Detecting Rigid Body Movements from TLS-Based Areal Deformation Measurements

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Key words: Terrestrial Laser Scanning (TLS), B-Spline, Control Points

SUMMARY

The use of Terrestrial Laserscanning for deformation measurements is well-established in engineering geodesy. However, the available processing techniques in order to obtain deformations from point clouds and for the respective statistical assessment are quite restrictive. This paper describes an approach to obtain rigid body movements of objects based on point clouds that were recorded in consecutive epochs. The introduced approach comprises two steps. In the first step for each epoch the object's geometry is modelled as a B-Spline surface. The main concepts of the object modelling using B-Splines are introduced in the paper. In the second step the elements of the rigid body movements are derived using a similarity transformation between the control points of the modelled B-Spline surfaces. One advantage of the introduced approach is that it uses well-known point-based methods of deformation analysis while allowing areal-based considerations at the same time. The applicability and the performance of the method are shown for measurements obtained at an aluminium-test specimen forming a B-spline surface with known parameters. The rigid body movement's components obtained from the point clouds are statistically compared with those obtained from point-based measurements of a lasertracker.

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1. INTRODUCTION

The development of terrestrial laser scanners has substantially increased the importance of areal measurements in engineering geodesy. In order to preserve the added value yielded by these techniques in comparison to the conventional point based ones, an areal data analysis is unavoidable. In this paper the modelling by means of freeform surfaces such as B-splines is discussed. The estimation of this type of surfaces requires the allocation of appropriate surface parameters to the observations as well as the specification of an appropriate number of control points. Setting up these elements of the surface's definition is in the focus of the 2nd chapter of this contribution. The mathematical derivation of the retrieval of rigid body movements from the control points of the approximating B-spline surfaces is given in the 3rd chapter. A simulated example and a concrete example based on real measurements illustrate the approach's application in the fourth chapter.

2. ESTIMATION OF B-SPLINE-SURFACES

2.1 Mathematical representations

The parametric representation of a B-Spline-curve is given by:

$$\hat{\boldsymbol{C}}(u) = \boldsymbol{C}(u) + \boldsymbol{e}(u) = \sum_{i=0}^{n} N_{i,p}(u)\boldsymbol{P_i} \text{ , } u = [0,..,1], \tag{1}$$

where:

- $\hat{\boldsymbol{C}}(u)$ – point on the B-Spline-curve corresponding to the parameter u,
- $\boldsymbol{C}(u)$ – measured point allocated to the parameter u,
- $\boldsymbol{e}(u)$ – residual of the measured point $C(u)$,
- $\boldsymbol{P}$ – vector of control points of the B-Spline-curve,
- $N_{i,p}$ – B-Spline basis functions of degree p,
- n+1 – number of control points.

Based on (1) a B-Spline-surface can be considered to result from a double-infinite set of B-Spline curves aligned in the direction of the parameter axis u and v. The tensor-product formulation of such a surface is given by the product of two one-dimensional basis functions $N_{i,p}(u)$ and $N_{j,q}(v)$ respectively (Piegl and Tiller, 1997):

$$\hat{\boldsymbol{S}}(u,v) = \boldsymbol{S}(u,v) + \boldsymbol{e}(u,v) = \sum_{i=0}^{n_u}\sum_{j=0}^{n_v} N_{i,p}(u) N_{j,q}(v)\boldsymbol{P}_{ij} \text{ , } u, v = [0,..,1]. \tag{2}$$

where:

- $\hat{\boldsymbol{S}}(u, v)$ – point on the B-Spline-surface corresponding to the parameters u and v,
- $\boldsymbol{S}(u, v)$ – measured point allocated to the parameters u and v,
- $\boldsymbol{e}(u, v)$ – residual of the measured point $\boldsymbol{S}(u, v)$,
- $\boldsymbol{P}$ – vector of control points of the B-Spline-surface,
- $N_{i,p}(u)$ – B-Spline basis function of degree p in the direction of parameter u,
- $N_{i,q}(v)$ – B-Spline basis function of degree q in the direction of parameter v,
- $n_u + 1$ – number of control points in u-direction,
- $n_v + 1$ – number of control points in v-direction.

