

Progress Towards a Centimeter Geoid for Dubai Emirate

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SUMMARY

A high-resolution and high-precision detailed gravimetric geoid has been computed for Dubai Emirate, ranging from $24^{\circ} 35'N$ to $25^{\circ} 21'N$ in latitude and $54^{\circ} 52'E$ to $56^{\circ} 13' E$ in longitude. The EGM96 geopotential model complete to degree and order 360 was combined with surface gravity data and Fast Fourier Transformation (FFT) algorithm to generate the geoid file. The surface data consists of 1km x 1 km gravity data and 20m X 20m Digital Terrain Elevation Model as well as GPS and leveling data. The method of least square collocation has also been used for an alternative preliminary geoid computation. The computed geoid has an estimated error of 2cm rms.

Comparison of the gravimetric geoid with the GPS/ leveling derived geoidal heights of **3750** stations all over Dubai Emirate shows that the absolute agreement with respect to the GPS/ leveling datum is generally better than 3-4 cm rms. Results show that combining both GPS heights and the Dubai geoid model can give orthometric heights accurate to 2-5 cm. The method can thus work as a good alternative to traditional levelling, particularly for third order levelling in large areas.

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1. INTRODUCTION

The positions of points derived from Global Positioning System (GPS) measurements are usually computed in a terrestrial three-dimensional Cartesian frame. The resulting X, Y and Z co-ordinates of the GPS points are then transformed, using a reference ellipsoid, into geodetic co-ordinates in terms of latitude (ϕ), longitude (λ), and ellipsoidal height (h). The GPS recovers the latitude and longitude differences to about 1 ppm and ellipsoidal height differences to about 1-2 ppm Ayhan (1993). Ellipsoidal heights (h) are heights measured from a defined reference ellipsoid. However, orthometric heights which are useful in most Surveying and Engineering applications are measured from the geoid. The difference between the two heights depends on the separation between the geoid and the reference ellipsoid, that is geoidal height (N). If the geoid undulation (N) of station is known, and the ellipsoidal height (h) is determined from GPS observations, then the absolute orthometric height (H) of the station can be determined directly from: $(N = h - H)$. Also in a relative mode, the orthometric height differences between two stations can be obtained without levelling by combining the computed ΔN and Δh determined by GPS interferometry from: $(\Delta H = \Delta h - \Delta N)$.

The determination of the geoid has been one of the prime objectives of geodesy. The knowledge of the geoid with respect to some reference ellipsoid, either on a global or local scale is valuable to geodesy, surveying and geophysics for a number of purposes such as the reduction of measured distances to a reference surface and the processing of satellite observations. The geoid represents the datum to which height differences and the gravity potentials are referred. The geoid heights are essential for verification of global datums and transformation of local datums to the world datum. Also the combination of an accurate geoid model with GPS co-ordinates plays a dominant role in achieving high accuracy levelling results. Spirit levelling is tedious, time consuming and costly in conventional surveying exercise. The knowledge of the geoid is also essential in geophysical explorations (reconnaissance survey), in control surveys, in large scale mapping, in engineering surveys, in height control and in understanding of the Earth's crustal structure.

The geoid solution was based on EGM96 model coefficient set complete to degree and order 360, 1 km x 1 km point gravity anomalies and 20m x 20m mean height blocks. Our intention is to generate a more accurate geoid file for Dubai Emirate by combining the above data set with EGM96 model coefficients set, to satisfy current geodetic requirements in the Emirate.

The lack of gravity measurement over much of the earth surface is still the major problem in gravimetric geodesy. Various methods do exist to compute geoid undulations from surface gravity data, namely, Stokes's integration, Fast-Fourier Transform (FFT) and Least Squares Collocation (LSC). The combination of spherical harmonics potential coefficient set with terrestrial gravity data in order to reduce the latter requirement to a localised region for geoid height computation has been carried out by many researchers (e.g Molodenskii et al,

1960;Paul, 1973;Rapp, 1981; Jekeli, 1981; Sjoberg, 1985;Vanicek et al, 1986). In this investigation, the combination of spherical harmonics potential coefficient set with terrestrial gravity data is used to compute the geoid. The modified spheroidal Stokes's kernel is used in the geoid computation instead of the conventional ellipsoidal Stokes's kernel. It is found that spheroidal function tapers off more rapidly than the ellipsoidal function for increasing spherical distances. Thus we can expect that a truncation of the spheroidal (modified) integration at a certain spherical distance lead to smaller truncation errors compared to the truncation of the ellipsoidal (original) Stokes's integration.

