A New Mathematical Approach to Cadastral Documents Processing for Parcel Boundaries Restoration

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Key words: cadastral document processing, boundaries restoration, least squares adjustment, constrained adjustment

SUMMARY

One of the most important tasks in cadastral activity is restoration of parcels boundaries that were measured and determined in the past. The restoration process is based on processing cadastral documents - field books and field sheets that include details of ground surveying of the parcels turning points. Generally, an observational redundancy occurs that gives rise to ambiguity in boundaries restoration. For the time being, some computational techniques enable obtaining turning points planar coordinates by applying straightforward calculations. The main drawback of these techniques is their inability to consider fully and optimally the information included in the field books and the field sheets. Furthermore, often certain additional information exists regarding positional constraints of the turning points, determined either in an explicit or implicit manner, which is presently not taken into consideration despite the fact that it may significantly affect the computational results.

The paper introduces a new mathematical approach aimed at defining an optimal way of calculating the position of parcels boundary turning points by considering all available information recorded in cadastral documents. The documents processing is executed based on the classic Least Squares Adjustment Method that enables both optimal adjustment of redundant observations and application of functional constraints.

The initial results of this new computational technique are very encouraging. The obtained results indicate a significant decrease of system noise originating in the inevitable errors that occur during ground surveying. Moreover, the proposed technique enables solving one of the critical problems of cadastral process – keeping adjusted observations and pre-defined conditions maximally close to their legal values (registered parcels fronts and areas, declared width of roads). The latter quality is of great importance for implementing the proposed technique for establishing a digital cadastre based on legal coordinates.

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1. INTRODUCTION

The Israeli cadastre consists of approximately 15,000 registered blocks containing about 700,000 land parcels (Gavish & Doytsher, 2002). According to the customary method of land property registration in Israel, the land parcel, as a subject of registration, has to be measured by ground surveying (Henssen, 1995). The ground surveying details are recorded in special field books, and the parcel itself is plotted on official cadastral block maps and field sheets. As a result of the registration process, the parcel's measured area and the distances measured on the ground between adjacent turning points (fronts) attain legal validity.

The customary ground surveying method in Israel during many decades of cadastral activity was the orthogonal method (Perelmuter & Steinberg, 1992). Due to its relative simplicity, the measured parcels turning points have been plotted directly on cadastral maps skipping the stage of analytical processing of the surveying results. The post factum processing during parcel boundary restoration (which may occur decades after the original surveying has been completed) often reveals numerous discrepancies. For example, differences between parcel fronts measured on the ground and calculated from coordinates, which are greater than allowed by the Surveying Regulations, or between the parcels registered and calculated areas. In such cases, some matching work is required regarding the turning points coordinates in order to achieve harmony between the different types of original cadastral information. Unfortunately, thus far the processing work and consequential matching are performed by means of approximate methods, which are very far from being optimal from a mathematical standpoint.

Therefore, a new approach to optimal processing of cadastral documents is required. The development of such a mechanism gained urgency recently due to considerable efforts made in Israel, as in many other countries, to establish a nationwide coordinated cadastre whose legal validity will be guaranteed by optimal processing of original cadastral documents.

It should be mentioned that so far most research and engineering developments were concentrated on improving digitized cadastral maps, by trying to increase the positional accuracy of parcels boundaries (Doytsher & Shmutter, 1991), or by application of geometrical constraints (Morgenstern et al., 1989, Cothren, 2005), or on proposing of alternative models (Gavish & Doytsher, 2002). Because of the complexity of original cadastral documents processing and the many ambiguities and discrepancies characterizing them, some techniques aimed at skipping this problematic stage were developed (Xiaohua Tong, 2005). The main drawback of these techniques is the difficulty to obtain results based on graphic materials (digitized maps) on one hand, and achieving juridical validity required for implementation of a nationwide legal coordinated cadastre on the other hand. In the past few years, some new ideas have been proposed aimed at processing the original cadastral documents in an optimal manner (Fradkin & Doytsher, 2002).

The paper depicts a new mathematical approach to optimal definition of parcels turning points position considering different kinds of legal cadastral information stored both in explicit and implicit form in the original cadastral documents.

