Tidal Harmonic Analysis at Bonga Field

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SUMMARY

Traditionally, water level observations are carried out with the aid of tide gauges at areas close to shoreline in Nigeria. However with oil exploration and prospecting activities being moved from swamp and shallow locations to deep offshore locations in Nigeria, there is a need to deploy appropriate tide measuring equipment deep offshore and also study the nature and effect of tides in deep offshore marine environment.

In this paper, 50 days water level data derived from pressure data recorded by AANDERAA Water Level Recorder WLR 7 at an average depth of 1,000m at Bonga field was used to do a well detailed tidal harmonic analysis.

In the analysis, the minimum water depth was set as the chart datum and water level above this datum was regarded as tidal data. Eighteen tidal constituents were used for the harmonic analysis. Astronomical arguments (V+U) and the nodal factor (f) were computed for the middle of the observation period with a programme written in Matlab environment.

The harmonic constants such as the amplitudes and the phase lags for each of the constituents were determined and prediction starting from the initial time of observation in 2008 to December 2013 was made at 10 minutes intervals.

Statistical analysis of the predicted tides with validation data was made and the maximum deviation of the predicted tides from the validation data was 0.08m. The accuracy of the harmonic analysis and prediction is high despite the fact that only 50 days data was used for the analysis.

Tidal data covering a period of at least one year should be collected at deep offshore locations. These tidal data should be analysed using harmonic analysis. The results of the analyses should be used to support deep marine operations in the country.

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1. INTRODUCTION

Tides are periodic variations in the water level. They occur as a result of gravitational attraction of the moon and sun on the earth's water bodies.

Traditionally, water level observations are carried out with the aid of tide gauges at areas close to shoreline in Nigeria. However with oil exploration and prospecting activities being moved from swamp and shallow locations to deep offshore locations in Nigeria, there is a need to deploy appropriate tide measuring equipment deep offshore and study the nature and effect of tides in deep offshore marine environment.

1.1 Water Level Recorder WLR 7

The Water Level Recorder is specially designed to measure ocean water levels. Placed on the seabed, the instrument records pressure, temperature and conductivity at regular intervals. The data is stored in a removable and reusable solid-state Data Storage Unit (DSU) 2990. Five channels of 10 bits each are recorded in sequence. The channels are:

i. Reference
ii. Temperature
iii. Pressure, most significant
iv. Pressure, least significant
v. Conductivity (optional)

The reference is a fixed reading that serves to indicate correct performance of the instrument and to identify data series from individual instruments. The temperature is measured by a thermistor fitted into a stud extending into the water. The pressure sensor is based on a pressure controlled quartz crystal oscillator. The measurement is a 20-bit word, but is divided into two 10-bit words recorded in two successive channels. Conductivity is measured by an electrodeless induction-type sensor. When the conductivity sensor is not installed the instrument will record a fixed value in channel 5. Figure 1.1 shows the Water Level Recorder WLR 7 used for this study.

Figure 1.1: Water Level Recorder WLR 7

1.2 Study Area

The Bonga Field is an oilfield in Nigeria. It is located in Oil Mining Lease (OML) 118. The lease covers an area of approximately 1167 km^2. The average water depth of Bonga field is 1,000 metres. The field was discovered in 1996, with government approval for its development given in 2002. The field began its first production in November 2005. The field produces both crude oil and natural gas through a floating production, storage and off-take (FPSO) vessel. The crude oil is offloaded to tankers through a single point mooring (SPM) buoy while the gas is exported through a series of pipelines to Bonny NLNG plant. The field is operated by Shell Nigeria and owned by Shell Nigeria (55%), Exxon (20%), Nigerian AGIP (12.5%) and Elf Petroleum (12.5%).

2. METHODOLOGY

This study is focused on tidal prediction to support deep water projects in Bonga field, offshore Nigeria. The techniques employed for this study include water level observation with Water Level Recorder WLR 7, tidal data extraction from records of the water level recorder, tidal harmonic analysis and prediction. This chapter describes the methodology and the steps taken in achieving the desired results.

2.1 Data Acquisition

A Water Level Recorder WLR 7 was used for data acquisition. The Water Level Recorder WLR7 is a high precision recording instrument for determining water level in the open sea. The water level is determined by measuring the hydrostatic pressure with an ultra-precise quartz pressure sensor. Knowing the density of water and atmospheric pressure, the water level can then be found. The operation depth is limited by the range of the pressure transducer. The mechanical parts of this equipment are strengthened to withstand a pressure down to 6000 meter depth.

A fifty day data water level data recorded by Water Level Recorder WLR 7 was provided by the Geomatics Department of the Shell Nigeria Exploration and Production Company (SNEPCo). The water level data were taken at ten minutes intervals between September 11th 2010 to November 1st , 2010. A total of 8987 datasets were acquired within this period. The 10 minute interval data was converted to a 1224 hourly data. These hourly readings were then converted to depth data using equation 2.1.

$$\text{Depth (m)} = 0.001*(P_{wd} - P_{atmos})/d*g \qquad 2.1$$

Where

P_{wd} = Pressure at water depth (Pa)

P_{atmos} = Atmospheric Pressure (Pa)

Atmospheric pressure of Bonga = 101000 (Pascal)

d = Density of water at the actual location

Density used for Bonga = 1.03017 (Kg/m^3)

g = Gravity of the earth (9.78334 m/s^2)

The minimum water depth was set as the chart datum and water level above this datum was regarded as tidal data.
Prior to the process of harmonic analysis of the tidal data, the observed tidal data was passed through a median filter to filter off spikes in the data. The main idea of the median filter is to run through the signal entry by entry, replacing each entry with the median of neighboring entries (www/en.wikipedia.org/wiki/Median_filter, 2012).