2. DATA USED IN THE COMPUTATIONS

2.1. Gravity data

Gravity data for Dubai were surveyed by the German contractor "Hansa Luftbild" in the period 2000-2001. The gravity net was referenced to three absolute gravity stations established in Dubai. The heights of the gravity points were measured with fast static GPS. As part of the gravity processing, gravity values and ellipsoidal heights of the gravity points were converted into conventional free-air and Bouguer gravity anomalies, using the EGM96 geoid model (with the new geoid model the gravity anomalies will change a fraction of a mgal, but with the GPS levelling fit applied this will have no consequence for the geoid). The Bouguer anomalies are smooth in mainland Dubai, but a very large gradient goes through Hatta, cf. Fig. 1.

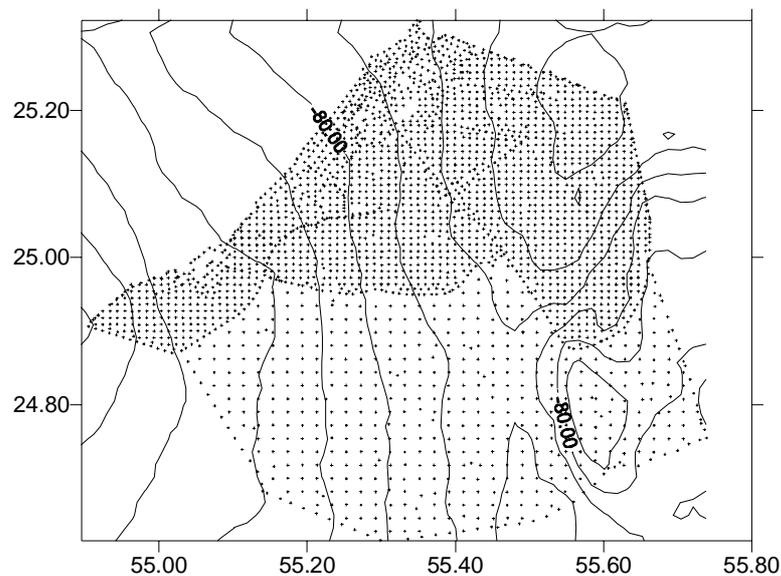


Fig.1a. Bouguer anomalies of Dubai main area (5 mgal contour interval)

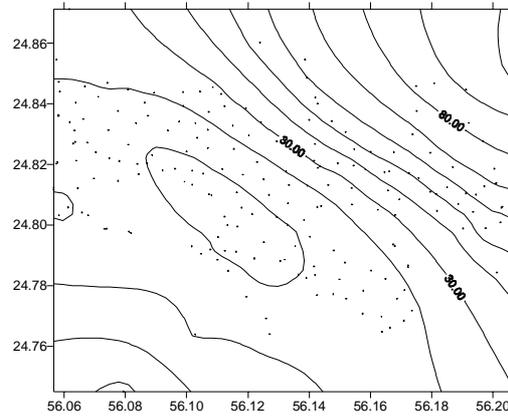


Fig 1b. Bouguer anomalies of Hatta (10 mgal contour interval)

The large horizontal gradient of the gravity anomalies through Hatta makes the gravimetric geoid computation from gravity in this region very difficult – the data area is just too small. Gravity data from neighbouring areas (other emirates of U.A.E. and Oman) were not available for this project. All gravity was checked for outliers, and the marine gravity data compared to the satellite altimetry, to check for possible datum errors. The comparison statistics is shown in Table 1. The statistics show that the region has a quite rough gravity field, and that the marine data fits well.

