Kinematic Analysis of Structural Deformation Using Kalman Filter Technique

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KEY WORDS: Structural deformation, Kinematic, Kalman Filter

SUMMARY

The main purpose of structural deformation monitoring scheme and analysis is to detect any significant movements of the structure. An effective approach is to model the structure by using well-chosen discrete points located on the surface of the structure which, when situated correctly, accurately depict the characteristics of the structure. It can then be said that any movements of those points represent deformations of the object. Large, aboveground oil storage tanks that are commonly used in oil and gas industries are examples of structures that must be routinely surveyed to monitor their stability and overall integrity.

This paper outlines the procedure of geodetic monitoring system of circular oil storage tanks and presents the analysis of the resulted observations to determine the values of their deformation.

In this study, deformation analysis by Kalman Filter technique of the measurement data obtained on Tank 6 with 22m high and 72m diameter using refletorless total Station at the Forcados Terminal is presented. The measurement system consisted of 3 controls and sixteen monitoring points carried. The data were collected during four measurement campaigns carried out between 2000 to 2008. The computation and least square adjustment of each epoch measurement were carried out using Carlson 2011 software. Analysis of the result indicated that the tank ovality was expanding with years as indicated by the horizontal velocity i.e. increased in diameter while the vertical component indicate settlement

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1. INTRODUCTION

As a result of tanks age, geological formation of the soil around Forcados tanks non uniform settlement of tanks foundations, loading and offloading of oil and temperature of the crude will cause stress and strain for tanks membrane and settlement of sediments. The tanks tend to undergo radial deformation or out of roundness, therefor monitoring the structural deformation of these circular oil storage tanks must be done by using accurate geodetic observations and analysis methods.

To develop a reliable and cost effective monitoring system of any structure, deformation monitoring scheme consists of measurements made to the monitored object from several monitoring stations that are referred to several reference control points (assumed to be stable). To obtain correct object point displacements (and thus deformation), the stability of the monitoring stations must be ensured. This is accomplished by creating a reference network of monitoring stations surrounding a particular structure.

The method involves dividing the tank into circular cross section. These monitoring points are suited at the outer surface of the tank and placed at the same level 2.0 m from the tank base.
The monitoring stations are connected to the existing control networks at Forcados terminal. However, it is important to state that the monitoring stations, surrounding the studied tank, were first established in 1999 by Geodetic Positioning Services Limited. All recent control established were referred to the control established in 1999 after confirming their integrity.

The surveillance of an object involved in a deformation process requires the object as well as modeling process. Geodetic modeling of object and its surrounding means dissecting the continuum by discrete points in such a way that the points characterize the object, and that the movements of the points represent the movements and distortions of the object. This means that only the geometry of the object is modeled. Furthermore, modeling the deformation process means conventionally to observe (by geodetic method) the characteristic points in certain time intervals in order to monitor properly the temporal course of the movements. This means that only the temporal aspect of the process is modeled.

2. STRUCTURAL DEFORMATION MODELING

Nowadays, different models have been developed for analysis and the interpretation of structural deformations. These models include static, kinematic and dynamic models. Static model is not time dependent but provides the deformation characteristic on points, area or the structure being monitored.

However, most of the current engineering applications require monitoring of movement behaviors. A kinematic deformation model determines displacements, velocities and acceleration and is time dependent.
In dynamic model, in addition to the kinematic model, the relationship between deformations and the influencing factors are also taken into consideration. Different deformation analysis algorithms are shown in (fig. 1.0).

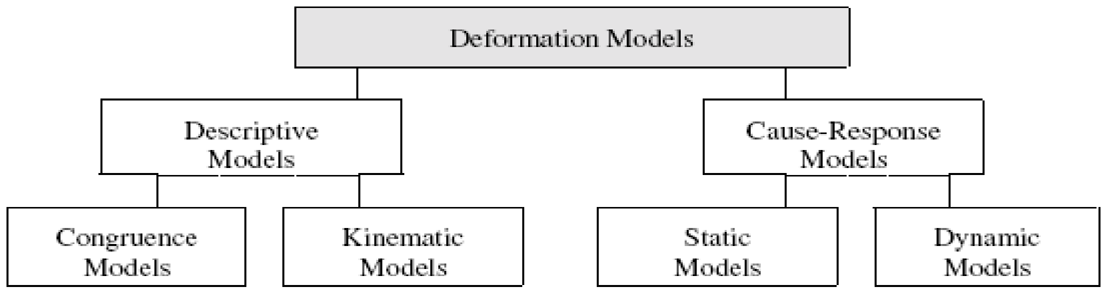

Figure 1.0 - Hierarchy of models in geodetic deformation analysis

In the following table (fig. 2.0), the four categories of deformation models are characterized by their capacity of taking the factors 'time' and 'load' into account.