2.2 Harmonic Analysis of Tides

The basic equation for tide modeling is given by Doodson and Warburg (1941) as:

$$h(t) = S_o + \sum_{i=1}^{n} H_i \cos(\omega_i t + \alpha_i) \qquad 2.2$$

Where;

ω_i = Tidal constituent frequency
H_i = Amplitude of tidal constituent i
A_i = Initial phase of the constituent
S_o = Height of mean water level above the datum used
t = Time
n = number of harmonic constituents

As a result of a slow rotation of the orbit of the moon with a period of about 18.61 years, the magnitude, H, and phase, α, of each harmonic constituent vary slowly on either side of the values they would have assuming the moon's orbit were constant. To account for this variation a nodal factor f and astronomical argument (v + u) are usually introduced to modify equation 2.2 (Eluwa, 1991).

$$h(t) = S_0 + \sum_{i=1}^{n} f_i H_i \operatorname{Cos}(\omega_i t + (v_i + u_i) - \alpha_i) \qquad 2.3$$

where
n = Number of harmonic constituents
v = Phase angle at time zero
u = Nodal angle
f = Nodal factor
ω = Constituent speed

Eighteen tidal constituents shown in table 2.1 were used for the harmonic analysis.

Table 2.1: Tidal Constituents used for Harmonic Analysis

S/N	Constituent Name	Constituent Speed (ω_i)
1.	M2	28.9841042
2.	S2	30.0000000
3.	N2	28.4397295
4.	K2	30.0821373
5.	K1	15.0410686
6.	O1	13.9430356
7.	P1	14.9589314
8.	MSf	1.0158958
9.	2N2	27.8953548
10.	MO3	42.9271400
11.	MK3	44.0251729
12.	MN4	57.4238337
13.	M4	57.9682084
14.	SN4	58.4397300
15.	MS4	58.9841042
16.	2MN6	88.4079380
17.	M6	86.9523127
18.	2MS6	87.9682080

Formulae for the computation of V and U are given in tables 2.2 and 2.3 according to Schureman (1958), with some minor changes due to the direct use of the original astronomical parameters (Stravisi, 1983).

Table 2.2 summarizes the fundamental astronomical parameters. The time dependent auxiliary coefficients (c) are introduced for their recurrent use. Their numerical values at the beginning of 1985 are given, together with the annual variations taken between 1980 and 1990; final values can be rounded to six decimal digits. The longitudes of lunar and solar elements (d) define the long period time dependence of the constituent arguments V; they are expressed as a function of

$$T = \{365\, n + \text{int}\,((n - 1)/4) + l/2\} /36525 \qquad 2.4$$

Where T is time expressed in Julian centuries (36 525 d), reckoned from Greenwich mean noon, December 31, 1899 (Gregorian calendar); m is time after 0 h, January 1, 1900 in years and the integer part of (n - 1)/4 accounts for the leap years. The time dependent elements of the moon's orbit (e) define, according to Schureman (1958), f and u.

Astronomical arguments (V+U) and the nodal factor (f) were computed for seven constituents at the middle of the observation period in matlab environment using equations in tables 2.2 and 2.3.