Table 1. Comparison of marine gravimetry and KMS-01 satellite altimetry

Unit: mgal	Mean	Std.dev.
Marine gravimetry	-54.4	39.8
KMS01 altimetry	-46.3	34.6
Difference	-0.9	10.6

2.2. Digital elevation models

The digital elevation models available consisted of a 30'' x 30'' DEM from NGDC, USA, covering the complete region, and a number of detailed heights provided by Dubai Municipality. The detailed heights were averaged into a 100 m x 100 m basic DEM, and the 30'' DEM used to fill in the missing regions.

2.3. GPS and leveling data

A set of approx. 3750 leveled benchmarks with GPS ellipsoidal heights were made available for the computation. The GPS data were tied into the ITRF base network of Dubai, and the levelling referred to a fundamental tide gauge at Port Rashid, Dubai. Most levelling was third order, with some points levelled by trigonometric methods, which was not for this Geoid computation purpose. Many GPS points were repeated RTK measurements (with a 5 cm acceptance limit), while other points in build-up areas were actually determined using classical techniques from nearby GPS points. At points with GPS and leveling a GPS geoid value was derived by:

$$N^{GPS} = h^{GPS} - H^{levelling}$$

It should be pointed out that these geoid heights, opposed to the geoid heights determined from global models and gravity data, refer to a *local* vertical datum, due to the sea-surface topography at the reference tide gauge.

In connection with the gravity observations, a levelling line was observed around the perimeter of the Dubai main area, and GPS observations (for gravity station heights) were done in connection with this. The eastern and southern part of the perimeter levelling line GPS was done using rapid static techniques. However, baselines were relatively short, and it appears that the accuracy was good enough also for geoid use (3-5 cm for most points). The perimeter GPS geoid data have therefore also been used for constraining the final geoid, after an iterative editing of outliers, outlined in detail later

3. COMPUTATION OF THE GRAVIMETRIC GEOID

The Dubai precise geoid has been computed in two steps:

- 1) A gravimetric geoid model, computed by spherical FFT in a global datum, and
- 2) A GPS-tailored local geoid, which fits the GPS observations and the Dubai vertical datum to a few cm. This step has involved an iterative editing of GPS-leveling outliers.

The advantage of the two-step method is that the second step, being less complex, can be readily repeated as future GPS-levelling observations detect errors in the original data, and hence the tailored geoid. The same method has been used for the 1-cm geoid of Denmark (Forsberg et al., 1996).

The method of least-squares collocation has also been used for an alternative preliminary geoid computation, primarily to assess the overall accuracy of the geoid. Collocation has the advantage that both steps can be done at once, with a semi-automatic rejection of outliers. The disadvantage is the computational expense, since a very large system of linear equations has to be solved.

The collocation error estimates showed that the geoid in the center of mainland Dubai is accurate to 2 cm r.m.s., while in Hatta expected errors are at the 10 cm level.

All computations outlined in the sequel have been done by programs of the GRAVSOFT package, © KMS and University of Copenhagen.

3.1 Gravimetric geoid computation: “remove” step

In the gravimetric geoid computation, the geoid signal N is constructed by subdividing it into three parts

$$N = N_1 + N_2 + N_3$$

Where first part comes from a spherical harmonic expansion complete to degree and order 360 (EGM96, cf. Lemoine et al, 1996), the second part from the topography, and the third part from the contributions of "residual" gravity (i.e., gravity anomalies minus the global field contribution and gravimetric terrain effects).

The EGM96 geopotential model values are computed in a grid (GRAVSOFT program “harmexp”), and observed free-air anomaly gravity data reduced using this linear interpolation in this grid using (“geoip”).

Terrain effects have been removed in a consistent residual terrain model (RTM) data reduction using prisms (“tc” program), taking into account the topographic irregularities

relative to a mean height surface, for details see Forsberg (1984). Table 2 shows the effects of including the 100 m DEM data, compared to the 1 km NGDC DEM. The difference is only significant in Hatta.