In this paper $N_{i,p}(u)$ and $N_{i,q}(v)$ are considered to be deterministic values which are calculated iteratively starting from the rectangular functions with support depending on the so called knot vector U (Schmitt and Neuner, 2015). An appropriate method to fix the locations of the knots is given by deBoor (1978) while Bureick et al. (2016) propose a method to estimate the knot locations. After having allocated appropriate parameters to the measured points and after having specified the number of control points ($n_u + 1$) and ($n_v + 1$), equation (2) can be formulated for each coordinate of a measured point and the resulting Gauss-Markov-model to estimate the control points' unknown location can be solved. In this paper the stochastic model of the adjustment corresponds to the unit weight matrix. Developments towards a fully occupied covariance matrix are given by Harmening et al. (2016) as well as Kauker and Schwieger (2016).

2.2 Parameterisation

Allocating the parameters u and v to the observed points is not straightforward since the measured points are not arranged along a regular grid. This task was treated extensively in Harmening and Neuner (2015). Here only an outline of their approach developed for surfaces delimited by four boundary curves is presented.

A common approach to parameterise unordered point clouds is the definition of a base surface with known parametric form and a subsequent projection of the observed points on this surface. Via this projection it is possible to allocate parameters belonging to the base surface's parameter space to the observations. The difficulty of this procedure lies in the definition of an appropriate base surface, which has to fulfil certain criteria according to Ma and Kruth (1995): On the one hand the base surface has to be as smooth as possible while being a good approximation of the point cloud. On the other hand, an unambiguous projection of the observed points on the surface has to be possible. As the base surface's parameterisation influences the parameterisation of the surface to be estimated, the former should furthermore reflect the point cloud's form. Regarding an automated analysis, the base surface's formulation should additionally be generally valid and consequently independent from the acquired data set.

In order to parameterise surfaces which are delimited by four boundary curves, Coons patch (Ma and Kruth, 1995) is used as a base surface. The first step for the construction of this patch is the determination of points at the edge of the surface. They form the basis for the estimation of four boundary B-spline-curves:

$$\hat{\boldsymbol{C}}_k(u) = \boldsymbol{C}_k(u) + \boldsymbol{e}_k(u) = \sum_{i_c=0}^{n_c} N_{i_c,p_c}(u)\boldsymbol{P}_{i_c}^k,\ k = 0,1,$$
$$\hat{\boldsymbol{C}}_l(v) = \boldsymbol{C}_l(v) + \boldsymbol{e}_l(v) = \sum_{j_c=0}^{m_c} N_{j_c,q_c}(v)\boldsymbol{P}_{j_c}^l,\ l = 0,1. \tag{3}$$

A ruled surface $\boldsymbol{R}_v(u, v)$ is generated by a linear interpolation between the curves $\hat{\boldsymbol{C}}_k(u)$. A second ruled surface $\boldsymbol{R}_u(u, v)$ is analogously generated from the facing curves $\hat{\boldsymbol{C}}_l(v)$. Finally, a third surface $\boldsymbol{B}(u, v)$ is constructed as the bilinear interpolant of the four corner points. The Coons patch results from the linear combination of these surfaces: $\boldsymbol{P}(u, v) = \boldsymbol{R}_v(u, v) + \boldsymbol{R}_u(u, v) - \boldsymbol{B}(u, v)$.

In order to project the measured points onto Coons patch, the base surface is subdivided into quadrilaterals which are supposed to be approximately planar. By means of a principal axis transformation the measured points are projected into the coordinate system of the nearest quadrilateral. Afterwards the requested parameters u and v can be computed by the inverse bilinear interpolation of the quadrilateral's corner points. The parameters determined in this manner can now be used to estimate a best-fitting B-spline surface $\boldsymbol{S}(u, v)$ according to (2).