Table 2.2: Astronomical Parameters of Use in Tidal Computations

a) Constants			
c	$= 3.844\,03 \times 10^{8}$ m	mean earth-moon distance	
c_1	$= 1.495\,042\,01 \times 10^{11}$ m	mean earth-sun distance	
S/E	$= 332\,488 \pm 43$	sun/earth mass ratio	
M/E	$= 12\,289 \pm 4 \times 10^{-6} = 1 / 81.37$	moon/earth mass ratio	
S/M	$= 2.705\,455 \times 10^{7}$	sun/moon mass ratio	
S'	$= (c/c_1)^3$ S/M $= 0.459\,875\,64$	solar factor	
e	$= 0.054\,900\,56$	eccentricity of moon's orbit	
i	$= 5.145\,376\,28°$	inclination of moon's orbit to plane of ecliptic	
b) Time dependent parameters			
n		time after 1900, in years	
e_1	$= 0.016\,751\,04 - 4.180 \times 10^{-7}\, n - 1.26 \times 10^{-11}\, n^2$	eccentricity of earth's orbit	
ω	$= 23.452\,294° - 1.301\,11° \times 10^{-4}\, n$	obliquity of the ecliptic	
c) Time dependent auxiliary coefficients		Numerical value, 1985	Increment, per year
A	$= S'\,(1 + 3/2\, e_1^2) / (1 + 3/2\, e^2)$		
A_1	$= \cos i \cos \omega$	0.913 771 493	+ 0.000 000 900
A_2	$= \sin i \sin \omega$	0.035 676 679	− 0.000 000 187
A_3	$= \cos \frac{1}{2}(\omega - i) / \cos \frac{1}{2}(\omega + i)$	1.018 819 128	− 0.000 000 108
A_4	$= \sin \frac{1}{2}(\omega - i) / \sin \frac{1}{2}(\omega + i)$	0.643 957 699	− 0.000 001 671
A_5	$= A \sin 2\omega$	0.334 316 893	− 0.000 001 429
A_6	$= A \sin^2 \omega$	0.072 478 792	− 0.000 000 761
B_1	$= \{\cos \frac{\omega}{2} \cos \frac{i}{2}\}^{-4}$	1.092 333 626	− 0.000 001 029
B_2	$= \{A_5 + (1 - 3/2 \sin^2 i) \sin 2\omega\}^{-2}$	0.897 663 509	+ 0.000 007 647
B_3	$= \{A_6 + (1 - 3/2 \sin^2 i) \sin^2 \omega\}^{-2}$	19.098 898 614	+ 0.000 400 363
B_4	$= \{\sin \omega \cos^2 \frac{\omega}{2} \cos^4 \frac{i}{2}\}^{-1}$	2.632 568 903	+ 0.000 012 547
B_5	$= 2\, A_5\, B_2$	0.600 208 150	+ 0.000 002 548
B_6	$= 2\, A_6\, B_3$	2.768 530 194	+ 0.000 028 978
B_7	$= \{1 + (1 - 3/2 \sin^2 i) / A\}^{-2} = B_2\, A_5^2 = B_3\, A_6^2$	0.100 329 862	− 0.000 000 003
d) Longitude of lunar and solar elements (3)			
T	time in Julian centuries (36 525 d), reckoned from Greenwich mean noon, December 31, 1899		
h	$= 279.696\,678° + 36\,000.768\,925°\, T + 3.025° \times 10^{-4}\, T^2$	mean longitude of sun	
s	$= 270.437\,422° + 481\,267.892\,000°\, T + 2.525° \times 10^{-3}\, T^2 + 1.89° \times 10^{-6}\, T^3$	mean longitude of moon	
p	$= 334.328\,019° + 4\,069.032\,206°\, T - 1.034\,4° \times 10^{-2}\, T^2 - 1.25° \times 10^{-5}\, T^3$	longitude of lunar perigee	
N	$= 259.182\,533° - 1\,934.142\,397°\, T + 2.106° \times 10^{-3}\, T^2 + 2.22° \times 10^{-6}\, T^3$	longitude of moon's node	
e) Time dependent elements of the lunar orbit (3)			
I	$= \arccos \{A_1 - A_2 \cos N\}$	obliquity of lunar orbit with respect to earth's equator	
C	$= \arctan \{A_3 \tan N/2\}$		
ν	$= C - \arctan \{A_4 \tan N/2\}$	right ascension of lunar intersection	
ν'	$= \arctan \{(\sin 2I \sin \nu) / (A_5 + \sin 2I \cos \nu)\}$	auxiliary term for K_1	
$2\nu''$	$= \arctan \{(\sin^2 I \sin 2\nu) / (A_6 + \sin^2 I \cos 2\nu)\}$	auxiliary term for K_2	
ξ	$= N + \nu - 2C$	longitude in moon's orbit of lunar intersection	

(1) American Ephemeris and Nautical Almanac
(2) Smithsonian Physical Tables
(3) Schureman (1958)

Culled from Stravisi, 1983

Table 2.3: Time Dependent Nodal Factors, Arguments and Speeds of Seven Major Harmonic Component Tides

	f	V (t) (1)	u	σ (2)
M_2	$B_1 \cos^4 \frac{I}{2}$	$2\tau - 2s + 2h$	$2\xi - 2\nu$	$28.984\,104\,214 - 10.14 \times 10^{-9}\,T = 28.984\,104\,205$
S_2	1	$2\tau \quad 0$	0	30
N_2	$B_1 \cos^4 \frac{I}{2}$	$2\tau - 3s + 2h + p$	$2\xi - 2\nu$	$28.439\,729\,516 - 28.16 \times 10^{-9}\,T = 28.439\,729\,492$
K_2	$\{B_3 \sin^4 I + B_6 \sin^2 I \cos 2\nu + B_7\}^{1/2}$	$2\tau + 2h$	$-2\nu''$	$30.082\,137\,278 + 1.38 \times 10^{-9}\,T = 30.082\,137\,279$
K_1	$\{B_2 \sin^2 2I + B_5 \sin 2I \cos \nu + B_7\}^{1/2}$	$\tau + h - 90°$	$-\nu'$	$15.041\,068\,639 + 0.69 \times 10^{-9}\,T = 15.041\,068\,640$
O_1	$B_4 \sin I \cos^2 \frac{I}{2}$	$\tau - 2s + h + 90°$	$2\xi - \nu$	$13.943\,035\,575 - 10.84 \times 10^{-9}\,T = 13.943\,035\,566$
P_1	1	$\tau - h + 90°$	0	$14.958\,931\,361 - 0.69 \times 10^{-9}\,T = 14.958\,931\,360$

(1) $\tau = 15°\,t + 180°$ is the hour angle of the mean sun; t is Greenwich time in hours.
(2) Angular speed in °/h; terms in T^2, computed for $T = 1$, are enclosed.
Constant speeds refer to $T = 0.85$, year 1985.