Deformation Model	Congruence Model	Kinematic Model	Static Model	Dynamic Model
Time	no modeling	movements as a function of time	no modeling	movements as a function of time and loads
Acting Forces	no modeling	no modeling	displacements as a function of loads	
State of the Object	sufficiently in equilibrium	permanently in motion	sufficiently in equilibrium under loads	permanently in motion

Figure 2.0 - Characterization and classification of deformation models

3. STRUCTURAL DEFORMATION ANALYSIS USING KINEMATIC MODEL

When automated measurement procedures came into use, the temporal course of deformation processes was more considered in models evaluation. If these models are restricted to the investigation and description of object movements and distortions in space and time, one speaks of kinematic models which have offered the opportunity to extend the classical purely geometrical deformation analysis in congruence models.

Kinematic models allow estimating the velocity and even the acceleration (by building double differences) of control point movements. Because this is done for every single point, this type of models is called “single point” deformation models. The unknown parameters of a single point deformation model are the velocity and the acceleration of control points. Therefore, a time-dependent function is required to estimate these parameters. In this paper we are only considering the velocity model.

The intention of kinematic models is to find a suitable description of point movements by time functions without regarding the potential relationship to causative forces. Polynomial approaches, especially velocities and accelerations, and harmonic functions are commonly applied.
A time-dependent 3-D kinematic model that contains position, velocity and acceleration can be expressed by the following formula:

$$X_j^{(k+1)} = X_j^{(k)} + (t_{k+1} - t_k)\, v_{xj} + \frac{1}{2}(t_{k+1} - t_k)^2 a_{xj}$$

$$Y_j^{(k+1)} = Y_j^{(k)} + (t_{k+1} - t_k)\, v_{yj} + \frac{1}{2}(t_{k+1} - t_k)^2 a_{yj} \quad (1.0)$$

$$Z_j^{(k+1)} = Z_j^{(k)} + (t_{k+1} - t_k)\, v_{zj} + \frac{1}{2}(t_{k+1} - t_k)^2 a_{zj}$$

Where $X_J^{K+1}, Y_J^{K+1}, Z_J^{K+1}$ – Coordinates of point *J* at time t_{k+1} (predicted values), $v_{X_J}^K, v_{Y_J}^K, v_{Z_J}^K$ – velocities of X, Y,Z coordinates of point *J* at time t_k; $a_{X_J}^K, a_{Y_J}^K, a_{Z_J}^K$ – accelerations of X, Y,Z coordinates of point *J* at time t_k. k=1, 2, . . . , m (m: measurement period number(number of epochs)). j =1, 2, n (n: number of points).

4. KALMAN FILTERING MODEL

Kalman filtering is an important tool for deformation analysis combining information on object behavior and measurement quantities. The intention of kinematic models is to find a suitable description of point movements by time functions without regarding the potential relationship to causative forces.

Kalman filtering technique is employed for the prediction of present state vector using state vector information of known motion parameters at period t_k and the measurements collected at period t_{k+1}. The state vector of motion parameters consists of position, motion and acceleration variables. The motion and acceleration parameters are the first and the second derivations of the position with respect to time. The matrix form of the motion model used for the prediction of motion parameters by Kalman filtering technique in 3-D networks can be given as follows:

$$\begin{bmatrix} X_J^{K+1} \\ Y_J^{K+1} \\ Z_J^{K+1} \end{bmatrix} = \begin{bmatrix} X_J^{K} \\ Y_J^{K} \\ Z_J^{K} \end{bmatrix} + (t_{K+1} - t_K) \begin{bmatrix} v_{X_J}^{K} \\ v_{Y_J}^{K} \\ v_{Z_J}^{K} \end{bmatrix} + \frac{1}{2}(t_{K+1} - t_K)^2 \begin{bmatrix} a_{X_J}^{K} \\ a_{Y_J}^{K} \\ a_{Z_J}^{K} \end{bmatrix}, \quad (2.0)$$

By analysis of equation (2.0) it is shown that the unknown displacement parameters consist of position, velocity (first derivative of position) and acceleration (second derivative of position). These unknown parameters can be calculated using the method of Kalman filter with four cycles of measurements at different times.