Table 2. Comparison of RTM gravity terrain effects computed from 100 m and 1 km DEM

RTM effects (unit: mgal)	Mean	Std.dev.	Min	Max
<i>Hatta area (199 points):</i>				
100 m grid	5.5	6.8	-5.2	60.3
1 km grid	6.3	8.3	-5.2	85.1
Difference	-0.7	2.0	-24.8	0.4
<i>Dubai main area (subset 356 pts):</i>				
100 m grid	-2.4	1.6	-6.2	1.5
1 km grid	-2.4	1.7	-6.2	1.6
Difference	-0.0	0.1	-0.6	0.6

Two different resolutions of the RTM mean height surface was tested: 50 km and 100 km (computed by “tgrid”). In addition a complete topographic-isostatic reduction (land only) was also tested, to see if the terrain- and EGM96-reduced data in Hatta could be sufficiently detrended or bias reduced. This was not possible, and it appears that the small Hatta enclave sits by chance on a major geological gravity anomaly, and that the topographic-isostatic gravity field do not provide a good fit to anomalies.

The statistics of the terrain reductions are shown in Table 3, for the DM data sets as well as the complete data set, including marine gravimetry and altimetry. An atmospheric correction has been applied to the gravimetry data, in accordance with theoretical requirements for solving the geodetic boundary values problem.

Table 3. Statistics of gravity data reductions

All data (9206 pts)

Unit: mgal	Mean	Std.dev.	Min	Max
Original data	-54.3	40.1	-128.1	132.3
- EGM	-5.0	13.0	-78.8	62.3
- EGM – RTM (60’)	-2.7	12.3	-70.6	66.9
- EGM – RTM (30’)	-3.9	12.4	-78.4	69.9
- EGM – Isostatic	-0.6	13.5	-69.1	66.7

Dubai main area gravimetry (2881 pts)

Unit: mgal	Mean	Std.dev.	Min	Max
Original data	-72.3	10.9	-95.6	51.0
- EGM	-8.8	11.2	-40.5	8.3
- EGM – RTM (60’)	-6.5	11.3	-44.4	10.9
- EGM – RTM (30’)	-8.2	10.9	-32.6	9.1
- EGM – Isostatic	-5.6	11.2	-52.5	13.7

Hatta gravimetry only (199 pts)

Unit: mgal	Mean	Std.dev.	Min	Max
Original data	66.5	19.0	44.7	132.3
- EGM	-18.3	18.1	-42.7	62.3
- EGM – RTM (60’)	-21.8	16.3	-45.4	32.7
- EGM – RTM (30’)	-10.7	15.8	-33.9	40.1
- EGM – Isostatic	-30.8	16.1	-54.4	22.6

The 30' RTM reduction was selected for the subsequent geoid processing, as it fits EGM96 best concerning data bias in Hatta, and should (in theory) be consistent with a perfect EGM.

3.2. Gravity to geoid conversion by FFT

Residual gravity anomalies are subsequently converted into residual height anomalies by spherical FFT (Strang van Hees, 1990). The method in principle evaluates Molodensky's integral

$$\zeta_3 = \frac{R}{4\pi\gamma} \iint_{\sigma} (\Delta g_3 + g_1^c) S(\psi) d\sigma$$

where g_1^c is the first term of the Molodensky series for the *terrain-reduced* field, and ζ the quasi-geoid. The integral is in practice identical to Stokes' integral, as the g_1^c -term is very small, and may for most practical purposes be neglected. For the computations here we have treated ζ and the classical geoid N as identical; the difference is less than 2 cm in Hatta, and since the leveling data have not been adjusted in geopotential numbers, the theoretical type of geoid is not an issue, especially considering that GPS-leveling is used to constrain the geoid in the end.

In the used FFT method, the Molodensky/Stokes' integral is written as a spherical convolution in latitude and longitude (φ, λ) for a given reference parallel φ_{ref} , and by utilization of a number of bands a virtually exact convolution expression may be obtained by a suitable linear combination of the bands. For each band the convolution expressions are evaluated by

$$\zeta_3 = S_{ref}(\Delta\varphi, \Delta\lambda) * [\Delta g_3(\varphi, \lambda) \sin\varphi] = F^{-1}[F(S_{ref})F(\Delta g_3 \sin\varphi)]$$

where S_{ref} is a (modified) Stokes' kernel function, and $*$ and F the two-dimensional convolution and Fourier transform, respectively, for details see Forsberg and Sideris (1993).