C

The Nodal factors f and astronomical arguments V and U for the remaining 11 tidal constituents were derived from the nodal factors and astronomical arguments V and U of the seven constituents given in table 2.3. Table 2.4 shows the relationships between the various nodal factors and astronomical arguments.

Table 2.4: Relationships Between Various Nodal Factors And Astronomical Arguments

S/N	**Constituent Name**	**Constituent Speed (ω_i)**	**Nodal Factor (f_i)**	**Astronomical Argument (V_i+U_i)**
1.	MSf	1.0158958	f of M_2	360-(v+u) of M_2
2.	2N2	27.8953548	f of M_2	2x(v+u) of N_2-(v+u) of M_2
3.	MO3	42.9271400	(f of M_2)x(f of O_1)	(v+u) of M_2+(v+u) of O_1
4.	MK3	44.0251729	(f of M_2)x(f of K_1)	(v+u) of M_2+(v+u) of K_1
5.	MN4	57.4238337	(f of M_2) Squared	(v+u) of M_2+(v+u) of N_2
6.	M4	57.9682084	(f of M_2) Squared	2x(v+u) of M_2
7.	SN4	58.4397300	f of M_2	(v+u) of N_2
8.	MS4	58.9841042	f of M_2	(v+u) of M_2
9.	2MN6	88.4079380	(f of M_2) Cubed	2x(v+u) of M_2+(v+u) of N_2
10.	M6	86.9523127	(f of M_2) Cubed	3x(v+u) of M_2
11.	2MS6	87.9682080	(f of M_2) Squared	2x(v+u) of M_2

The tidal harmonic and prediction model in equation 2.3 can be expanded using the trigonometric identity as:

$$h(t) = S_0 + \sum_{i=1}^{n} f_i H_i \mathrm{Cos}(\omega_i t + (V_i + U_i)) \mathrm{Cos} X_i + \sum_{i=1}^{n} f_i H_i \mathrm{Sin}(\omega_i t + (V_0 + U_i)) \mathrm{Sin} X_i \qquad 2.5$$

Let $A_i = H_i \mathrm{Cos} X_i$ and

$B_i = H_i \mathrm{Sin} X_i$

The tidal harmonic and prediction model becomes:

$$h(t) = S_0 + \sum_{i=1}^{n} (A_i f_i \mathrm{Cos}(\omega_i t + (V_i + U_i)) + B_i f_i \mathrm{Sin}(\omega_i t + (V_i + U_i)) \qquad 2.6$$

Vandermonde Matrix will therefore be created in the following form:

$$A = \begin{bmatrix} S_0 & A_1 & B_1 & A_2 & B_2 \ldots \\ 1 & f_1 \mathrm{Cos}(\omega_1 t_1 + (V_1 + U_1)) & f_1 \mathrm{Sin}(\omega_1 t_1 + (V_1 + U_1)) & f_2 \mathrm{Cos}(\omega_2 t_1 + (V_2 + U_2)) & f_2 \mathrm{Sin}(\omega_2 t_1 + (V_2 + U_2)) \ldots \\ 1 & f_1 \mathrm{Cos}(\omega_1 t_2 + (V_1 + U_1)) & f_1 \mathrm{Sin}(\omega_1 t_2 + (V_1 + U_1)) & f_2 \mathrm{Cos}(\omega_2 t_2 + (V_2 + U_2)) & f_2 \mathrm{Sin}(\omega_2 t_2 + (V_2 + U_2)) \ldots \\ . & . & & . & \\ & . & & . & \\ 1 & f_1 \mathrm{Cos}(\omega_1 t_n + (V_1 + U_1)) & f_1 \mathrm{Sin}(\omega_1 t_n + (V_1 + U_1) & f_2 \mathrm{Cos}(\omega_2 t_n + (V_2 + U_2)) & f_2 \mathrm{Sin}(\omega_2 t_n + (V_2 + U_2) \ldots \end{bmatrix} \ldots \quad 2.7$$

A total or sum of thirty seven constants ((2*18) + 1) would be determined for the trigonometric polynomial.

A total of 1200 hourly data (50 days data) were used for the adjustment. 1200 tidal observations form the vector of observations. The Vandermonde designed matrix A (1200 x 37) is then formed.

Least squares method was used to solve the unknown parameters. The least squares solution is given as:

$$X = (A^T PA)^{-1} A^T PL \qquad 2.8$$

$X = N^{-1}U$

Where $N = (A^T PA)$

$U = A^T PL$

$$X = [S_0, A_1, B_1, \ldots A_n, B_n]^T \qquad 2.9$$

$$L = [h(t_1), h(t_2) \ldots h(t_n)]^T \qquad 2.10$$

Where A is called the Vandermonde design matrix,

The normal equation N is near singular and thus the unknown parameters X were determined by using the conjudate gradient method. This method was discussed extensively in Badejo *et al* (2012).