The estimated surface obtained in this way can be used in a further iteration step as a new base surface as it approximates the measured points in a better way than Coons patch does. However, iterating the parameter allocation procedure and the subsequent adjustment of the surface may lead to divergence. It was observed that this divergence occurs at boundary locations with sparse distributions of measurements. The main idea to counteract this divergence is to introduce restrictions of the form, that the surface's outer isolines are similar to the boundary curves estimates in (3).

The introduction of constraints at the level of identities between curve equations of the form (1) is difficult to handle, especially if different number of control points for the surface isolines and for the boundary curves (3) are allowed. As the curve's characteristics are determined by the control polygons and as these polygons can be handled much easier than the continuous curves, the constraints are based on the control polygons. Due to the fact that the control polygons cannot be assumed to be identical in case of different number of control points, similarity measures for polygonal lines have to be formulated. Furthermore, a minimization of these measures has to be included as a constraint into the adjustment process in order to stabilise the parameterisation. A comparative analysis of different similarity measures given by Harmening and Neuner (2015) revealed that a method which is based on the parametrisation of the polygonal lines is very appropriate. The main idea of the approach is to introduce correspondences between points of the polygonal lines which have assigned the same parametric value and to minimize the distance between these corresponding points in terms of constraints in the adjustment. The main concept is illustrated in figure 1:

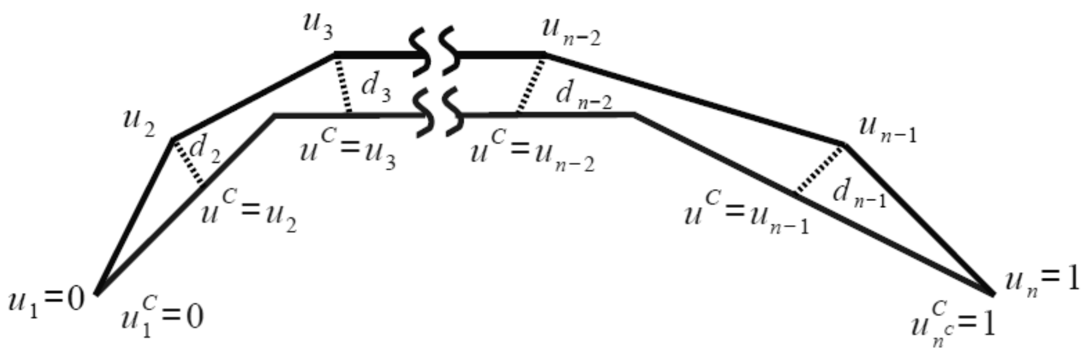

Fig 1.: Basic concept of the introduced constraints

Introducing the constraints in this way leads in general to satisfying approximation results. By stabilising the boundary regions of the surface the process of parameter allocation and surface adjustment converges after few iterations.

2.3 Model selection

The specification of the number of control points ($n_u + 1$) and ($n_v + 1$) in equation (2) is a problem of model selection. In the present paper the Bayesian Information Criterion (BIC) is used as a model selection criterion. The Bayesian/Schwarz Information Criterion (BIC/SIC) was introduced by Schwarz (1978) and is an asymptotic consistent criterion. It evaluates the posteriori probabilities of the candidate models M_j (j = 0,…,n) and chooses that one which seems to be the most likely according to the given data (Cavanaugh and Neath (1999)). Usually the unconditional likelihood of the data is equal for all candidate models such that the difference between posteriors is determined by the models' marginal likelihoods $\lambda_{n,j}$(data). As a closed calculation of this quantity is impossible in general, the marginal likelihood is approximated by means of the Laplace approximation, which results in the final computation of BIC (Claeskens and Hjort, 2008):

$$BIC_j \approx -2\log\left(L\left(\boldsymbol{\theta}_j|\text{data}\right)\right) + K_j \log\left(n_{obs}\right); \;\; j = 0..n, \tag{4}$$

where:

- L – log-likelihood,
- θ_j – coordinate vector of the unknown control points in the model M_j,
- K_j – number of estimated parameters of the candidate model M_j,
- n_{obs} – number of measured points used for the approximation.