2. EXISTING METHOD

Most measurements for cadastral mapping were performed by using the chain surveying method (Kavanagh, 2000). It is based on measuring two corresponding distances – longitudinal and perpendicular to the chain survey measurement lines connecting geodetic control points (Figure 1).

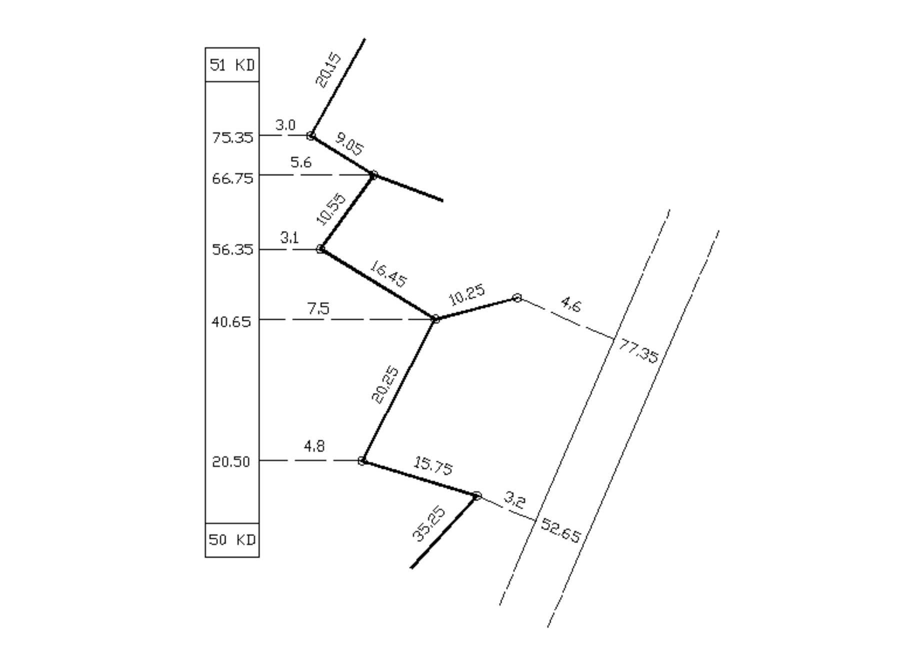

Figure 1. Chain surveying method **-** bold lines are the surveyed parcels boundaries, 50KD, 51KD are the geodetic control points

On the basis of these pairs of distances, measured independently for defining parcels boundaries turning points or physical objects on the ground, and based on the known coordinates of geodetic control points, the coordinates of measured points are calculated as:

$$Y_P = Y_A + a_P * (Y_B - Y_A) / L_{obs} + b_P * (X_B - X_A) / L_{obs} \qquad (1)$$
$$X_P = X_A + a_P * (X_B - X_A) / L_{obs} - b_P * (Y_B - Y_A) / L_{obs}$$

where

Y_P, X_P - calculated coordinates of measured point P
Y_A, X_A, Y_B, X_B - known coordinates of control points A, B
a_P - measured distance longitudinal to chain survey measurement line
b_P - measured distance perpendicular to chain survey measurement line
L_{obs} - observed length of measurement line (generally, different from the calculated one)

The common improvement achieved in the existing processing is the correction of distance a proportionally to the ratio between the length of chain survey measurement line calculated from the known coordinates of control points and its observed length: $a' = a * \frac{L_{calc}}{L_{obs}}$. If the consequent parcel turning points, connected by a measured front, have been measured on the same chain survey measurement line, the additional correction may be made to preserve measured front: correction of a according to Pythagoras theorem. No correction may be made in consequent parcels turning points, connected by a measured front, if they have been measured on different chain survey measurement lines. There is no optimal technique for considering more than two fronts connected to one point.

