Egyptian Geoid using Ultra High-Degree Tailored Geopotential Model

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Data

1. Gravity Data

Data

2. Digital Height Models (EGH13) (Abd-Elmotaal and Ashry, 2013)

Coarse DHM resolution is 30” × 30”
Fine DHM resolution is 3” × 3”
Data

3. GPS/Levelling Stations

Remove/Restore Technique

1. Traditional Technique

- Double consideration of the topographic-isostatic masses within the radius $R_2$ (double hatched area)
Remove/Restore Technique

2. Window Technique

No double consideration of the topographic-isostatic masses (no double hatching)

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Harmonic Analysis of the Topographic-Isostatic Potential

\[
\frac{C_w}{S_w} = \frac{R^2 \Delta \omega}{M(2n+1)(n+3)} \sum \sum \rho \left[ 1 + H \frac{R}{n+3} \right]^{-1} \\
+ \Delta \rho \left[ 1 + \frac{D}{R} \right] \left[ 1 - \frac{H}{R - D} \right]^{-1} [\cos \lambda_1, \cos \phi] \hat{P}_{nm} \cos \phi
\]

\[
\rho_0 = \rho_0 \quad \text{for } H \geq 0
\]

\[
\rho_0 = \rho_0 - \rho_u \quad \text{for } H < 0
\]

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Methodology

\[ \Delta g_{red} = \Delta g_F - \Delta g_{TC \, win} - \Delta g_{GM \, Adapt} \]
\[ = \Delta g_F - \Delta g_{TC \, win} - (\Delta g_{GM} - \Delta g_h) \]

\[ \Delta g_{GM} = \Delta g_{GM}\big|_{0 \leq n \leq 360} + \Delta g_{GM}\big|_{361 \leq n \leq 2160} \]

\[ \Delta g_{red} = \Delta g_F - \Delta g_{TC \, win} - \Delta g_{GM}\big|_{0 \leq n \leq 360} - \Delta g_{GM}\big|_{361 \leq n \leq 2160} + \Delta g_h \]

\[ \Delta g_{red} = 0 \]

Preparation Gravity Anomalies for Tailored Geopotential Model Computation

1.1. SRTM 30' × 30' Global Digital Height Model

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Preparing Gravity Anomalies for Tailored Geopotential Model Computation

1.2. EGM2008TI Topographic-Isostatic Geopotential Model

\[
\begin{align*}
\{ \overline{C}_{nm} \} _{EGM \ 2008TI} &= \{ \overline{C}_{nm} \} _{EGM \ 2008} - \{ \overline{C}_{nm} \} _{TI}
\end{align*}
\]

Min = -337 mgal       Max = 273 mgal       Mean = -1 mgal       St. dev. = 27 mgal
Preparing Gravity Anomalies for Tailored Geopotential Model Computation

3. Local Isostatic (Free-air – window topo-iso) Gravity Anomalies (Egypt)

Min = -190 mgal  Max = 202 mgal  Mean = 0 mgal  St. dev. = 37 mgal

4. Merged Isostatic Gravity Anomalies (EGM2008TI + Egypt)

Min = -337 mgal  Max = 273 mgal  Mean = -1 mgal  St. dev. = 27 mgal
Harmonic Analysis using FFT Technique

- Expanding \( f(\theta, \lambda) \rightarrow \)

\[
f(\theta, \lambda) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} F_{nm} Y_{nm}(\theta, \lambda)
\]

- Orthogonality →

\[
F_{nm} = \frac{1}{4\pi} \int_{\sigma} f(\theta, \lambda) Y_{nm}(\theta, \lambda) d\sigma
\]

- HRCOFITR Program (Abd-Elmotaal, 2004 b) has been used
- Iteration to obtain the best coefficients accuracy and minimum residual field

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Tailored Reference Geopotential Model for Egypt (EGTGGM2014)
(lower harmonics only till \( n=360 \); model is available till \( n=2160 \))

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Residual Field (Data – Model)

Min = -32.4 mgal           Max = 27.6 mgal            Mean  = 0 mgal          St. dev. = 5.1 mgal

Gravity Reduction
(102418 points)

<table>
<thead>
<tr>
<th>Reduced gravity</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Average</th>
<th>St. dev.</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta g_F$</td>
<td>-210.60</td>
<td>314.99</td>
<td>-27.58</td>
<td>50.65</td>
<td>2565.1</td>
</tr>
<tr>
<td>$\Delta g_{\text{Airy window}}$ (EGM2008)</td>
<td>-99.15</td>
<td>122.49</td>
<td>-0.26</td>
<td>20.46</td>
<td>418.6</td>
</tr>
<tr>
<td>$\Delta g_{\text{Airy window}}$ (EGTGM2014)</td>
<td>-134.30</td>
<td>112.69</td>
<td>-0.45</td>
<td>16.45</td>
<td>270.5</td>
</tr>
</tbody>
</table>
Airy window isostatic anomalies for Egypt using the EGTGM2014 tailored geopotential model