By allocating the parameters according to the strategy described above as well as by specifying the number of control points using the BIC both prerequisite steps for solving the Gauss-Markov-model given by (2) are done. Thus, a B-spline-surface estimated from the respective point cloud is available for each epoch in a deformation measurement project.

3. RIGID BODY MOVEMENTS

In this chapter we will show that the rigid body movements of an object modelled by B-spline surfaces can be retrieved from the similarity transformation of the control points. The equivalence is exemplarily shown for one point on the B-Spline surface. The relation between the spatial location of the point coordinates determined in two epochs I and II is given by:

$$\begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix}_{II} = m\boldsymbol{R}\begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix}_{I} + \boldsymbol{t}\,, \qquad (5)$$

where:

- $\boldsymbol{R}$ – rotation matrix,
- $\boldsymbol{t}$ – translation vector,
- m – scale factor.

The B-Spline surface in equation (2) can be written for the point location in epoch I as:

$$\begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix}_{I} = \Big(\underset{(1,n_u n_v)}{\big(N_{0,p}N_{0,q} \quad N_{0,p}N_{1,q} \quad N_{0,p}N_{2,q} \quad \cdots \quad N_{n_u,p}N_{n_v,q} \big)} \otimes \underset{(3,3)}{\boldsymbol{I}} \Big) \underset{(3n_u n_v,1)}{\begin{pmatrix} x_{\boldsymbol{P}_{00}} \\ y_{\boldsymbol{P}_{00}} \\ z_{\boldsymbol{P}_{00}} \\ \vdots \\ x_{\boldsymbol{P}_{n_u n_v}} \\ y_{\boldsymbol{P}_{n_u n_v}} \\ z_{\boldsymbol{P}_{n_u n_v}} \end{pmatrix}_{I}} . \qquad (6)$$

For clarity reasons and without loss of generality the parameters *u* and *v* allocated to the considered point where omitted in (6). Introducing the transformation (5) in (6) the relation for the point's location in the second epoch results as a function of the elements of the B-Spline-surface adjusted in Epoch I:

$$\begin{aligned} \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix}_{II} &= m\boldsymbol{R}\begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix}_{I} + \boldsymbol{t} \\ &= m \underset{(3,3)}{\boldsymbol{R}} \Big(\underset{(1,n_u n_v)}{\big(N_{0,p}N_{0,q} \quad N_{0,p}N_{1,q} \quad N_{0,p}N_{2,q} \quad \cdots \quad N_{n_u,p}N_{n_v,q} \big)} \otimes \underset{(3,3)}{\boldsymbol{I}} \Big) \underset{(3n_u n_v,1)}{\begin{pmatrix} x_{\boldsymbol{P}_{00}} \\ y_{\boldsymbol{P}_{00}} \\ z_{\boldsymbol{P}_{00}} \\ \vdots \\ x_{\boldsymbol{P}_{n_u n_v}} \\ y_{\boldsymbol{P}_{n_u n_v}} \\ z_{\boldsymbol{P}_{n_u n_v}} \end{pmatrix}_{I}} + \underset{(3,1)}{\boldsymbol{t}} . \end{aligned} \qquad (7)$$

