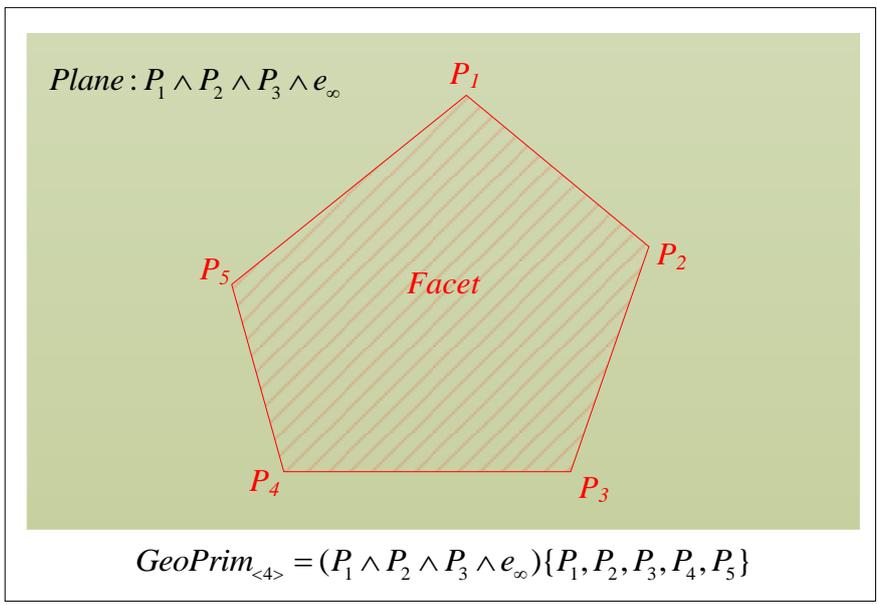
Based on existing research results in CGA applications, we introduced CGA to the area of 3D cadastral development (Zhang et al 2016). A 3D cadastral spatial data model which can realize the

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unified expression form for different dimensional cadastral primitives is developed based on CGA. Cadastral spatial units are decomposed into three types of primitive elements including points, boundary lines and boundary faces. All above three primitive elements together with the original Euclidean points are embedded into conformal space. Points, boundary lines and boundary faces in conformal space are represented by vector, bivector and quadvector, respectively. Primitive elements with different dimensions in CGA can be expressed by outer product uniformly. Considering that results of outer products such as bivectors do not have boundary constraint, the sequence of point sets that compose the corresponding cadastral objects are employed to restrict their boundary. Example can be seen in figure 1 which shows how we define a cadastral facet in conformal space. Cadastral spatial units in conformal space are stored and organized in the multivector structure, which is a basic mathematical element that can simultaneously integrate multiple dimensional primitives in CGA. All the dimensional component elements that construct the corresponding cadastral spatial unit are integrated in the multivector structure and organized according to different dimensions. To be understandable, figure 2 shows the process of representing a 3D cadastral spatial unit denoted by cube. Relative concepts and definitions in figure 2 can be found in reference Zhang et al 2016.  $\{ \}$  in figure 2 represents the sets of geometric elements' algebraic expression with the same dimensions.



**Figure 1. A primitive facet represented by outer products in CGA (Source: Zhang 2016)**

From the definition of cadastral spatial unit in figure 2 we can know that the algebraic expressions in CGA are hybrid dimensional structure which integrated in the multivector structure. The conformal algebraic expressions of cadastral objects enable us to handle them as an entity. It should be noted that all the different dimensional components are independent. This means that all the dimensional elements contained in the multivector structures can be employed to compute the spatial relations independent when we need to analyze the spatial relations between two cadastral

objects. In this paper, we utilize powerful computation ability of CGA to define the method for merging cadastral objects.

### 3. SPATIAL MERGING ALGORITHM BASED ON CGA

#### 3.1 Relative concepts in CGA

##### 3.1.1 Blade

The blade, generated by outer product, is a k-dimensional object that used to express the corresponding dimensional subspace in geometric algebra. This is different from linear algebra in which objects in all dimensions are represented by vector. For example, 1-blades (vectors) in geometric algebra used to denote one dimensional subspace and 2-blades (bivectors, generated by outer product of two vectors) denote two dimensional subspace. In the 3D cadastral data model that based on CGA, blades with corresponding dimensions are the geometric container for the basic cadastral components of GeoPrim (Zhang 2016).