Kalman Filter is designed for recursive estimation to the state vector of a priori known dynamical system. To determine the current state of the system, the current measurement must be known, as well as the previous state of the filter. Thus, the Kalman filter is implemented in the time representation, rather than in frequency. Using the Kalman filter, the kinematic model of movement of any observable point J on the surface of circular oil storage tanks can be written in matrix form as following:

$$\bar{Y}_{K+1} = \begin{bmatrix} X \\ Y \\ Z \\ v_X \\ v_Y \\ v_Z \\ a_X \\ a_Y \\ a_Z \end{bmatrix}_{K+1} = \begin{bmatrix} I & I(t_{K+1}-t_K) & I\frac{(t_{K+1}-t_K)^2}{2} \\ 0 & I & I(t_{K+1}-t_K) \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ v_X \\ v_Y \\ v_Z \\ a_X \\ a_Y \\ a_Z \end{bmatrix}_{K} + \begin{bmatrix} I\frac{(t_{K+1}-t_K)^2}{2} \\ I(t_{K+1}-t_K) \\ I \end{bmatrix} \xi_K , \tag{3.0}$$

Then,

$$\bar{Y}_{K+1} = R_{K+1,K}\overset{\frown}{Y}_K + S_{K+1,K}\ \xi_K , \tag{3.1}$$

Where $R_{K+1,K}$ – transition matrix from time t_K to t_{K+1} (matrix of prediction);

$\bar{Y}_{K+1}$ – state vector at time t_{K+1} containing prediction values and its noise;

$\overset{\frown}{Y}_K$ – state vector at time t_K;

$S_{K+1,K}$ – noise (error) matrix;

ξ_K – vector of stochastic effects (the vector of noise) during t_K;

I – unit matrix.

Expression (3.1) is the basic equation of the Kalman filter model when performing kinematic analysis of measurement results. Therefore, the covariance matrix of the state vector during t_{K+1} has the form

$$C_{\bar{Y}_{K+1}} = R_{K+1,K}\ C_{\overset{\frown}{Y}_K}\ R^T_{K+1,K} + S_{K+1,K}\ C_{\xi_K}\ S_{K+1,K} , \tag{3.2}$$

where $C_{\overset{\frown}{Y}_K}$ – variance-covariance matrix of vector $\overset{\frown}{Y}_K$; C_{ξ_K} – variance–covariance matrix of vector ξ_K.

The variance-covariance matrix C_{ξ_K} can be determined as following:

$$C_{\xi_K} = 4(t_{K+1} - t_K)^{-4} C_{\overset{\frown}{Y}_K}, \tag{3.3}$$

Using the method of least squares, the corrected state vector at the time t_{K+1} is formed as follows:

$$l_{K+1} + V_{l,K+1} = A_{K+1} \overset{\frown}{Y}_{K+1}, \tag{3.4}$$

где l_{K+1} – observations at time t_{K+1};

$V_{l,K+1}$ – vector of corrections to measurements results;

A_{K+1} – matrix of coefficients.