Successively one obtains:

$$
\begin{pmatrix}\hat{x}\\ \hat{y}\\ \hat{z}\end{pmatrix}_{II} = m\left(\underset{(1,n_u n_v)}{\begin{pmatrix}N_{0,p}N_{0,q} & N_{0,p}N_{1,q} & \cdots & N_{n_u,p}N_{n_v,q}\end{pmatrix}} \otimes \underset{(3,3)}{\boldsymbol{R}}\right)\underset{(3n_u n_v,1)}{\begin{pmatrix} x_{\boldsymbol{P}_{00}}\\ y_{\boldsymbol{P}_{00}}\\ z_{\boldsymbol{P}_{00}}\\ \vdots \\ x_{\boldsymbol{P}_{n_u n_v}}\\ y_{\boldsymbol{P}_{n_u n_v}}\\ z_{\boldsymbol{P}_{n_u n_v}}\end{pmatrix}_I} + \left(\sum_{i=0}^{n_u}\sum_{j=0}^{n_v} N_{i,p}N_{j,q}\right)\underset{(3,1)}{\boldsymbol{t}},
$$

$$
= \left(\underset{(1,n_u n_v)}{\begin{pmatrix}N_{0,p}N_{0,q} & N_{0,p}N_{1,q} & \cdots & N_{n_u,p}N_{n_v,q}\end{pmatrix}} \otimes \underset{(3,3)}{\boldsymbol{I}}\right)\begin{pmatrix} m\boldsymbol{R}\,\boldsymbol{x}_{\boldsymbol{P}_{00}}\\ \vdots \\ m\boldsymbol{R}\,\boldsymbol{x}_{\boldsymbol{P}_{n_u n_v}}\end{pmatrix} + \left[\underset{(1,n_u n_v)}{\begin{pmatrix}N_{0,p}N_{0,q} & N_{0,p}N_{1,q} & \cdots & N_{n_u,p}N_{n_v,q}\end{pmatrix}}\underset{(n_u n_v,1)}{\boldsymbol{e}}\right]\underset{(3,1)}{\boldsymbol{t}}, \tag{8}
$$

$$
= \left(\underset{(1,n_u n_v)}{\begin{pmatrix}N_{0,p}N_{0,q} & N_{0,p}N_{1,q} & \cdots & N_{n_u,p}N_{n_v,q}\end{pmatrix}} \otimes \underset{(3,3)}{\boldsymbol{I}}\right)\begin{pmatrix} m\boldsymbol{R}\,\boldsymbol{x}_{\boldsymbol{P}_{00}} + \boldsymbol{t}\\ \vdots \\ m\boldsymbol{R}\,\boldsymbol{x}_{\boldsymbol{P}_{n_u n_v}} + \boldsymbol{t}\end{pmatrix}.
$$

In the first equation of (8) the partition-of-unity-property of the B-Spline basis functions is used in the second summand. In the second equation of (8) $\boldsymbol{x}_{P\,ij}$ denotes the coordinate triple of the corresponding control point $\boldsymbol{P}_{ij}$. As can be noted from the third equation the control points of the surface approximation in epoch II results directly from the similarity transformation of the control points estimated in epoch I.

4. IMPLEMENTATION CONSIDERATIONS

4.1 Simulation studies

To exemplify the retrieval of a rigid body movement from the control points of B-Spline surfaces a measured point cloud was translated with the nominal parameters [100, 250, -50] and rotated with the nominal angles [-7°, 15°, 45°] around the x-, y- and z-axis respectively. The scale remained unchanged. The approximation procedure for B-Spline-surfaces described in the 2nd chapter was independently applied for the original and for the transformed point cloud as well. The obtained surfaces are shown in figure 2. By solving (5) the translation parameters [99.9999908, 249.999979, -50.000011] and the rotation parameters [-7.0000034°, 15.0000042°, 44.9999974°] were obtained from the sets of control points. These parameters differ from the nominal ones applied to the point cloud only by numerical inaccuracies, which demonstrates the practical applicability of (8).