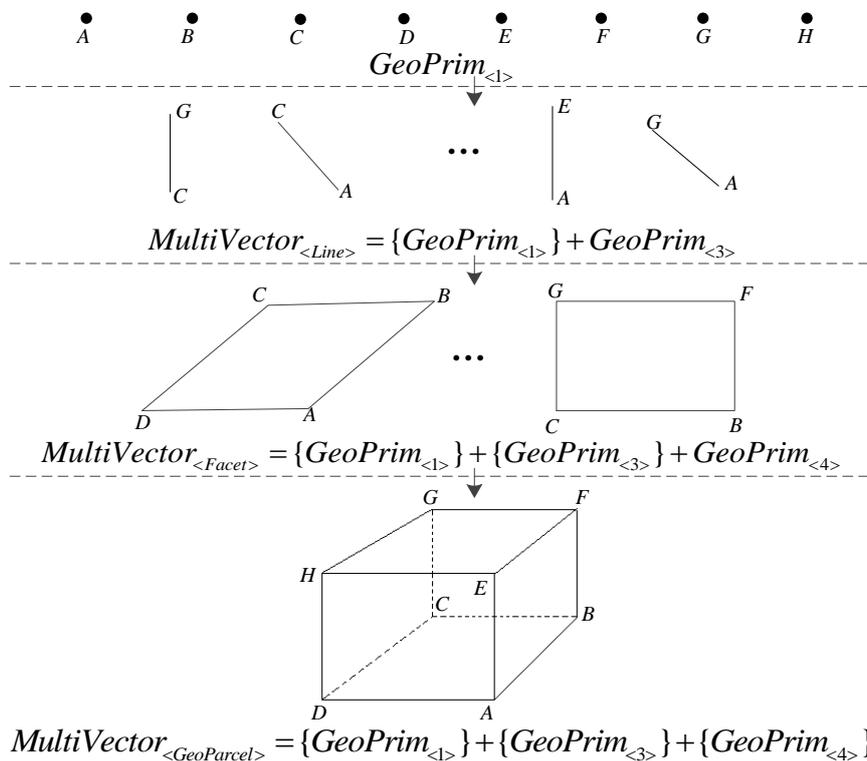


Figure 2. A 3D cadastral spatial unit algebraic representation in CGA (Source: Zhang 2016)

##### 3.1.2 Inner product

Inner product in CGA is a kind of subtraction operator for subspaces. It is different from the point product in vector algebra whose operation objects are limited to vectors. The definition of inner product is shown as follows.

Definition 2: If  $A$  and  $B$  are two blades with grade  $s$  and  $t$  in  $Cl_{4,1}$ , the inner product of them is defined as follow:

$$\langle A \rangle_s \cdot \langle B \rangle_t = \langle AB \rangle_r, \text{ where } r = \begin{cases} 0 & s < t \\ s-t & s \geq t \end{cases}$$

Inner product in CGA has a clear geometric meaning. The result of inner product of two blades can be used to denote the distance or angle between them.

### 3.1.3 Pseudo-scalar

The pseudo-scalar denoted by  $I$  is the highest dimensional blade in relative geometric space. For example, in  $Cl_2$  the pseudo-scalar is  $I = e_1 \wedge e_2$  while in  $Cl_3$  the pseudo-scalar is  $I = e_1 \wedge e_2 \wedge e_3$  and in the conformal space  $Cl_{4,1}$ , the pseudo-scalar is  $I = e_1 \wedge e_2 \wedge e_3 \wedge e_0 \wedge e_\infty$ .

## 3.2 Cadastral objects merging algorithm based on CGA

In this paper, the merging algorithm for cadastral objects expressed in CGA is developed. The algorithm mainly includes two parts: the spatial relations determination for cadastral objects and the merging method. As cadastral objects are represented by multivector in CGA, the spatial relations among them can be determined in a more algebraic way. As we discussed, the cadastral objects in CGA are represented and organized in multivector structure in which the different dimensional components are independent. As a result, the determination of topological relations among cadastral objects can be transferred to judge their different dimensional components that organized in multivector. The meet operation between two cadastral objects that need to be merged is defined as follows.

Definition 3: If  $A$  and  $B$  are two blades in  $Cl_{4,1}$ , the meet operator for  $A$  and  $B$  is defined as follow:

$$M = Meet(A, B) = A \cap B = A^* \cdot B$$

Where  $M$  is the common part of  $A$  and  $B$ ,  $A^*$  is the dual of blade  $A$  (Jaap 2003). The meet operator has two possible results which depend on the result of  $M^2$ , which is shown as follows:

$$M^2 = \begin{cases} = 0 & \text{no intersection} \\ > 0 & \text{intersection exists} \end{cases}$$

Consider that 3D cadastral objects mainly represent by lines and planes in CGA, the condition of  $M^2 < 0$  is not occur in this case which may exist in circle or sphere. It should be noted that the blades in CGA is infinite. The intersection of two blades do not equal to the intersection of GeoPrims which are used to represent cadastral objects. The intersection relations of GeoPrims need to be determined based on the reintersection results of blades by further boundary operator which is defined as follows.