With the values of the unknown parameters in equation 2.9 computed, and the values of f_i and $(v_i + u_i)$ obtained from tables 2.2 and 2.3, we can solve for the harmonic constant α_i as follows:

$B_i/A_i = H_i \text{ Sin } \alpha_i \ / \ H_i \text{ Cos } \alpha_i = \text{Tan } \alpha_i$ 2.11a

$\alpha_i = \text{Tan}^{-1} (B_i/A_i)$ 2.11b

H_i can also be determined from the following relationship:

$B_i = H_i \text{ Sin } \alpha_i$ 2.12a

$H_i = B_i / \text{ Sin } \alpha_i$ 2.12b

3. RESULTS AND ANALYSIS OF RESULTS

The results and analysis of results of the work done are presented in sections 3.1 and 3.2.

3.1 Results

The results of the least squares harmonic analysis are presented in this section. Table 3.1 shows the least squares solution and the residuals from the least squares adjustment, while table 3.2 shows the tidal characteristics of the eighteen constituents used for the least squares adjustment.

Table 3.1: Least Squares Solution and Residuals from Least Squares Adjustment

S/N	Least Squares Solution (X)	Residuals from Adjustment (V=AX-L)
1	0.8955061328	-3.56E-14
2	-0.4934201491	-9.05E-14
3	-0.0596395818	-1.10E-13
4	-0.1089335765	-1.13E-15
5	0.1336482689	4.01E-14
6	-0.1033088931	-2.64E-14
7	0.0592944342	-3.16E-13
8	-0.0297730983	-4.80E-15
9	-0.0266593669	-3.64E-14
10	0.0924520282	1.30E-13
11	-0.1013088968	2.29E-14
12	0.0178565621	1.19E-13
13	0.0237078003	-7.13E-13
14	0.0344841015	-2.22E-14
15	-0.0117571831	6.27E-14
16	-0.0111300358	-5.99E-13
17	-0.0229343292	5.06E-13
18	0.0091126804	-3.46E-13
19	0.0320225152	3.99E-15
20	0.0001806800	2.03E-13
21	0.0007702764	1.62E-13
22	-0.0013186013	-9.43E-13
23	-0.0015664692	-2.31E-12
24	0.0066212762	-1.09E-12
25	0.0055185843	-7.28E-13

26	-0.0095449268	-8.98E-13
27	0.0104535647	-3.89E-13
28	0.0011717711	8.44E-13
29	0.0023108752	6.81E-13
30	-0.0034879694	2.92E-13
31	0.0082143343	7.42E-13
32	0.0005404965	-8.43E-13
33	-0.0000773116	1.98E-13
34	0.0015285740	2.25E-13
35	0.0002063698	5.28E-13
36	0.0016164270	-8.08E-13
37	-0.0000454650	1.45E-13

Table 3.2: Tidal Characteristics of the Eighteen Constituents used for Least Squares Adjustment

S/N	Constituent Name	Constituent Speed (m/s)	Amplitudes (H) (m)	Nodal Factor (F)	V+U (Deg)	Phase Lag (Deg)
1	M2	28.9841042000	0.1433380234	1.0063923852	1.2854605886	186.8918952806
2	S2	30.0000000000	0.2466006241	1.0000000000	360.0000000000	129.1826711369
3	N2	28.4397295000	0.2778514447	1.0063923852	281.6303114864	150.1462078758
4	K2	30.0821373000	0.0308251429	0.9682220264	83.5240317641	221.8418364774
5	K1	15.0410686000	0.1623680664	0.9953671368	228.0175372109	312.3828149372
6	O1	13.9430356000	0.0269270663	0.9920402910	93.4467282115	53.0131767945
7	P1	14.9589314000	0.0688045195	1.0000000000	98.6537475120	341.1734450712
8	MSf	1.0158958000	0.0671200035	1.0063923852	358.7145394114	244.1126892071
9	2N2	27.8953548000	0.0320332562	1.0063923852	201.9751623842	74.1151819934
10	MO3	42.9271400000	0.0007911065	0.9983817946	94.7321888001	76.7990608860
11	MK3	44.0251729000	0.0016101879	1.0017299069	229.3029977995	229.9104510050
12	MN4	57.4238337000	0.0057122694	1.0128256329	282.9157720751	39.8099325798
13	M4	57.9682084000	0.0494229282	1.0128256329	2.5709211773	132.3985370134
14	SN4	58.4397300000	0.0023374243	1.0063923852	281.6303114864	63.1118871266
15	MS4	58.9841042000	0.0158124501	1.0063923852	1.2854605886	113.0070406986
16	2MN6	88.4079380000	-0.0001086132	1.0193000044	284.2012326637	351.8597318230
17	M6	86.9523127000	0.0002364713	1.0193000044	3.8563817659	7.6889020828
18	2MS6	87.9682080000	-0.0000578705	1.0128256329	2.5709211773	358.3888753874

3.2 Analysis of Results

Out of the 1224 hourly datasets available, only 1220 datasets were used for the harmonic analysis. The remaining 24 hourly data were reserved for validating the predicted tides. Predicted tides were made for 1224 hours starting from the initial time of the observed data. The 1220 observed data used for the harmonic analysis with the 24 reserved tidal data were used to validate the tidal prediction from the model. A sample of 48 datasets from the validation data and predicted data are shown in table 3.3 below.