Min = -134.3 mgal  Max = 112.7 mgal  Mean = -0.5 mgal  St. dev. = 16.5 mgal

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Stokes’ Integral with 1D-FFT Technique

\[
N = \frac{R}{4\pi\gamma} \int \int \Delta g S(\psi) d\sigma
\]

\[
N(\phi_p, \lambda_p) = \frac{R \Delta \phi \Delta \lambda}{4\pi\gamma} \sum_q (\Delta g_q \cos \phi_q) S(\psi_{pq})
\]

\[
N_{\phi_q}(\lambda) = \frac{R \Delta \phi \Delta \lambda}{4\pi\gamma} \sum_{\phi_q} \cos \phi_q \sum_{\lambda_q} \Delta g_q S(\psi_{pq})
\]

\[
S(\psi_{pq}) = S_{\phi,\phi_q}(\lambda_p - \lambda_q) = S_\phi(\lambda_p - \lambda_q)
\]

\[
N_{\psi_p}(\lambda) = \frac{R \Delta \phi \Delta \lambda}{4\pi\gamma} \sum_{\phi_p} \cos \phi_p \sum_{\lambda_q} \Delta g_q S_\psi(\lambda_p - \lambda_q)
\]

\[
N_{\phi_p}(\lambda) = \frac{R \Delta \phi \Delta \lambda}{4\pi\gamma} \sum_{\phi_q} (\Delta g_q \cos \phi_q) [S_\phi(\lambda_p - \lambda_q)]
\]
Absolute Geoid Difference to GPS/levelling

\[ N_{GPS} - N_{Airy\,win\,EGM\,2008} \]

Min = -10.65 m          Max = 3.78 m          Mean = -1.20 m          St. dev. = 3.00 m

Min = 0.80 m          Max = 10.24 m          Mean = 5.44 m          St. dev. = 2.33 m
### Statistics of the Remaining Differences at the 27 GPS Stations used for the Geoid Fitting after Removing a Kriging Trend Function (Internal Precision)

<table>
<thead>
<tr>
<th>Geoid type</th>
<th>Minimum [cm]</th>
<th>Maximum [cm]</th>
<th>Average [cm]</th>
<th>Standard deviation [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{\text{Airywin EGM 2008}} )</td>
<td>-9.2</td>
<td>9.8</td>
<td>0.2</td>
<td>3.4</td>
</tr>
<tr>
<td>( N_{\text{Airywin EGTGM 2014}} )</td>
<td>-7.6</td>
<td>9.8</td>
<td>0.2</td>
<td>3.3</td>
</tr>
</tbody>
</table>

### Statistics of the Remaining Differences after Removing a Kriging Trend Function at the 3 GPS Stations which were not used for the Geoid Fitting (External Accuracy)

<table>
<thead>
<tr>
<th>Geoid type</th>
<th>Minimum [m]</th>
<th>Maximum [m]</th>
<th>Average [m]</th>
<th>Standard deviation [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{\text{Airywin EGM 2008}} )</td>
<td>-1.40</td>
<td>2.46</td>
<td>0.29</td>
<td>1.97</td>
</tr>
<tr>
<td>( N_{\text{Airywin EGTGM 2014}} )</td>
<td>-0.88</td>
<td>2.13</td>
<td>0.30</td>
<td>1.60</td>
</tr>
</tbody>
</table>
Summary and Conclusions

- An ultra high-degree tailored reference geopotential model for Egypt, complete to degree and order 2160, has been developed.
- The tailored geopotential model created in this investigation gives better residual gravity anomalies (unbiased and have much less variance). The variance has dropped by about 35%.
- Gravimetric geoids for Egypt have been computed in this investigation using both the EGM2088 and the EGTGM2014 tailored geopotential models in the framework of the window remove-restore technique using 1D-FFT technique.
- The computed geoids have been fitted to the GPS-derived geoid by removing a trend surface.
- A kriging trend function has been computed using only 27 GPS stations among the available 30 GPS stations in Egypt.
- The internal precision of the fitted geoids is very good (about 3 cm) and it is nearly equal for both geoids.
- Using the EGTGM2014 tailored geopotential model improves the external geoid accuracy by about 20%, and the range of the remaining differences has dropped by about 22%.
Thank You!

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