KINENATIC ANALYSIS OF STRUCTURAL DEFORMATION USING KALMAN FILTER TECHNIQUE

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INTRODUCTION

❖Geological formation of the soil, age, non uniform settlement of foundations, loading and offloading of oil and temperature of the crude often result to stress and strain for reservoir. The reservoirs tend to undergo radial deformation or out of roundness, therefore monitoring the structural deformation of these circular oil storage reservoirs must be done by using accurate geodetic observations and analysis methods

INTRODUCTION CONTINUES

❖ To develop a reliable and cost effective monitoring system of any structure, deformation monitoring scheme consists of measurements made to the monitored object from several monitoring stations that are referred to as reference control points (assumed to be stable). To obtain correct object point displacement (and thus deformation), the stability of the monitoring stations must be ensured. This is accomplished by creating a reference network of monitoring stations surrounding a particular structure.

Structural Deformation Modeling

❖ Nowadays, different models have been developed for analysis and the interpretation of structural deformations. These models include static, kinematic and dynamic models. Static model is not time dependent but provides the deformation characteristic on points, area or the structure being monitored.

❖ However, most of the current engineering applications require monitoring of movement behaviors. A kinematic deformation model determines displacements, velocities and acceleration and is time dependent.

Structural Deformation Modeling

❖In dynamic model, in addition to the kinematic model, the relationship between deformations and the influencing factors are also taken into consideration. Different deformation analysis algorithms are shown in (fig. 1.0).

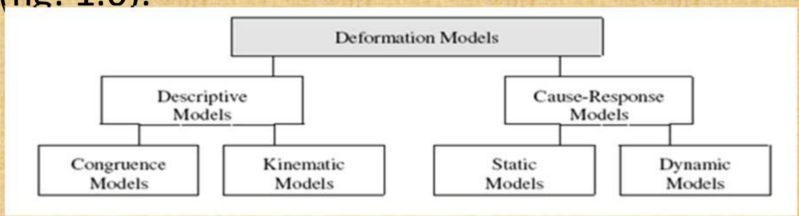

Figure 1.0 - Hierarchy of models in geodetic deformation analysis

Structural Deformation Modeling

❖In the following table (fig. 2.0), four categories of deformation models are characterized by their capacities of taking the factors 'time' and 'load' into account.

Deformation Model	Congruence Model	Kinematic Model	Static Model	Dynamic Model
Time	no modeling	movements as a function of time	no modeling	movements as a function of time
Acting Forces	no modeling	no modeling	displacements as a function of loads	and loads
State of the Object	sufficiently in equilibrium	permanently in motion	sufficiently in equilib-rium under loads	permanently in motion

Figure 2.0 - Characterization and classification of deformation models

Structural deformation analysis using Kinematic model

- The intention of kinematic models is to find a suitable description of point movement as a function of time without regarding the potential relationship to causative forces. Polynomial approaches, especially velocities, accelerations, and harmonic functions are commonly applied.
- A time-dependent 3-D kinematic model that contains position, velocity and acceleration can be expressed by the following equation:

Structural deformation analysis using Kinematic model

$$
\begin{aligned}
X_j^{(k+1)} &= X_j^{(k)} + (t_{k+1} - t_k)\, v_{xj} + \frac{1}{2}(t_{k+1} - t_k)^2 a_{xj} \\
Y_j^{(k+1)} &= Y_j^{(k)} + (t_{k+1} - t_k)\, v_{yj} + \frac{1}{2}(t_{k+1} - t_k)^2 a_{yj} \\
Z_j^{(k+1)} &= Z_j^{(k)} + (t_{k+1} - t_k)\, v_{zj} + \frac{1}{2}(t_{k+1} - t_k)^2 a_{zj}
\end{aligned}
\qquad (1.0)
$$

Where $X_J^{K+1}, Y_J^{K+1}, Z_J^{K+1}$ - Coordinates of point *J* at time t_{k+1} (predicted values), $v_{X_J}^K, v_{Y_J}^K, v_{Z_J}^K$ – velocities of X, Y,Z coordinates of point *J* at time t_k; $a_{X_J}^K, a_{Y_J}^K, a_{Z_J}^K$ – accelerations of X, Y,Z coordinates of point *J* at time t_k. k=1, 2, . . . , m (m: measurement period number(number of epochs)). j =1, 2, n (n: number of points).

Kalman Filter

- Kalman filtering technique is employed for the prediction of present state vector using state vector information of known motion parameters at period t_k and the measurements collected at period t_{k+1}. The state vector of motion parameters consists of position, motion and acceleration variables. The motion and acceleration parameters are the first and the second derivations of the position with respect to time. The matrix form of the motion model used for the prediction of motion parameters by Kalman filtering technique in 3-D networks can be given as follows:

Kalman Filtering Model

$$\begin{bmatrix} X_J^{K+1} \\ Y_J^{K+1} \\ Z_J^{K+1} \end{bmatrix} = \begin{bmatrix} X_J^{K} \\ Y_J^{K} \\ Z_J^{K} \end{bmatrix} + (t_{K+1} - t_K) \begin{bmatrix} v_{X_J}^{K} \\ v_{Y_J}^{K} \\ v_{Z_J}^{K} \end{bmatrix} + \frac{1}{2}(t_{K+1} - t_K)^2 \begin{bmatrix} a_{X_J}^{K} \\ a_{Y_J}^{K} \\ a_{Z_J}^{K} \end{bmatrix},$$

By analysis of equation (2.0) it is shown that the unknown displacement parameters consist of position, velocity (first derivative of position) and acceleration (second derivative of position). These unknown parameters can be calculated using the method of Kalman filter with four cycles of measurements at different times.