The calculation of single point P positional accuracy may be performed by using the error propagation formula assuming independence of observations (Anderson & Mikhail, 1998):

$$\sigma_P{}^2 = (\frac{\partial F_P}{\partial a_P})^2 \sigma_{a_P}{}^2 + (\frac{\partial F_P}{\partial b_P})^2 \sigma_{b_P}{}^2 \qquad (2)$$

where $F_P = F(a_P, b_P)$ according to (1) and $\sigma_{a_P}{}^2$, $\sigma_{b_P}{}^2$ are MSE of the distances a and b

Based on calculated points coordinates, the calculated parcels fronts are

$$L_{P_1P_2} = \sqrt{(Y_{P_2} - Y_{P_1})^2 + (X_{P_2} - X_{P_1})^2} \qquad (3)$$

where $Y_{P_1}, X_{P_1}, Y_{P_2}, X_{P_2}$ - calculated coordinates of parcels turning points

Generally, the calculated values of parcels fronts differ from their measured values on the ground ("legal fronts"). In many cases, the differences even exceed the values allowed by Surveying Regulations. The existing method does not offer any optimal way of handling this discrepancy, rather than just a simple "game" with orthogonal surveying elements aimed at bringing the aforementioned differences closer to their allowed values. Naturally, the results of such a "game" cannot be optimal.

3. PROPOSED METHOD

The classic processing method uses minimum information for calculation of parcels turning points coordinates. Each pair of distances measured longitudinally and perpendicularly to the chain survey measurement lines, enables finding a pair of point planar coordinates. On the other hand, parcels fronts measured on the ground independently of other observations are providing the redundant information about turning point positions. This situation obligates performing some sort of matching process aimed at adjusting the different kinds of ground observations.

The presently proposed approach to the required processing and matching is the Least Squares Adjustment model of redundant observations (see Appendix A): pairs of distances of orthogonal surveying and distances between parcels turning points (parcels fronts).

One of the most critical factors in the proposed adjustment method is the correct selection of observations weights. In order to keep more precise observations maximally unchanged at the expense of less precise ones, the observations weights have to be defined inversely to their accuracies - $P = \Sigma^{-1}$. Matrix Σ has on its main diagonal a priori variances of single observations (fronts and pairs of distances of orthogonal surveying method) and outside it - zeros which express the absence of correlation between different observations. The question is - what are the actual accuracies of observations performed many decades ago? Study of this issue today is a very difficult task and deserves separate study. Having currently no better choice, we assumed that all original measurements were performed in the past meeting the requirements of Surveying Regulations valid at that time.

4. CONSTRAINTS APPLICATION

The proposed solution is based on adjustment of redundant observations. In addition to the measured data, implicit and explicit information on the relationship between the position of parcels boundaries can be used. This information is commonly referred to as functional constraints due to its mathematical form, which defines functional connections between turning points coordinates (see Appendix B). The functional constraints are divided into two main groups - geometric and cadastral. The constraints of the first group define mutual geometric positions of the points and serve for preservation of the geometric form of the parcels. The constraints of the second group define point positions and serve for preservation of legally declared values (not measured during ground surveying). The constraints of the first group are not defined explicitly (and therefore their identification is not a simple task), for example – collinearity of parcels turning points (points belonging to one line) or parallel parcels boundaries. The constraints of the second group are defined explicitly, parcels registered areas and declared width of roads - both generally differing from their calculated values based on the coordinates of the parcels boundaries.

4.1. Constraints Examples

4.1.1. Collinear Lines

Parcels segments belonging to one line are a typical example of collinear lines. The task is to calculate parcels turning points coordinates based on original cadastral documents and provide the geometrical constraint of collinearity.

The analytical formula of the straight line is:

$X = m*Y + n$, where m is the slope and n is X-intercept of the line

This is a combined constraint: parcels boundaries have to be (i) parallel and (ii) intercept axis X, or any vertical line parallel to axis X, at the same point. The conditions (i) and (ii) for straight lines 1 and 2 may be expressed by:

$$\begin{aligned} m_1 - m_2 &= 0 \\ n_1 - n_2 &= 0 \end{aligned} \qquad (4)$$

The analytical parameters m and n of segments $P_1 - P_2$ and $P_3 - P_4$ may be expressed as:

$$m_1 = \frac{X_{P_2} - X_{P_1}}{Y_{P_2} - Y_{P_1}},\ m_2 = \frac{X_{P_4} - X_{P_3}}{Y_{P_4} - Y_{P_3}} \qquad (5)$$
$$n_1 = \frac{X_{P_1} * Y_{P_2} - Y_{P_1} * X_{P_2}}{Y_{P_2} - Y_{P_1}},\ n_2 = \frac{X_{P_3} * Y_{P_4} - Y_{P_3} * X_{P_4}}{Y_{P_4} - Y_{P_3}}$$

By inserting (14) into (13), we obtain the constraints functions (9) and may continue with the adjustment process according to the model (12).