Equations (3.1) and (3.4) can be combined together to create a functional and probabilistic model of the Kalman filter and stored in a matrix form as follows:

$$\begin{bmatrix} \bar{Y}_{K+1} \\ l_{K+1} \end{bmatrix} = \begin{bmatrix} I \\ A_{K+1} \end{bmatrix} \overset{\frown}{Y}_{K+1} - \begin{bmatrix} V_{\bar{Y},K+1} \\ V_{l,K+1} \end{bmatrix}. \tag{3.5}$$

By this model, motion parameters and cofactor matrix are computed. The first step in solving the equation (3.5) using the Kalman filter is the computation of the Kalman gain matrix G as follows

$$\left.\begin{aligned} G_{K+1} &= C_{\bar{Y},K+1} A_{K+1}^T (C_{l,K+1} + A_{K+1} C_{\bar{Y},K+1} A_{K+1}^T)^{-1}, \\ G_{K+1} &= C_{\bar{Y},K+1} A_{K+1}^T D_{K+1}^{-1}. \end{aligned}\right\} \tag{3.6}$$

Therefore, the adjusted state vector for the monitoring point in the surface of the tank at time t_{K+1} can be calculated by the formula:

$$\overset{\frown}{Y}_{K+1} = \bar{Y}_{K+1} + G_{K+1}(l_{K+1} - A_{K+1} \bar{Y}_{K+1}). \tag{3.7}$$

In solving the system of equations (3.6) and (3.7) using the software MATLAB, the observed movements of points on the surface of the Tank and its velocity were determined using four cycles of measurements, as a minimum. It should be noted that the advantages of using the Kalman filter in comparison with the classical least squares method is that in this model of filtration; the number of observations can be less than the number of unknown parameters.

It is important to note that the velocity of deformation of any point J between two periods of time can be determined by:

$$\left.\begin{array}{l} v_{xyz_J}^{k+1} = \dfrac{\sqrt{(X_J^{K+!} - X_J^K)^2 + (Y_J^{K+1} - Y_J^K)^2 + (Z_J^{K+1} - Z_J^K)^2}}{\Delta t_{(K+1),K}} \\ \\ v_{xyz_J}^{k+1} = \dfrac{\sqrt{((Z_J^{K+1} - Z_J^K)^2}}{\Delta t_{(K+1),K}} \end{array}\right\} \qquad (4.0)$$

The calculated values of velocities for horizontal and vertical deformation values for tank № 6 are presented in the following tables 1.

Table 1 – Velocity of Tank 6

Monitoring point	**Velocity, mm/year**					
	Horizontal values, mm year			**Vertical values, mm/year**		
	t= 3 years from 5/2000 to May-03	**t= 4.25 year from 5/2000 to Aug -04**	**t= 8 years from 5/2000 to May-08**	**t= 3 years from 5/2000 to 5/2003**	**t= 4.25 year from 5/2000 to 8/2004**	**t= 8 years from 5/2000 to May-08**
STUD1	*21.58*	*17.26*	*19.39*	*3.84*	*3.68*	*2.87*
STUD9	*26.90*	*19.11*	*14.87*	*5.82*	*7.08*	*4.43*
STUD16	*33.19*	*24.94*	*20.88*	*4.67*	*4.75*	*3.69*
STUD8	*32.62*	*17.44*	*14.82*	*4.14*	*4.60*	*3.52*
STUD2	*25.91*	*16.35*	*17.92*	*3.69*	*3.99*	*3.18*
STUD10	*0.00*	*5.60*	*6.13*	*5.60*	*6.97*	*4.46*
STUD4	*32.92*	*12.36*	*19.94*	*0.00*	*0.64*	*1.24*
STUD12	*43.29*	*30.85*	*23.11*	*5.44*	*7.14*	*4.41*
STUD3	*13.79*	*9.52*	*16.08*	*0.00*	*0.76*	*1.32*
STUD11	*0.00*	*1.40*	*4.75*	*5.60*	*7.07*	*4.47*
STUD5	*22.31*	*5.12*	*14.25*	*1.33*	*2.35*	*2.07*
STUD13	*0.00*	*-2.15*	*9.03*	*4.97*	*6.60*	*4.26*
STUD7	*44.14*	*17.96*	*21.78*	*1.30*	*2.35*	*2.20*

STUD15	*28.60*	*17.69*	*20.05*	*3.46*	*5.84*	*3.88*
STUD6	*20.40*	*9.53*	*11.36*	*1.07*	*2.19*	*2.04*
STUD14	*27.78*	*22.35*	*17.40*	*4.10*	*6.42*	*4.15*

5. ANALYSIS OF RESULTS

Table I gives the horizontal and vertical deformation values for tank № 6. The first epoch of observation was year 2000, this serve as the reference observation. From the above, in term of horizontal component for year 2000 and 2003, the minimum deformation was at studs 10, 11 and 13 with value zero. By this we mean that no displacement at theses monitoring point for the year under study. The maximum deformation occurred at stud 7 with numerical value of 44.14mm. For year 2000 and 2004, the minimum deformation was found to be -2.15mm at stud 13 and maximum at stud 12 with a numerical value of 30.85mm. For 200 and 2008, the minimum displacement was at stud 5 with value of 4.75mm and maximum at stud 12 with value 23.11mm.