In the actual implementation of the method, the data are gridded by least-squares collocation ("geogrid"), and a 100% zero padding is used to limit the periodicity errors of FFT ("spfour"). A Wong-Gore kernel modification has been used for spherical harmonic degrees less than 60 to limit the long-wavelength errors. The collocation gridding was done using a correlation length of 20 km assuming a free-air anomaly noise of 0.5 mgal. Fig. 2 shows the empirical covariance models determined from the reduced data.

The gravimetric geoid has been computed in the region 22.5-27.5 N, 52.5 – 58.5 W at a 1' resolution in latitude and longitude. Use of a more detailed grid spacing do not improve the geoid since the residual geoid signal at very short wavelengths is essentially nil.

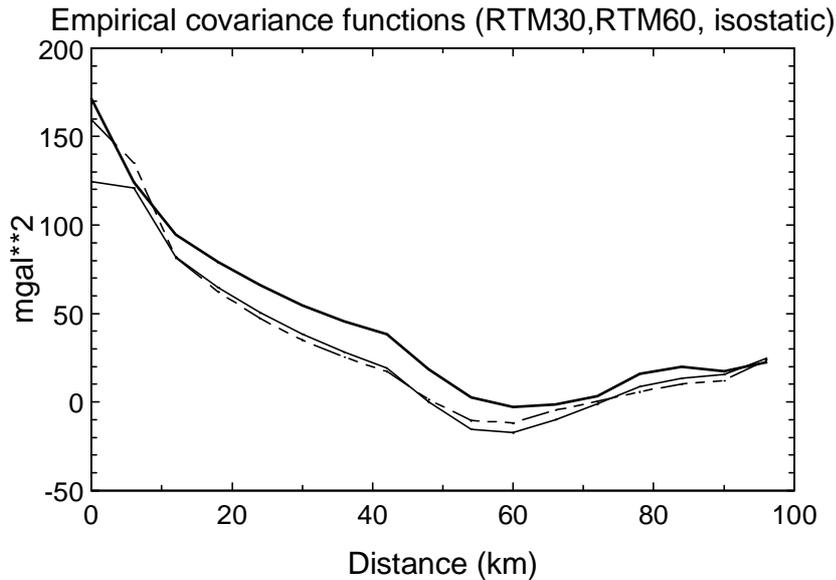


Fig. 2. Covariance function of reduced gravity data (dashed line isostatic).

3.3. Restoring of topography and EGM96 for the final gravimetric geoid

The topographic RTM restore signal (ζ_2) is evaluated by FFT methods, using the first-order mass-layer approximation to the RTM geoid effect, cf. Forsberg, 1985 (“tcfour”). Table 4 shows the statistics for the restore steps. Table 5 shows a comparison to DM GPS-leveling, using a slightly edited data set where obvious outliers greater than 0.5 m were removed. It is seen that the gravimetric geoid indeed makes a large improvement over EGM96 (the mean value is not of significance, as the GPS geoid heights refers to a local datum).

Table 4. Statistics for gravimetric geoid restore steps

Unit: m	Mean	Std.dev.	Min	Max
Reduced geoid (after FFT)	0.16	0.25	-0.66	1.43
Do, Wong-Gore mod.deg. 60	0.00	0.20	-1.02	1.25
RTM-60 terrain effects	-0.02	0.28	-0.69	1.89
RTM-30 terrain effects	0.00	0.13	-0.26	1.69
Final geoid (RTM30)	-29.10	3.72	-33.57	-18.66

Table 5. Comparisons of gravimetric geoid to Dubai Municipality GPS leveling

Unit: m	Mean	Std.dev.
<i>Dubai area (3157 GPS pts)</i>		
Gravimetric geoid	-1.40	0.09
EGM96 only	-0.83	0.16
<i>Hatta area (110 pts)</i>		
Gravimetric geoid	-1.39	0.12
EGM96	-1.52	0.25

4. GPS-LEVELING FITTING OF THE GEOID

The computed gravimetric geoid needs to be fitted to local GPS-leveling data for operational GPS height use, to eliminate datum shift, residual long-wavelength gravity errors, and as well possible systematic errors in the levelling. The basic principle used here is to model the gravimetric and GPS geoid difference by a smooth function consisting of a trend function f (a polynomial) and a residual ϵ' , to be modelled by least/squares collocation

$$\epsilon = N_{\text{grav}} - N_{\text{GPS}} = f(\varphi, \lambda) + \epsilon'$$

4.1. GPS-levelling outlier rejection

It was apparent that a number of outliers were present in the GPS leveling data. In the first step a linear regression was performed on the difference (trend function as a third order polynomial in N and E, no collocation), and the residuals used to reject outliers. The outlier detection was done for mainland Dubai and Hatta, separately. Table 6 shows the number of apparent outliers, based on the cut-off criterion.