Table 3.3: Sample Validation Data

YEAR	MONTH	DAY	HOUR	MIN	SEC	OBSERVED TIDE	PREDICTED TIDE	DIFFERENCE	REMARK
2010	10	30	0	0	0	1.139	1.117	-0.022	Prediction within observed data
2010	10	30	1	0	0	0.939	0.913	-0.026	Prediction within observed data
2010	10	30	2	0	0	0.749	0.729	-0.020	Prediction within observed data
2010	10	30	3	0	0	0.665	0.613	-0.051	Prediction within observed data
2010	10	30	4	0	0	0.643	0.586	-0.058	Prediction within observed data
2010	10	30	5	0	0	0.675	0.642	-0.033	Prediction within observed data

2010	10	30	6	0	0	0.802	0.765	-0.037	Prediction within observed data
2010	10	30	7	0	0	0.949	0.924	-0.025	Prediction within observed data
2010	10	30	8	0	0	1.118	1.085	-0.034	Prediction within observed data
2010	10	30	9	0	0	1.224	1.208	-0.015	Prediction within observed data
2010	10	30	10	0	0	1.266	1.257	-0.009	Prediction within observed data
2010	10	30	11	0	0	1.224	1.203	-0.021	Prediction within observed data
2010	10	30	12	0	0	1.055	1.045	-0.010	Prediction within observed data
2010	10	30	13	0	0	0.865	0.822	-0.043	Prediction within observed data
2010	10	30	14	0	0	0.654	0.597	-0.058	Prediction within observed data
2010	10	30	15	0	0	0.496	0.428	-0.068	Prediction within observed data
2010	10	30	16	0	0	0.401	0.355	-0.046	Prediction within observed data
2010	10	30	17	0	0	0.433	0.388	-0.044	Prediction within observed data
2010	10	30	18	0	0	0.549	0.519	-0.030	Prediction within observed data
2010	10	30	19	0	0	0.749	0.721	-0.028	Prediction within observed data
2010	10	30	20	0	0	0.992	0.958	-0.033	Prediction within observed data
2010	10	30	21	0	0	1.203	1.187	-0.016	Prediction within observed data
2010	10	30	22	0	0	1.371	1.359	-0.013	Prediction within observed data
2010	10	30	23	0	0	1.445	1.428	-0.017	Prediction within observed data
2010	10	31	0	0	0	1.403	1.374	-0.029	Prediction beyond observed data
2010	10	31	1	0	0	1.234	1.209	-0.026	Prediction beyond observed data
2010	10	31	2	0	0	1.023	0.981	-0.042	Prediction beyond observed data
2010	10	31	3	0	0	0.802	0.754	-0.048	Prediction beyond observed data
2010	10	31	4	0	0	0.643	0.585	-0.059	Prediction beyond observed data
2010	10	31	5	0	0	0.559	0.508	-0.051	Prediction beyond observed data
2010	10	31	6	0	0	0.591	0.534	-0.057	Prediction beyond observed data
2010	10	31	7	0	0	0.696	0.651	-0.045	Prediction beyond observed data
2010	10	31	8	0	0	0.865	0.831	-0.034	Prediction beyond observed data
2010	10	31	9	0	0	1.055	1.032	-0.023	Prediction beyond observed data
2010	10	31	10	0	0	1.224	1.204	-0.020	Prediction beyond observed data
2010	10	31	11	0	0	1.298	1.296	-0.001	Prediction beyond observed data
2010	10	31	12	0	0	1.276	1.274	-0.003	Prediction beyond observed data
2010	10	31	13	0	0	1.150	1.134	-0.016	Prediction beyond observed data
2010	10	31	14	0	0	0.918	0.909	-0.008	Prediction beyond observed data
2010	10	31	15	0	0	0.665	0.661	-0.004	Prediction beyond observed data
2010	10	31	16	0	0	0.475	0.454	-0.020	Prediction beyond observed data
2010	10	31	17	0	0	0.369	0.343	-0.027	Prediction beyond observed data
2010	10	31	18	0	0	0.390	0.351	-0.039	Prediction beyond observed data
2010	10	31	19	0	0	0.517	0.478	-0.039	Prediction beyond observed data
2010	10	31	20	0	0	0.738	0.700	-0.039	Prediction beyond observed data
2010	10	31	21	0	0	1.002	0.974	-0.028	Prediction beyond observed data

2010	10	31	22	0	0	1.255	1.244	-0.011	Prediction beyond observed data
2010	10	31	23	0	0	1.456	1.449	-0.007	Prediction beyond observed data

3.2.1 Root Mean Square Error

The Root-Mean-Square error (RMSE) of the observed and predicted hourly tides was found using equation 2.13 given by (www.nauticalcharts.noaa.gov/csdl/skillassess.html, 2012).

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N} e_i^2} \qquad 3.1$$

ved tide at time i
N = 1224
RMSE = 0.02143595

3.2.2 Standard Deviation of Observed and Predicted Tides

The standard deviations for the observed and predicted tides are given by Keller and Warrack (2003) as:

$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})^2}, \qquad 3.2$$

).893386
Mean of 1224 tidal predictions = 0.892834
The Standard Deviation of the Observed and Predicted Tides were calculated respectively as 0.000120 and 0.000119.