In term of settlement, the vertical displacement for year 2000 and 2003 was minimum at studs 3 and 4 with a zero value which is an indication that there was no displacement at these monitoring points for that year. The maximum displacement occurred at stud 9 with a numerical value of 5.53mm. For year 2000 and 2004, the minimum value occurred at stud 4 with value of 0.64mm and maximum at stud 12 with value of 7.1mm. From the table, maximum displacement of 4.47mm occurred at stud 1 and minimum at stud 4 with value of 1.24mm between year 2000 and 2008.

It is important to note that no observation was carried out in year 2001, 2002, 2005, 2006 and 2007 because of the unrest in the Niger delta of Nigeria.

7. CONCLUSION

Based on the presented analysis, the value of movement of 44.14mm horizontal at stud 7 for year 2003 appears high and it is now left for the structural Engineer to decide whether to put off the Tank for further investigation and this depend on the Tank elastic parameter.

Further observations need to be carried out to see the behavior of this and other points where movement appears to be abnormal and before final conclusion can be drawn about the behavior of the structure as a rigid body.

REFERENCES

1. Ehigiator – Irughe, R. and Ehigiator M. O.(2010)
 "Estimation of the centre coordinates and radius of Forcados Oil Tank from Total Station data using least square Analysis" International Journal of pure and applied sciences. A pan – African Journal Series 2010)

2. Ehigiator – Irughe, R. Development of Methodology and deformation analysis of Geodetic Monitoring of crude oil Reservoir Engineering Structure (Forcados Terminal - Nigeria a case Study). Ph.D. Thesis. - Department of Engineering Geodesy and GeoInformation Systems, Siberian State Geodesy Academy, Novosibirsk Russia. – 2012.

3. Gairns, C. Development of semi-automated system for structural deformation monitoring using a reflector less total station. M.Sc. Thesis. – Department of Geodesy and Geomatics Engineering – University of New Brunswick, 2008. –

4. Ehigiator – Irughe, R. Ashraf A. A. Beshr, and Ehigiator M. O.(2010)
"Structural deformation analysis of cylindrical oil storage tank using geodetic observations" (Paper Presented at Geo –Siberia 2010, International Exhibition and scientific conference VI page 34 - 37, Novosibirsk Russia Federation)

6 Radia MIR, Salem Kahlouche and Said Touam (2011)
"Investigation of Deformation in North of Algeria with GPS data and Kinematic model" (Paper Presented at FIG working week Marrakeck Morocco.

7 Temel Bayrak and Mualla (2003)
"A kinematic analysis program for deformation Monitoring" (Paper Presented at proceeding, 11th FIG symposium on Deformation Measurement Santorini, Greece.

8 Ehigiator – Irughe, R. Ashraf A. A. Beshr, J.O. Ehiorobo and O.M. Ehigiator (2012)
 "Modification of Geodetic Methods for Determining the Monitoring Station Coordinates on the Surface of Cylindrical Oil Storage Tank" Research Journal of Engineering and Applied Sciences (RJEAS) 1 (1) pp. 58 - 63. A United State Academy publications USA

9 Vladimir A, Seredovich, Ehigiator – Irughe, R. Ehigiator M. O. and Oriakhi Henry (2012)"Deformation Prediction Using Exponential Polynomial Functions" (Paper Presented at Geo – Siberia, Novosibirsk Russia Federation) vol. 1 pp.148 -154.

10 Vladimir A, Seredovich, Ehigiator – Irughe, R. Ehigiator M. O. and Oriakhi Henry (2012) Determining the Deformation Values Using Structural Data Error Analysis. Paper published at Nigeria Association of Mathematical Physics July Issue 2012 vol. 21, pp.389-394.