4.1.2. Parallel Lines

A road parcel with two parallel opposing sides is a typical example of a constraint parallelism situation. This example may be considered as a particular case of the previous constraint when there is no common intercept point to the boundary lines, and the only condition is $m_1 - m_2 = 0$. Again, by inserting (14) into (13), taking into consideration only the m condition, we obtain the constraints functions (9) and may continue the adjustment process according to the model (12).

4.2. Identification of Geometrical Constraints

As mentioned previously, the identification of geometrical constraints is a difficult task due to their implicit existence form. The preliminary identification may be performed by visual analysis of the cadastral map aiming to define regions where the constraints imposition may take place. We propose to make a decision about the possible usage of constraints by performing the statistical F test (Hamilton, 1964, Snow, 2002). The test is performed on two sets of turning points coordinates: one set of coordinates calculated according to the proposed method without constraints and another - coordinates calculated with constraints. The test task is to define whether two sets may or may not belong statistically to one set.

During the test, the calculated F value is compared with its table value $F_{\alpha,m,n-m}$. The F value is calculated according to the following formula (Hamilton, 1964):

$$F = \frac{1}{m} * (\hat{\beta}_c - \hat{\beta})^T * \Sigma_\beta^{-1} * (\hat{\beta}_c - \hat{\beta}) \qquad (6)$$

where

$\hat{\beta}_c$ - turning point coordinates estimates calculated with constraints

$\hat{\beta}$ - turning point coordinates estimates calculated without constraints

Σ_β - turning point positional variance estimates calculated without constraints

By performing the test, the null hypothesis $H_0 : \beta_c = \beta$ is checked vs. an alternative hypothesis $H_1 : \beta_c \neq \beta$. If $F < F_{\alpha,m,n-m}$, the null hypothesis is accepted. In this case, it may be assumed that according to α significance level, two sets may statistically be defined as one set, and therefore the constrained coordinates may be accepted. If the test fails, there is no statistical basis for concluding that the turning points coordinates satisfy the constraints conditions.

5. SIMULATION OF PROPOSED METHOD

In order to be able to analyze the results and the accuracy of the proposed method, it has been tested on synthetic data that actually is a generalized chart of real situation. The simulation of the synthetic sample was composed of four parcels surrounded by geodetic control points (see Figure 2). Starting from "true" coordinates of the parcel turning points, all observations of the synthetic data have been calculated to simulate the chain survey measurement values fitting completely the geometry of the sample. Afterwards, these "true" observations were "spoiled" by applying a normal (Gauss) distribution error mechanism to the extent of three situations (see Table 1) - (1) assumed observations accuracy, (2) low observations accuracy and (3) large discrepancies between calculated and measured fronts.

Table 1. Error application to the "true" observations

Situation No.	σ_a of longitudinal distance	σ_b of perpendicular distance	σ_L of fronts
1	0.08	0.05	0.05
2	0.32	0.20	0.20
3	0.08	0.05	0.20