Kalman Filtering Model

❖ Kalman Filter is designed for recursive estimation to the state vector of a priori known dynamical system. To determine the current state of the system, the current measurement must be known, as well as the previous state of the filter. Thus, the Kalman filter is implemented in the time domain, rather than in frequency domain. Using the Kalman filter, the kinematic model of movement of any observable point J on the surface of circular oil storage tanks can be written in matrix form as following:

Kalman Filtering Model

$$\bar{Y}_{K+1} = \begin{bmatrix} X \\ Y \\ Z \\ v_X \\ v_Y \\ v_Z \\ a_X \\ a_Y \\ a_Z \end{bmatrix}_{K+1} = \begin{bmatrix} I & I(t_{K+1}-t_K) & I\dfrac{(t_{K+1}-t_K)^2}{2} \\ 0 & I & I(t_{K+1}-t_K) \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ v_X \\ v_Y \\ v_Z \\ a_X \\ a_Y \\ a_Z \end{bmatrix}_{K} + \begin{bmatrix} I\dfrac{(t_{K+1}-t_K)^2}{2} \\ I(t_{K+1}-t_K) \\ I \end{bmatrix} \xi_K;$$

Then,

$$\bar{Y}_{K+1} = R_{K+1,K}\hat{Y}_K + S_{K+1,K}\,\xi_K, \qquad (3.1)$$

Where $R_{K+1,K}$ – transition matrix from time t_K to t_{K+1} (matrix of prediction);

$\bar{Y}_{K+1}$ – state vector at time t_{K+1} containing prediction values and its noise;

$\hat{Y}_K$ – state vector at time t_K;

$S_{K+1,K}$ – noise (error) matrix;

ξ_K – vector of stochastic effects (the vector of noise) during t_K;

I – unit matrix.

Velocity By Kalman Filtering

❖It is important to note that the velocity of deformation of any point J between two periods of time can be determined thus:

$$v_{X_J}^{K+1} = \frac{X_J^{K+1} - X_J^K}{\Delta t_{k-1,k}};$$

$$v_{Y_J}^{K+1} = \frac{Y_J^{K+1} - Y_J^K}{\Delta t_{k-1,k}};$$

$$v_{Z_J}^{K+1} = \frac{Z_J^{K+1} - Z_J^K}{\Delta t_{k-1,k}}.$$

The calculated values of velocities for horizontal and vertical deformation values for tank № 6 are presented in the following tables 1.

Table 1 – Velocity of Tank 6

Monitoring point	Velocity, mm/year					
	Horizontal values, mm year			Vertical values, mm/year		
	t= 3 years from 5/2000 to May-03	t= 4.25 year from 5/2000 to Aug -04	t= 8 years from 5/2000 to May-08	t= 3 years from 5/2000 to 5/2003	t= 4.25 year from 5/2000 to 8/2004	t= 8 years from 5/2000 to May-08
STUD1	21.58	17.26	19.39	3.84	3.68	2.87
STUD9	26.90	19.11	14.87	5.82	7.08	4.43
STUD16	33.19	24.94	20.88	4.67	4.75	3.69
STUD8	32.62	17.44	14.82	4.14	4.60	3.52
STUD2	25.91	16.35	17.92	3.69	3.99	3.18
STUD10	0.00	5.60	6.13	5.60	6.97	4.46
STUD4	32.92	12.36	19.94	0.00	0.64	1.24
STUD12	43.29	30.85	23.11	5.44	7.14	4.41
STUD3	13.79	9.52	16.08	0.00	0.76	1.32
STUD11	0.00	1.40	4.75	5.60	7.07	4.47
STUD5	22.31	5.12	14.25	1.33	2.35	2.07
STUD13	0.00	-2.15	9.03	4.97	6.60	4.26
STUD7	44.14	17.96	21.78	1.30	2.35	2.20
STUD15	28.60	17.69	20.05	3.46	5.84	3.88
STUD6	20.40	9.53	11.36	1.07	2.19	2.04
STUD14	27.78	22.35	17.40	4.10	6.42	4.15

5.0 ANALYSIS OF RESULTS

- Table I gives the horizontal and vertical deformation values for tank № 6. The first epoch of observation was year 2000, this serve as the reference observation. From the above, in terms of horizontal component for year 2000 and 2003, the minimum deformation was at studs 10, 11 and 13 with value zero. By this we mean that no displacement at these monitoring point for the year under study. The maximum deformation occurred at stud 7 with numerical value of 44.14mm. For year 2000 and 2004, the minimum deformation was found to be -2.15mm at stud 13 and maximum at stud 12 with a numerical value of 30.85mm. For 2000 and 2008, the minimum displacement was at stud 5 with value of 4.75mm and maximum at stud 12 with value 23.11mm.

5.0 ANALYSIS OF RESULTS

- In term of settlement, the vertical displacement for year 2000 and 2003 was minimum at studs 3 and 4 with a zero value which is an indication that there was no displacement at these monitoring points for that year. The maximum displacement occurred at stud 9 with a numerical value of 5.53mm. For year 2000 and 2004, the minimum value occurred at stud 4 with value of 0.64mm and maximum at stud 12 with value of 7.1mm. From the table, maximum displacement of 4.47mm occurred at stud 1 and minimum at stud 4 with value of 1.24mm between year 2000 and 2008.
- It is important to note that no observation was carried out in year 2001, 2002, 2005, 2006 and 2007 because of the unrest in the Niger delta of Nigeria.

CONCLUSION

- ❖ Based on the presented analysis, is possible to determine the kinematic behavior of deformable structure.
- ❖ It is also possible to determine the acceleration as well.
- ❖ If the velocity and acceleration are known, then is possible to predict when structures may likely fail or advise when the structure should be put out of use. Models like linear polynomial function for linear objects, quadratic and exponential polynomial functions for polygon objects are available .