Sx_1 = 0.000120 (Observed)
Sx_2 = 0.000119 (Predicted)

$$S_{X_1X_2} = \sqrt{\frac{(n_1-1)S^2_{X_1} + (n_2-1)S^2_{X_2}}{n_1+n_2-2}}. \qquad 3.3$$

The objective of the statistical test is to compare the two populations of observed tide and predicted tides (Keller and Warrack, 2003). The parameter is the difference between the two means, μ_1 and μ_2 (where μ_1= mean of observed tide and μ_2 is the mean of the predicted tide).

$H_0: (\mu_1 - \mu_2) =$ 0
$H_1: (\mu_1 - \mu_2) >$ 0

The *t* statistic to test whether the means are different can be calculated as follows:
The number of degrees of freedom of the test statistic is
$v = n_1 + n_2 - 2$ = 1224 + 1224 -2 = 2446

The rejection region for a 5% significance level is $t > t_{\alpha,y} = t0.05,2446 = 1.645$
The rejection region for a 95% significance level is $t < t_{\alpha,y} = t0.95,2446 = -1.645$
$S_{x1x2} = \sqrt{\{(1223 * 0.0000000144 + 1223 * 0.0000000142)/2446\}}$
$t = 0.0000000143$
Since $t < t_{\alpha,y}$ we therefore accept the null hypothesis that mean of the observed tide is equal to the mean of the predicted tide.

3.2.3 Charts of Observed and Predicted Tides

Charts were made in the analysis of the results of this work. Figure 3.1 shows the chart of the observed and predicted tides. From the chart in figure 3.1, it can be seen that the observed tides and the predicted tides match to a greater extent.

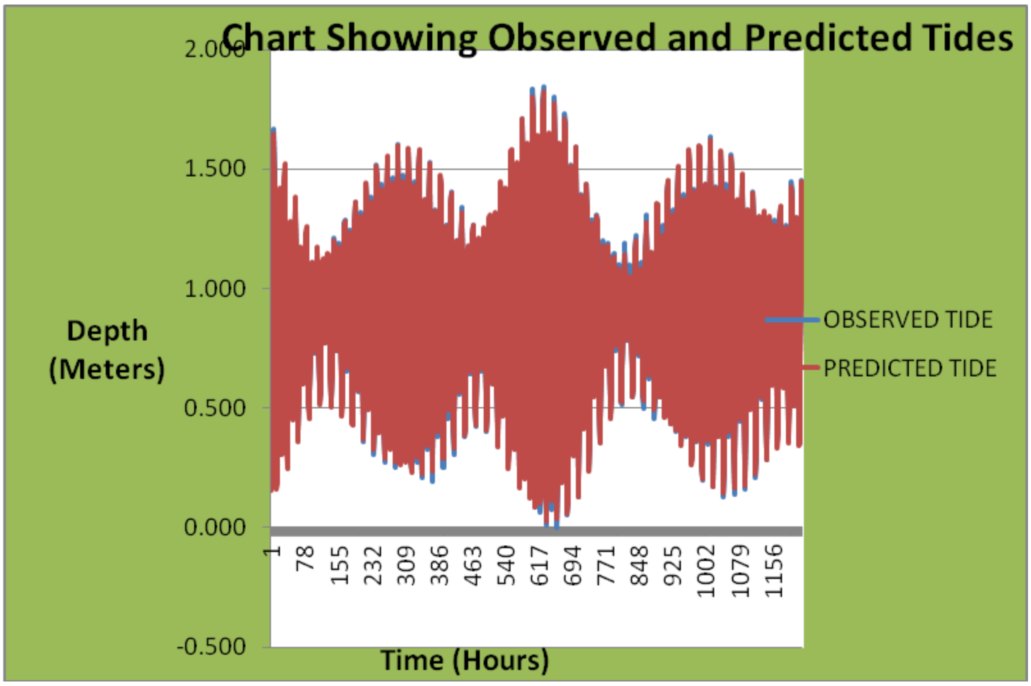

Figure 3.1: Chart of Observed and Predicted Tides

The difference between the observed and the predicted tides is the residuals. Figure 3.2 shows the residuals of the observed and predicted tides.