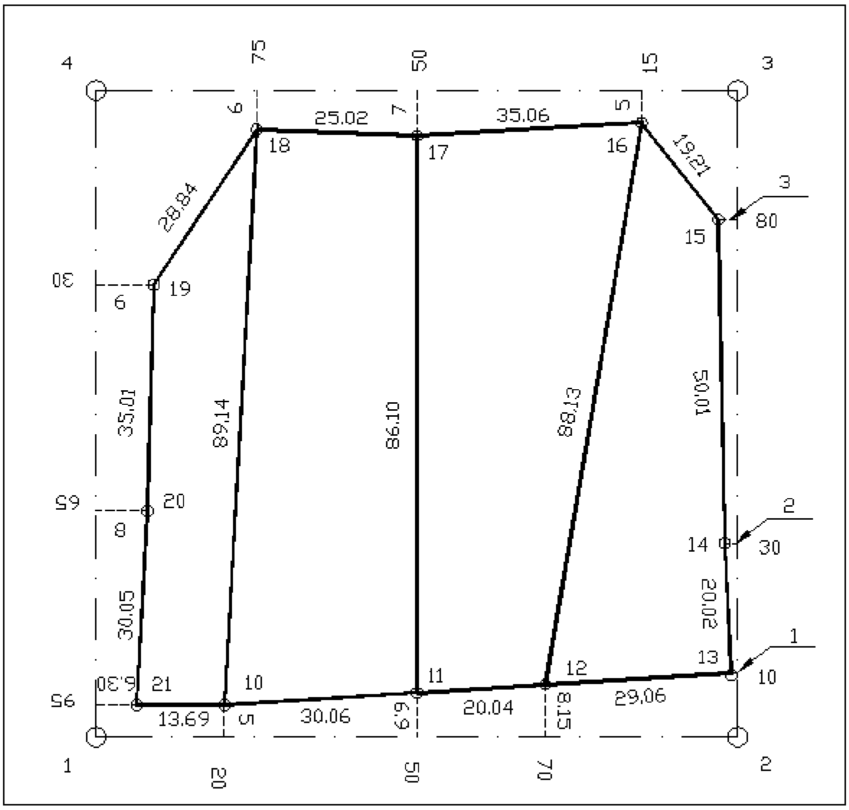

Figure 2. Simulation of proposed method: 1-4 – control points, 10-21 – surveyed parcels turning points

These simulated observations have been processed according to the existing and proposed methods. First, the preliminary processing was carried out according to the common existing method (see chapter 2). Then, the proposed adjustment method was applied in the following order:

– Observation adjustment without constraints (see chapter 3)
– Stochastic constraints application (see chapter 4)
– Fixed constraints application (see chapter 4)

– The following constraints have been imposed:
– points No. 10-13 are collinear (straight line constraint) (see chapter 4.1.1);
– lines 10-18 & 20-21 are parallel (parallel roadsides) (see chapter 4.1.2);

The iterative adjustment process was stopped when the incremental coordinates changes converged to zero over two consecutive iterations. Based on the obtained results, error ellipses (Table 2), coordinate differences (Table 3) and observations residuals (Table 4) were computed and compared to the results computed according to the common existing method.

Table 2. MSE of point position error ellipses (meters) computed based on the existing processing method (Chapter 2) and the proposed method (Chapters 3 & 4)

Situation No.	Existing method		Proposed method without constraints		Proposed method with stochastic constraints		Proposed method with fixed constraints	
	major	minor	major	minor	major	minor	major	minor
1	0.08	0.05	0.04	0.03	0.04	0.03	0.04	0.03
2	0.32	0.20	0.14	0.12	0.13	0.04	0.19	0.08
3	0.08	0.05	0.08	0.07	0.09	0.06	0.09	0.06

Table 3. MSE of differences (meters) between the adjusted points coordinates and the "true" coordinates

Situation No.	Existing method		Proposed method without constraints		Proposed method with stochastic constraints		Proposed method with fixed constraints	
	dY	dX	dY	dX	dY	dX	dY	dX
1	0.08	0.08	0.05	0.06	0.05	0.06	0.05	0.05
2	0.20	0.29	0.12	0.20	0.14	0.22	0.13	0.21
3	0.08	0.08	0.10	0.08	0.10	0.07	0.10	0.07

Table 4. Observations mean residuals (meters**)**

Situation No.	Existing method	Proposed method without constraints	Proposed method with stochastic constraints	Proposed method with fixed constraints
1	0.09	0.03	0.03	0.04
2	0.33	0.13	0.14	0.14
3	0.17	0.07	0.07	0.08

See the graphical presentation of the obtained results in following charts (situation No. 1 – blue, situation No. 2 – red, situation No. 3 – yellow):