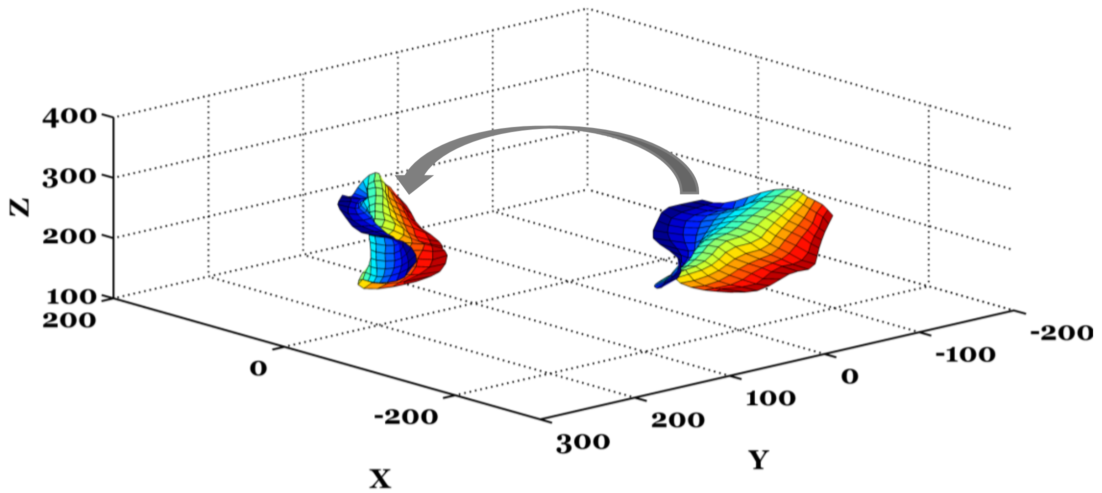

Fig. 2: B-Spline surfaces obtained from the original and from the transformed point cloud

4.2 Practical measurements

The determination of a rigid body movement from practical measurements was performed by means of a specimen, which is shown in figure 3. The object was designed by the authors and represents a B-Spline surface. All elements characterising the B-Spline surface e.g. parameters, knots and control points are known from the design and the manufacturing process. The specimen was scanned at two different locations using the Leica MS50.

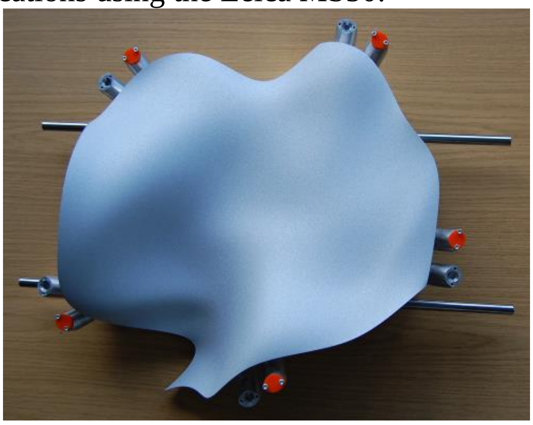

Fig. 3: Specimen representing an aluminium B-Spline surface

External points war mechanically linked to the specimen in the vicinity of the surface. The mark of these points allows a definite placement of a lasertracker reflector. Thus, the nominal transformation parameters of the rigid body movement are obtained by solving (5) with the two sets of coordinate triples of the external points determined by lasertracker measurements. The statistically significant transformation parameters are given in table 1.

t_x [m]	-0.263	0.003
t_y [m]	0.100	0.002
t_z [m]	0.000	0.002
r_z [gon]	- 2.2115	0.0311

Tab.1 Nominal transformation parameters and their standard deviations

As a main difference to the study in 4.1, in this experiment two completely different point clouds obtained from the scanning at each specimen's location are used. Thus, the points detected at the edge of the surface are different for the two locations, leading to different boundary curves and in consequence to a modified course of the parameter lines relative to the specimen. However, a main assumption of (8) is that the parameter lines remain unchanged by the rigid body movement.

To fulfil this assumption, the following procedure was adopted. The points at the edge of the surfaces were matched using the ICP algorithm in order to detect correspondences. Thus, the boundary curve parameters of the first epoch were allocated to the corresponding edge point of the second epoch. For both point clouds, the four boundary B-spline-curves were estimated by using only the corresponding points. By this procedure the course of the parameter lines is nearly kept unchanged. After this step the approximation of the B-spline surfaces was done as described in the 2[nd] chapter. The resulting surfaces can be seen in figure 4.