Definition 4: If  $GeoPrim_{\langle A \rangle}$  and  $GeoPrim_{\langle B \rangle}$  are two primitive components that contained in blades  $A$  and  $B$  in  $Cl_{4,1}$  with the same dimensions, the boundary operator for  $GeoPrim_{\langle A \rangle}$  and  $GeoPrim_{\langle B \rangle}$  is defined as follow:

$$\begin{aligned} & Boundary(GeoPrim_{\langle A \rangle}, GeoPrim_{\langle B \rangle}) \\ &= Boundary(Ptas, GeoPrim_{\langle B \rangle}) \& Boundary(Ptbs, GeoPrim_{\langle A \rangle}) \\ &= (mPtas, nPtbs), (0, 0) \end{aligned}$$

Where  $Ptas$  and  $Ptbs$  are the boundary point sets that construct  $GeoPrim_{\langle A \rangle}$  and  $GeoPrim_{\langle B \rangle}$ , respectively.  $mPtas$  is the number of points belonging to  $GeoPrim_{\langle A \rangle}$  that contained in  $GeoPrim_{\langle B \rangle}$  and  $nPtbs$  is the number of points belonging to  $GeoPrim_{\langle B \rangle}$  that contained in  $GeoPrim_{\langle A \rangle}$ . We suppose that  $m$  and  $n$  denote the total number points belonging to  $GeoPrim_{\langle A \rangle}$  and  $GeoPrim_{\langle B \rangle}$ , respectively. Consider the characteristics of cadastral expression that all the  $GeoPrims$  between cadastral objects with different ownerships should be independent. Then we can conclude as follow: (1) if  $mPtas = nPtbs = m = n$ , this mean that  $GeoPrim_{\langle A \rangle}$  and  $GeoPrim_{\langle B \rangle}$  are the common parts which have to disappear in the process of cadastral objects merging; (2) if  $0 < mPtas = nPtbs < m = n$  and the normal direction of  $GeoPrim_{\langle A \rangle}$  and  $GeoPrim_{\langle B \rangle}$  is the same, this mean that  $GeoPrim_{\langle A \rangle}$  and  $GeoPrim_{\langle B \rangle}$  are the two parts that possibly need to merge; (3) if  $mPtas = nPtbs = 0$ , this mean that  $GeoPrim_{\langle A \rangle}$  and  $GeoPrim_{\langle B \rangle}$  are disjoint. To the second condition that  $GeoPrims$  possibly need to merge, we need to confirm whether exist other  $GeoPrims$  with the opposite direction that shared the same constructive points with current  $GeoPrims$  that need to be merged. If there are no other  $GeoPrims$ , then we can merge current two  $GeoPrims$ . Otherwise, the current two  $GeoPrims$  need to keep independent.

We have discussed the method for determination of cadastral components that need to disappear and merge in the process of cadastral objects merging. Now we will show how to merge two cadastral objects which is expressed based on CGA by the multivector structure. Since cadastral objects can be expressed by  $GeoPrim_{\langle k \rangle}$  (Zhang 2016), the combination of cadastral objects can be transferred to the problems of  $GeoPrim_{\langle k \rangle}$  combination. As the range of components is limited by the sequence of constructive points, if we completed the combination of cadastral facet which is expressed by  $GeoPrim_{\langle 4 \rangle}$  then we can obtain other components such as  $GeoPrim_{\langle 3 \rangle}$  from the

limitative sequence points. In conclusion, the essence of combination of cadastral objects that are expressed by multivector is to realize the combination of corresponding cadastral facets. The combination of two cadastral facets can be defined as follows.