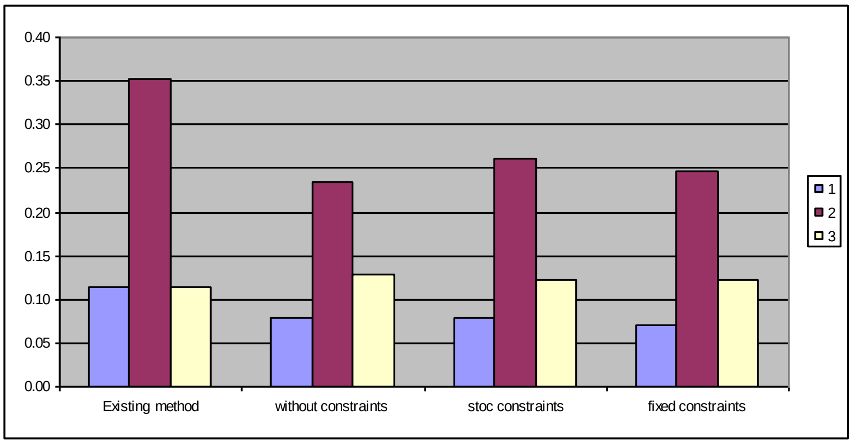

Chart 1. Point position error ellipses

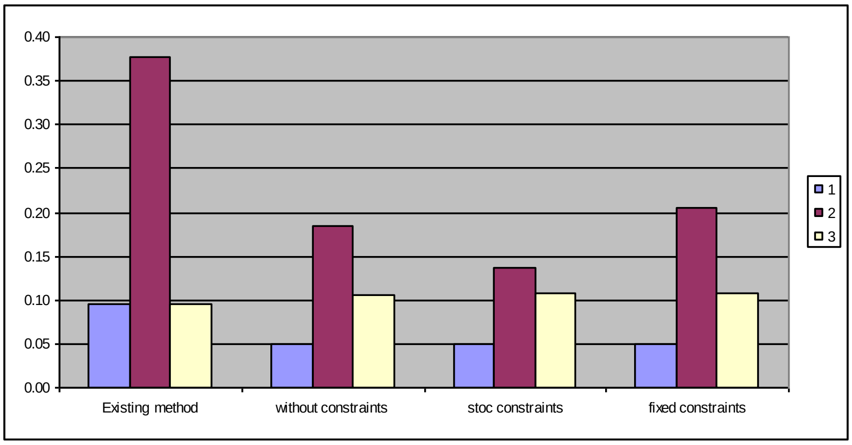

Chart 2. Differences between the adjusted points coordinates and the "true" coordinates

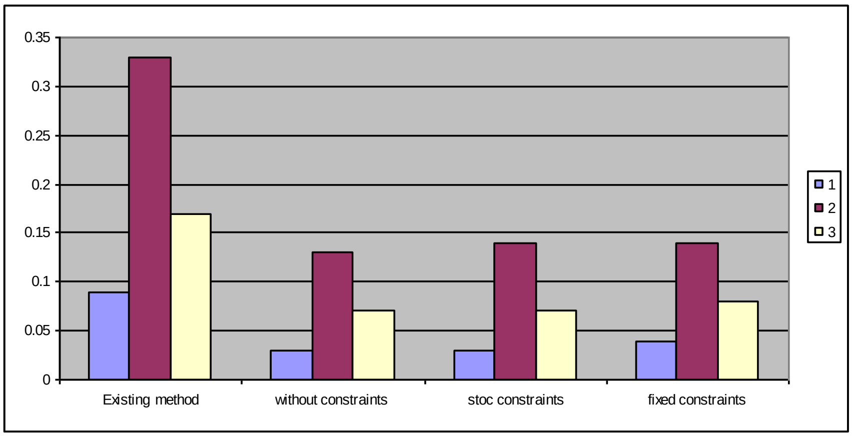

Chart 3. Observations mean residuals

As can be observed, the implementation of the proposed method in situations (1) and (2) enabled us to (i) reduce the positional error of the turning points (Table 2, Chart 1); (ii) obtain turning points coordinates closer to the "true" coordinates (Table 3, Chart 2); and, (iii) in all situations - considerably reduce the observations residuals (Table 4, Chart 3) in comparison to the existing method. Despite the fact that large discrepancies in situation (3) slightly increased the positional errors and coordinate differences, it was still compensated by gaining reduced observations residuals. These reduced residuals are the main purpose of our method – keeping the processed observations maximally close to their values having juridical validity. Obviously, the obtained results might have been even better, had the real observation accuracies been known.

An additional test has been carried out regarding the F-test implementation to the geometrical constraints identification (see chapter 4.2). The additional constraint, not existing in the original "ideal" situation, has been imposed on synthetic data - lines 10-18 & 20-19 are parallel (parallel roadsides). The previously processed results (without additional constraint) and the new ones (with additional constraint) have been tested by F-test (see Table 5).