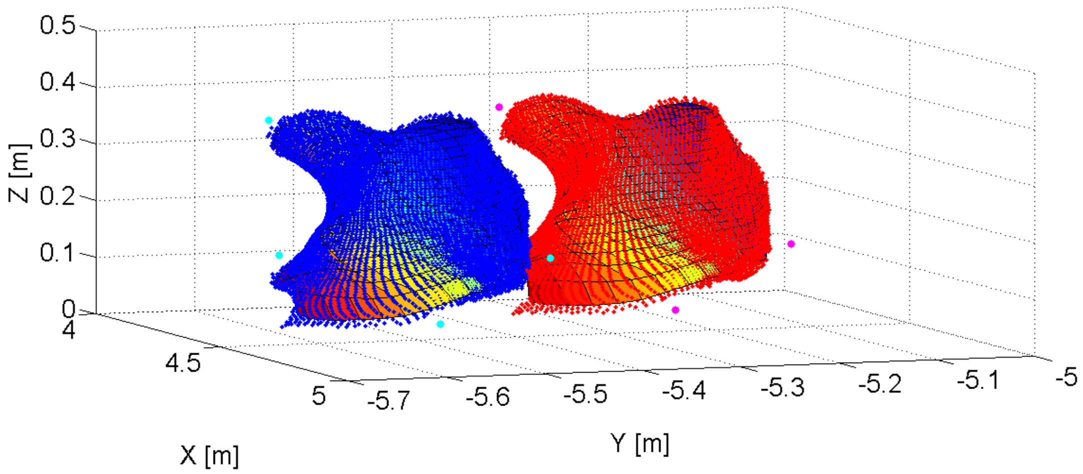

Fig. 4: Measurements of the test specimen at two different locations (blue and red point cloud), the estimated surfaces and the reference points, which were measured by the lasertracker (cyan and magenta coloured points)

The transformation between the estimated control points led to a standard deviation of unit weight of 0,002, which agrees very well to the performance of the used laser scanner. The obtained transformation parameters and their standard deviations are given in table 2. The last column of this table indicates the statistical significance at the level of 95% of the difference between the nominal transformation parameters and the ones obtained from the control points of the approximated B-spline surfaces.

t_x [m]	-0.251	0.011	No
t_y [m]	0.110	0.009	No
t_z [m]	0.001	0.001	No
r_z [gon]	- 2.0675	0.1252	No

Tab.2 Transformation parameters obtained from control points of approximated B-spline surfaces

As can be noticed from table 2 all differences to the nominal rigid body movement are statistically not significant. To analyse the impact of the functional differences between the transformation parameters given in table 1 and 2 respectively, the control points of the B-spline surface corresponding to the second location of the specimen were transformed into the system of the first location. The differences between the two sets of transformed control points were below 2 mm, which again agrees very well to the performance of the used laser scanner. Therefore, it can be concluded that in the practical case the rigid body movements can be retrieved from the control points of the approximated B-spline surfaces at the quality level of the used measurement instrument. However, in future better results are expected due to more refined procedures for preserving the course of the parameter lines relative to the object.

4.3 Summary

In this paper an areal approach to determine rigid body movements of point clouds was presented. The approach is based on the modelling of each acquired point cloud by means of B-spline surfaces. The respective adjustment requires on the one hand the a priori determination of appropriate surface parameters, which are obtained by means of an iterative and constrained parameterization approach, and on the other hand the choice of the optimal number of control points, which is determined by means of the model selection BIC.

It could be proven that the rigid body movement of the point cloud can be retrieved from the rigid body movement of the estimated B-spline control points. This finding was demonstrated on a simulated transformation of a point cloud as well as on a rigid body movement based on real measurements.

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