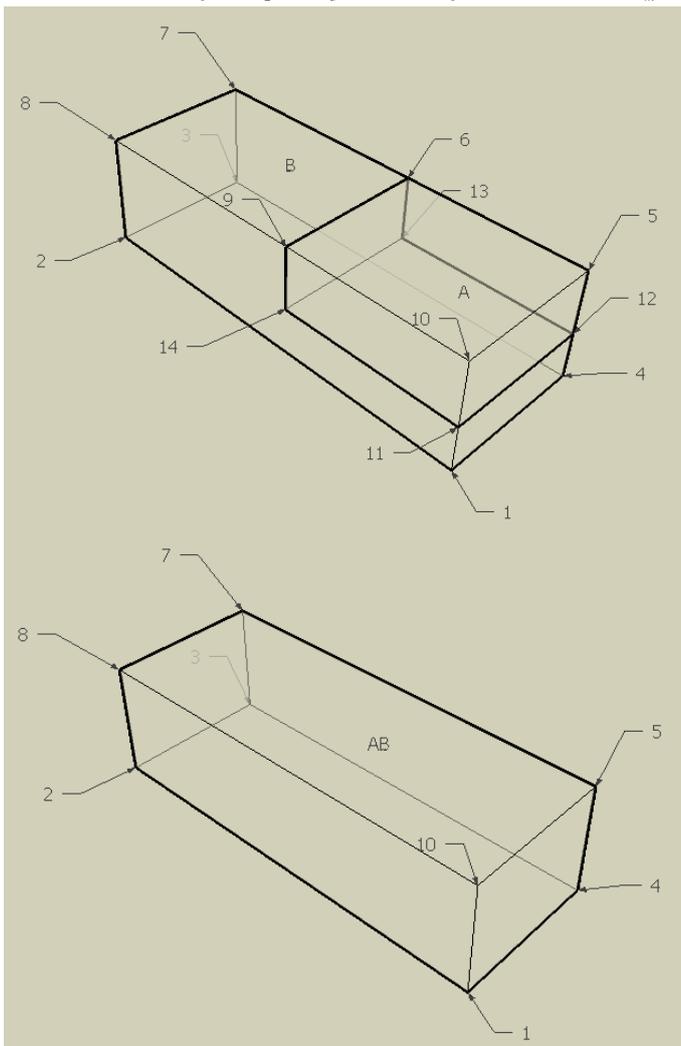
Definition 5:  $Facet_{\langle A \rangle}$  and  $Facet_{\langle B \rangle}$  are two constructive facets in  $Cl_{4,1}$  for cadastral  $A$  and  $B$ , their conformal representation are shown as following:

$$Facet_{\langle A \rangle} = (P_{a_1} \wedge P_{a_2} \wedge P_{a_3} \wedge e_\infty) \{P_{a_1}, P_{a_2}, P_{a_3}, \dots, P_w, P_x, \dots, P_y, P_z, \dots, P_{a_m}\} \text{ and}$$

$$Facet_{\langle B \rangle} = (P_{b_1} \wedge P_{b_2} \wedge P_{b_3} \wedge e_\infty) \{P_{b_1}, P_{b_2}, P_{b_3}, \dots, P_s, P_y, \dots, P_x, P_t, \dots, P_{b_n}\} .$$

The point sets  $P_x, \dots, P_y$  in  $Facet_{\langle A \rangle}$  are same with the inversion of the point sets  $P_y, \dots, P_x$  in  $Facet_{\langle B \rangle}$ . Now if we merge  $GeoPrim_{\langle A \rangle}$  into  $GeoPrim_{\langle B \rangle}$ , the result of combination can be defined as follows:

$$Facet_{\langle AB \rangle} = (P_{b_1} \wedge P_{b_2} \wedge P_{b_3} \wedge e_\infty) \{P_{b_1}, \dots, P_s, P_z, \dots, P_{a_m}, P_{a_1}, \dots, P_w, P_t, \dots, P_{b_n}\}$$



**Figure 3. An example for cadastral facet combination based on CGA representation**

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#### 4. CASE STUDIES

In order to evaluate the combination algorithm for cadastral objects expressed based on CGA, we design a simple case study. The experimental data is created by SketchUp software. We select two polyhedrons to denote the cadastral objects as examples. Results are shown in figure 3. We select the facet constructed by points 9, 10, 11, 14 in cadastral object *A* and 1, 2, 8, 9, 14, 11 in cadastral object *B* to merge as an example. The representations in CGA for above two selected facets are  $Facet_{\langle 9,10,11,14 \rangle} = (P_9 \wedge P_{10} \wedge P_{11} \wedge e_\infty)\{P_9, P_{10}, P_{11}, P_{14}\}$  and  $Facet_{\langle 1,2,8,9,14,11 \rangle} = (P_1 \wedge P_2 \wedge P_8 \wedge e_\infty)\{P_1, P_2, P_8, P_9, P_{14}, P_{11}\}$ , respectively. If we merge facet *A* into facet *B*, the combination result should be  $Facet_{\langle 1,4,12,11 \rangle} = (P_1 \wedge P_2 \wedge P_8 \wedge e_\infty)\{P_1, P_2, P_8, P_{10}\}$  according to definition 5 which is shown in figure 3. The components' expression of  $Facet_{\langle 1,4,12,11 \rangle}$  in multivector such as constructive arcs and points can be obtained from its limitative points sequence in CGA expression.

#### 5. CONCLUSION

In this paper, we introduced the basic information of geometric algebra and relative research results based on geometric algebra. The 3D cadastral data model based on CGA was mainly introduced. This study is a subsequent research based on Zhang et al 2016. We discussed the method to determine topological relations among cadastral objects which were represented based on CGA. Then we designed the algorithm for combination of cadastral facets which were organized and expressed within multivector structure. And other dimensional components in multivector for cadastral objects can be obtained easily from the relative facet's CGA expression. Finally, we designed a simple case study to demonstrate our combination algorithm for cadastral objects.

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