Table 5. F-test of geometrical constraint identification

Calculated F_{calc}	Proposed method with stochastic constraints	Proposed method with fixed constraints	Tabular $F_{0.05,24,15}$	Test
Without additional constraint	0.425	0.655	2.288	Accepted
With additional constraint	8.637	19.602	2.288	Denied

As can be observed, if the imposed constraint exists in the original situation, the test indicates its acceptance as a part of the processing. If not, the constraint is denied.

6. CONCLUSION AND FUTURE WORK

Applying the proposed approach to original cadastral documents processing enabled us (i) to coordinate optimally between different kinds of cadastral information, (ii) to keep ground observations, which have juridical validity, maximally close to their adjusted values, and (iii) to increase position accuracy of parcels boundary turning points compared with the existing method. Furthermore, it has been shown that by imposing constraints on turning point positions during the adjustment process enabled us to force parcels to retain their original geometrical form, compared with the results of the existing method. In addition, applying the proposed approach ensures obtaining parcels turning points positions that are much closer to their "true" position than to those achieved by applying the existing method.

An additional study should be made to analyze the issue of the right (correct) weighting of original ground observations. The right weighting issue may drastically affect the optimal turning points positions and their accuracy. While applying stochastic constraints, some preliminary considerations have to be implemented regarding the correct constraints weighting and the order in which they are imposed. This topic deserves special study. Practice shows that despite implementation of an optimal computation process to separate blocks of parcels, the issue of their connection into homogeneous seamless space remains to be solved.

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APPENDIX A. APPLICATION OF LEAST SQUARES THEORY TO PROPOSED METHOD - NON-CONSTRAINED SOLUTION

The observations Y and the unknown parameters β (turning points coordinates) have a functional relation that may be expressed as:

$$Y = F(\beta) \tag{7}$$

Relation (7), named also the observations equation, actually expresses in compact form formulae (1) and (3) represented in form of: $a, b, L = F(Y_P, X_P)$

As an adjustment model, we use the Gauss-Markov model (Koch, 1999), which provides obtaining the best unbiased solution regarding the estimation of unknown parameters (turning points coordinates) and adjusted observations connected by the following relation:

$$\hat{Y} = X\hat{\beta} + \hat{\varepsilon} \tag{8}$$

where
X - Jacobian matrix of partial derivatives of relations (7)
ε - vector of observations residuals
^ - stands for Least Squares Estimator

From a mathematical viewpoint, the proposed method is a nonlinear adjustment process, which goes through linearization by expansion of functions (7) to Taylor series ignoring the second and higher order derivatives:

$$Y_b = F(\beta_o) + \frac{\partial F(\beta_o)}{\partial \beta_o} * d\beta + ... \quad (9)$$

In the last expression, index "o" stands for approximate values of unknown coordinates and Y_b stands for the vector of actual observations performed on the ground.

Trivial operation transforms (9) to the form of the aforementioned Gauss-Markov model (8):

$$Y_b - F(\beta_o) = \frac{\partial F(\beta_o)}{\partial \beta_o} * d\beta + ... \quad (10)$$

where

$Y_b - F(\beta_o) = \hat{Y}$, $\frac{\partial F(\beta_o)}{\partial \beta_o} = X$ and $d\beta = \hat{\beta}$ according to the notations of (8)

The adjustment process is performed as a series of sequential iterations. The formulae (1) are providing the initial (approximate) values of parcels turning points coordinates β_o which are close to their true values. By using the chosen adjustment model in each iteration, we obtain the least squares estimates of unknown incremental changes $d\hat{\beta}$ of the initial values of turning points coordinates (in the first iteration) or of the corrected ones (in the following iterations) from the following expression (Koch, 1999):

$$d\hat{\beta} = (X'PX)^{-1} X'PY \quad (11)$$

where P stands for the observations weight matrix. Then we calculate the estimates of turning points coordinates by $\hat{\beta}_a = \hat{\beta}_o + d\hat{\beta}$, which become the approximate values for the next iteration or the final (adjusted) ones if the process is stopped (when $d\hat{\beta}$ values converge to zero or fall below a predefined threshold).