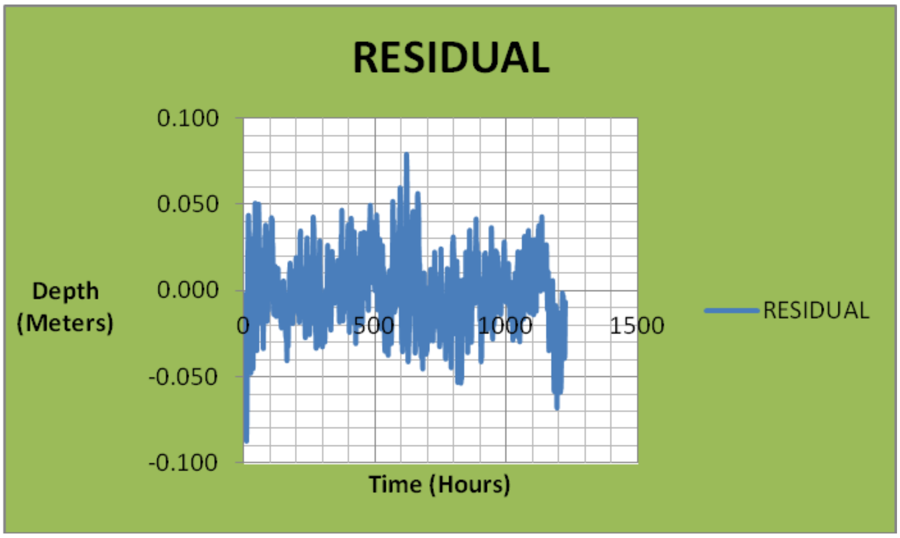

Figure 3.2: Chart Showing Residuals of Observed and Predicted Tides

The tidal heights for the 51st day have been predicted. This was carried out in order to predict tides outside the data used for adjustment within the Matlab program. The result provides a maximum residual of 0.059 meters.

5. CONCLUSION AND RECOMMENDATION

5.1 Conclusion

In this work, 50 days water level data derived from pressure data recorded by WLR 7 water level recorder at an average depth of 1,000m at Bonga field was used to do a well detailed tidal harmonic analysis.

In the analysis, the harmonic constants such as the amplitudes and the phase lags for eighteen tidal constituents were determined and prediction starting from the initial time of observation in 2008 to December 2013 was made at 10 minutes intervals. Fifty one days hourly predictions were however made to validate the work done.

Statistical analysis of the predicted tides with validation data was made and the maximum deviation of the predicted tides from the validation data was 0.08m. The accuracy of the harmonic analysis and prediction is high despite the fact that only 50 days data was used for the analysis.

The results of this work should be used to support deep marine operations in the country.

5.2 Recommendation

The following recommendations are made as a result of the work carried out:

i. Tidal data covering a period of at least one year should be collected at deep offshore and shallow water locations for harmonic analysis and prediction so as to support marine operations in oil industries.

ii. Tide Gauge Stations or buoys capable of being tracked by satellites should be placed along the Nigerian coast for further tidal studies on the Nigerian coastal waters.

iii. Effort should also be made by governmental and non-governmental agencies to observe and analyse water current (using classical oceanographic equipment and satellite data) at various offshore and near shore locations around the Nigerian coastal waters to better understand the hydrodynamic forces operating in the Nigerian coastal environment.

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BIOGRAPHICAL NOTES

DR. OLUSEGUN TEMITOPE BADEJO

Dr. O.T. Badejo graduated from the University of Lagos with a Bachelor of Science (B.Sc.) degree in Surveying in 1992. He also obtained a Master of Science (M.Sc.) degree in Surveying, in University of Lagos in 1996. His B.Sc. Project was on Sea Level Variation in a Coastal Seaport, while his M.Sc. research work was on Tidal Prediction Using Least Squares Approach. Dr. Badejo also has a Ph.D in Surveying and Geoinformatics. Dr. Badejo has worked with the Office of the Surveyor General of the Federation of Nigeria, and he is a

Senior Lecturer in Department of Surveying and Geoinformatics, University of Lagos, Nigeria. He is presently on sabbatical with Shell Nigeria Exploration and Production Company (SNEPCo). He is working on oil spill pollution transport and coastal processes. Dr. O.T. Badejo has over 25 publications, and he is a member of the Nigerian Institution of Surveyors (NIS) and Nigerian Hydrographic Society.

SURV. PETER EVARIE

Peter Evarie graduated with a B.Sc. (Surveying) and M.Sc. (Hydrographic Surveying) from the University of Lagos in 1986 and 1990 respectively. In 2004, he enrolled for the Masters in Business Administration (MBA) program at the University of Liverpool in the United Kingdom and graduated in 2006 specializing in investment strategies. During his B. Sc program at the University of Lagos, Peter won the best graduating student's prizes in Geodesy (The Adekunle Kukoyi's Prize) and Photogrammetry (The Adegboyega Ajayi Memorial Prize).

Peter has spent 21 years working for Shell in the land, swamp and offshore environments and he is currently the Head of Offshore Surveys and Metocean for the Shell Nigeria Exploration and Production Company Limited (SNEPCo) based in Lagos. He is a SURCON Registered Surveyor with extensive local and international experience and exposure. Prior to joining Shell, Peter lectured in a polytechnic in the 1980s/early 1990s, worked as a contractor staff at Shell and he is currently a part-time lecturer in Hydrographic Surveying at the University of Lagos.

In his Shell career, Peter has worked in Nigeria, United Kingdom and The Netherlands and has been exposed to the latest technologies in offshore surveys/construction, GIS applications and Metocean. For example, he was the GPS focal point for Shell Nigeria in the early 1990s and was responsible for the implementation of GPS technology in Shell Nigeria operations. The delivery of a major component that made GPS implementation a success at the time - "The determination of the 7-Shift Transformation Parameters from WGS 84 to Nigerian Local Datum" are still in use today for Southern Nigeria.

Peter has attended several local and international conferences and workshops and he is a member of the Nigerian Institution of Surveyors (NIS), Nigerian Hydrographic Society and the Institute of Navigation (ION). He is a certified Quality Management Systems (QMS) Auditor.

He is married with a daughter and has a passion for developing young surveyors, health/safety and travelling.

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