A New Mathematical Approach to Cadastral Documents Processing for Parcel Boundaries Restoration

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INTRODUCTION

- The Israeli cadastre:
  - 15,000 registered blocks
  - 700,000 parcels
- The customary method of ground surveying:
  - the orthogonal method
- The main cadastral documents:
  - special field books containing ground surveying details
  - official cadastral block maps containing parcels boundaries
ORTHOGONAL METHOD

Chain Surveying

Processing

\[
Y_p = Y_1 + a_p \frac{(Y_B - Y_A)}{L_{AB}^{obs}} + b_p \frac{(X_B - X_A)}{L_{AB}^{obs}}
\]

\[
X_p = X_1 + a_p \frac{(X_B - X_A)}{L_{AB}^{obs}} - b_p \frac{(Y_B - Y_A)}{L_{AB}^{obs}}
\]

\[
\sigma_p^2 = \left( \frac{\partial F_p}{\partial a_p} \right)^2 \sigma_{a_p}^2 + \left( \frac{\partial F_p}{\partial b_p} \right)^2 \sigma_{b_p}^2
\]

\[
L_{calc} = \sqrt{(Y_B - Y_1)^2 + (X_B - X_1)^2}
\]

\[
L_{calc} \neq L_{obs}
\]

MAIN DRAWBACKS

Lack of possibility

- to consider redundant observations
- to provide optimal solution to large discrepancies:
  - between measured and calculated fronts
  - between registered and calculated parcels areas
- to apply geometrical and cadastral constraints
PROPOSED METHOD

- Cadastral documents processing by means of Least Squares Adjustment of redundant observations
- Applying geometrical and cadastral constraints to parcel turning points position

NON-CONSTRAINED SOLUTION

- Functional relation between observations and $\beta$ unknown parameters $Y = F(\beta)$
- The Gauss-Markov adjustment model $\hat{Y} = X\hat{\beta} + \hat{\epsilon}$
- Linearization of the nonlinear adjustment process $Y_b - F(\beta_o) = \left( \frac{\partial F(\beta_o)}{\partial \beta} \right) d\beta + ...$
- Iterative solution $d\hat{\beta} = (X'PX)^{-1}X'PY$ $\rightarrow \hat{\beta}_a = \hat{\beta}_o + d\hat{\beta}$
CONSTRANDED SOLUTION (1)

- Functional constraints
  \[ G(\beta) = 0 \]

- Constraints linearization
  \[ G(\beta_0) + \frac{\partial G(\beta_0)}{\partial \beta_0} \cdot d\beta + \ldots = 0 \]

- The model of simultaneous adjustment
  \[
  \begin{bmatrix}
  X^T P X & H^T \\
  H & P^{-1}
  \end{bmatrix}
  \begin{bmatrix}
  d\beta \\
  k
  \end{bmatrix}
  =
  \begin{bmatrix}
  X^T P y \\
  w
  \end{bmatrix}
  \]

CONSTRANDED SOLUTION (2)

- Geometrical (implicit) constraints identification
  \[ F = \frac{1}{m} \sum (\hat{\beta}_c - \hat{\beta})^T \cdot \sum^{-1} \cdot (\hat{\beta}_c - \hat{\beta}) \]
  - \( \hat{\beta}_c \) - point coordinates calculated with constraints
  - \( \hat{\beta} \) - point coordinates calculated without constraints
  - \( \sum^{-1} \) - point positional variance calculated without constraints

- Hypothesis test (statistical F-test)
  \[ H_0 : \hat{\beta}_c = \hat{\beta} \ \text{vs.} \ H_1 : \hat{\beta}_c \neq \hat{\beta} \]
  (null hypothesis vs. alternative hypothesis)
  If \( F < F_{\alpha, m, n-m} \) - \( \hat{\beta}_c \) may be accepted
SIMULATION – Idea

1. The source: “true” original turning points coordinates and “true observations” fitting completely the geometry of the sample
2. “Spoiling” of “true observations” by applying a normally distributed errors
3. Processing according to the existing and proposed methods

SIMULATION - Details

- 3 situations of observations “spoiling”
  1. Assumed observations accuracy
  2. Low observations accuracy
  3. Large discrepancies between calculated and measured fronts
- Proposed adjustment method application
  1. Observations adjustment without constraints
  2. Stochastic constraints application
  3. Fixed constraints application
- Constraints
  1. Points co-linearity
  2. Lines parallelism
SIMULATION – Processing Results (1)

Point position error ellipses

3 situations:
- assumed observations accuracy
- low observations accuracy
- large discrepancies between calculated and measured fronts

SIMULATION – Processing Results (2)

Differences between the adjusted points coordinates and the "true" coordinates
SIMULATION – Processing Results (3)

Observations mean residuals

SIMULATION – Comparison

Proposed method vs. existing one:

- In situations (1) and (2) [assumed & low accuracy]
  - points positional errors are reduced
  - points coordinates are obtained closer to their "true" position
- In all three situations
  - observations residuals are considerably reduced
**SIMULATION – F-test**

**Implementation:**
- Additional constraint, not existing in the original situation, has been imposed

**Results:**
- Constraints existing in the original situation, are accepted; not existing – are denied

<table>
<thead>
<tr>
<th>Calculated $F_{0.05,24,15}$</th>
<th>Proposed method with stochastic constraints</th>
<th>Proposed method with fixed constraints</th>
<th>Tabular $F_{calc}$</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without additional constraints</td>
<td>0.425</td>
<td>0.655</td>
<td>2.288</td>
<td>Accepted</td>
</tr>
<tr>
<td>With additional constraints</td>
<td>8.637</td>
<td>19.602</td>
<td>2.288</td>
<td>Denied</td>
</tr>
</tbody>
</table>

**PROPOSED METHOD ADVANTAGES**
- Optimal coordination between different kinds of cadastral information
- Keeping ground observations, having juridical validity, maximal closeness to their adjusted values
- Increasing positional accuracy of parcels turning points
- Forcing parcels to retain their original geometrical form
FUTURE WORK

- Analysis of the weights of the original ground observations (achieving “correct” weights)
- Analysis of “correct/true” constraints weights and order of their implementation
- Connection of separate blocks of parcels to a homogeneous seamless cadastral space
You are invited to attend the

2009 FIG working week in Eilat, Israel

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Thank You