Since, in our case we are dealing with redundant and uncorrelated observations, matrix X has a full column rank and accordingly matrix $X'PX$ can be inverted.

As a result of high order derivatives omission in the Taylor expansion (9) and because of inevitable occurrence of random errors during performance of ground surveying, there will always exist residuals ε in the adjustment process whose estimates are calculated according to the Gauss-Markov model (8) by $\hat{\varepsilon} = \hat{Y} - X\hat{\beta}$ or, considering convergence of $\hat{\beta}$ to zero at the end of the adjustment process, by $\hat{\varepsilon} = \hat{Y} = Y_b - F(\hat{\beta}_a)$.

The Gauss-Markov adjustment model is characterized by the minimum sum of weighted squared residuals of observations ($\sum p_i \hat{\varepsilon}_i^{\,2} = \min$). Therefore, it enables meeting one of the essential requirements in cadastral documents processing – keeping adjusted values of observations $F(\hat{\beta}_a)$ maximally close to their measured on the ground values Y_b.

The turning points positional accuracy estimation is calculated as $\sum_{\beta} = \hat{\sigma}_0^{\,2} * Q$, where Q is a cofactor matrix $Q = N^{-1} = (X' PX)^{-1}$ and $\hat{\sigma}_0^{\,2}$ is a posteriori unit weight reference variance estimation calculated as $\hat{\sigma}_0^{\,2} = \frac{\hat{\varepsilon}^T P \hat{\varepsilon}}{r}$, where $r = n - m$ is the system redundancy, n - number of observations, m - number of unknowns (equals the number of parcels turning points multiplied by two).

APPENDIX B. APPLICATION OF LEAST SQUARES THEORY TO PROPOSED METHOD - CONSTRAINED SOLUTION

The mathematical model of functional constraints may be expressed as (Mikhail & Ackerman, 1976):

$$G(\beta) = 0 \qquad (12)$$

Generally, the model (12) is not linear and should be linearized according to the Taylor expansion:

$$G(\beta_o) + \frac{\partial G(\beta_o)}{\partial \beta_o} * d\beta + ... = 0 \qquad (13)$$

As an adjustment model, we use the Gauss-Markov model with constraints (Koch, 1999):

$$H\beta = w \qquad (14)$$

where

H - matrix of constraints equations coefficient terms calculated as partial derivatives of constraints functions: $H = \frac{\partial G(\beta_o)}{\partial \beta_o}$

w - vector of constraints functions values calculated as: $w = -G(\beta_0)$

The model of simultaneous adjustment of redundant observations (7) and functional constraints (12) is:

$$\begin{bmatrix} X^T P X & H^T \\ H & P_c^{-1} \end{bmatrix} * \begin{bmatrix} \beta \\ k \end{bmatrix} = \begin{bmatrix} X^T P y \\ w \end{bmatrix} \qquad (15)$$

where

k - vector of Lagrange multipliers of constraints functions

P_c - matrix of constraints weights

In the case of stochastic constraints (partial fit to constraints), P_c diagonal terms have finite values defined according to a predetermined relation between them. In the case of fixed constraints (complete fit to constraints), the diagonal terms have infinite values and P_c^{-1} turns to zero.

BIOGRAPHICAL NOTES

Mr. Michael Klebanov received an Engineer Degree (cum laude) in Civil Engineering from the Technical University of Cheliabinsk, Russia, in 1985. In 2000-2002, he completed advanced studies at the Technion - Israel Institute of Technology, Division of Geodetic Engineering, towards a Licensed Surveyor Degree. He is currently a candidate in an M.Sc./Ph.D. direct track in Mapping and Geo-Information Engineering at the Technion. Since 1991, he is with the Survey of Israel – the Israeli national agency for geodesy, cadastre, mapping and geographic information systems.

Prof. Yerach Doytsher graduated from the Technion - Israel Institute of Technology in Civil Engineering in 1967. He received a M.Sc. (1972) and D.Sc. (1979) in Geodetic Engineering also from the Technion. Until 1995 he was involved in geodetic and mapping projects and consultation within the private and public sectors in Israel. Since 1996 he is a faculty staff member in Civil and Environmental Engineering at the Technion, and is currently the Dean of the Faculty of Architecture and Town Planning. He is also heading the Geodesy and Mapping Research Center at the Technion.

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