Table 6. Number of GPS outliers relative to cut-off limit (DM and Hansa GPS data)

Cut-off limit	Number of points rejected	Fraction of total
<i>Dubai main area (3452 pts)</i>		
6 cm	456	13.2%
12 cm	185	5.3%
20 cm	90	2.6%
<i>Hatta (291 pts)</i>		
15 cm	25	13.1%
18 cm	17	8.9%
24 cm	7	3.6%

For the final geoid fitting it was chosen to use a cut-off of 20 cm for Dubai and 24 cm for Hatta, in order not to loose data in regions with a poor gravimetric geoid (especially at the outer limits of the area). Fig. 3 shows the covariance function for the plane-fit detrended GPS geoid data. It is apparent that the residual errors are close to white noise.

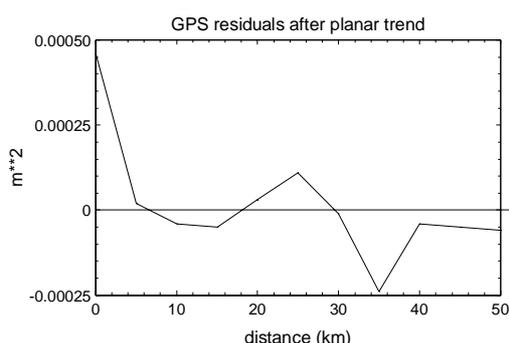


Fig. 3. Covariance function of GPS geoid residuals after planar fit (DM data).

4.2. Final geoid computation

For the final geoid computation, the residual ε' is modelled by least-squares collocation, using a 2nd order Markov covariance function (“geogrid”)

$$C(s) = C_0(1+ks)e^{-ks}$$

where k is a constant, determined by correlation length, and s the distance. The factor C_0 is determined from the data, whereas k and the apriori noise on the GPS leveling may be determined by the user. These factors effectively act like a smoothing parameter, making sure the final geoid on average fits the GPS data, but is not affected by individual, random errors. After a number of tests, a correlation length ($x_{1/2}$) of 25 km and GPS geoid noise (sigma) of 0.035 m seemed to give good results. The polynomial trend function used was a constant, to prevent the geoid to diverge too much outside the area of GPS coverage. Table 7 shows the fit of the GPS leveling data to the tailored geoid. It should be stressed that the 3.5 cm is not the final accuracy of the geoid – the geoid will affectively average a large number of GPS data, and we believe that the collocation error estimate of 2 cm r.m.s. (1-sigma) is a realistic estimate for the error of the geoid in the Dubai main area? In Hatta the geoid is difficult to compute due to lack of gravity data in the neighbouring regions (Oman and other UAE emirates), and due to the apparent larger noise in the GPS leveling. The geoid might only be accurate to 5-10 cm. here.

Fig. 4. GPS corrector signal to the gravimetric geoid. It is seen that the major corrections – except for a bias – is occurring along the edges (and at Hatta), in accordance with expectations.

Table 7. R.m.s. fit of GPS leveling data to the tailored geoid “DUBAIGEO”

<i>Unit: m</i>	<i>Mean</i>	<i>Std.dev.</i>	<i>Min</i>	<i>Max</i>
<i>All data (3540 pts.)</i>	<i>-.001</i>	<i>.041</i>	<i>-.269</i>	<i>.245</i>
<i>DM GPS data, Dubai main (3212 pts)</i>	<i>.000</i>	<i>.036</i>	<i>-.215</i>	<i>.198</i>
<i>Hansa GPS data, Dubai (150 pts)</i>	<i>.000</i>	<i>.050</i>	<i>-.115</i>	<i>.210</i>
<i>Hatta area (110 pts.)</i>	<i>-.008</i>	<i>.110</i>	<i>-.423</i>	<i>.461</i>

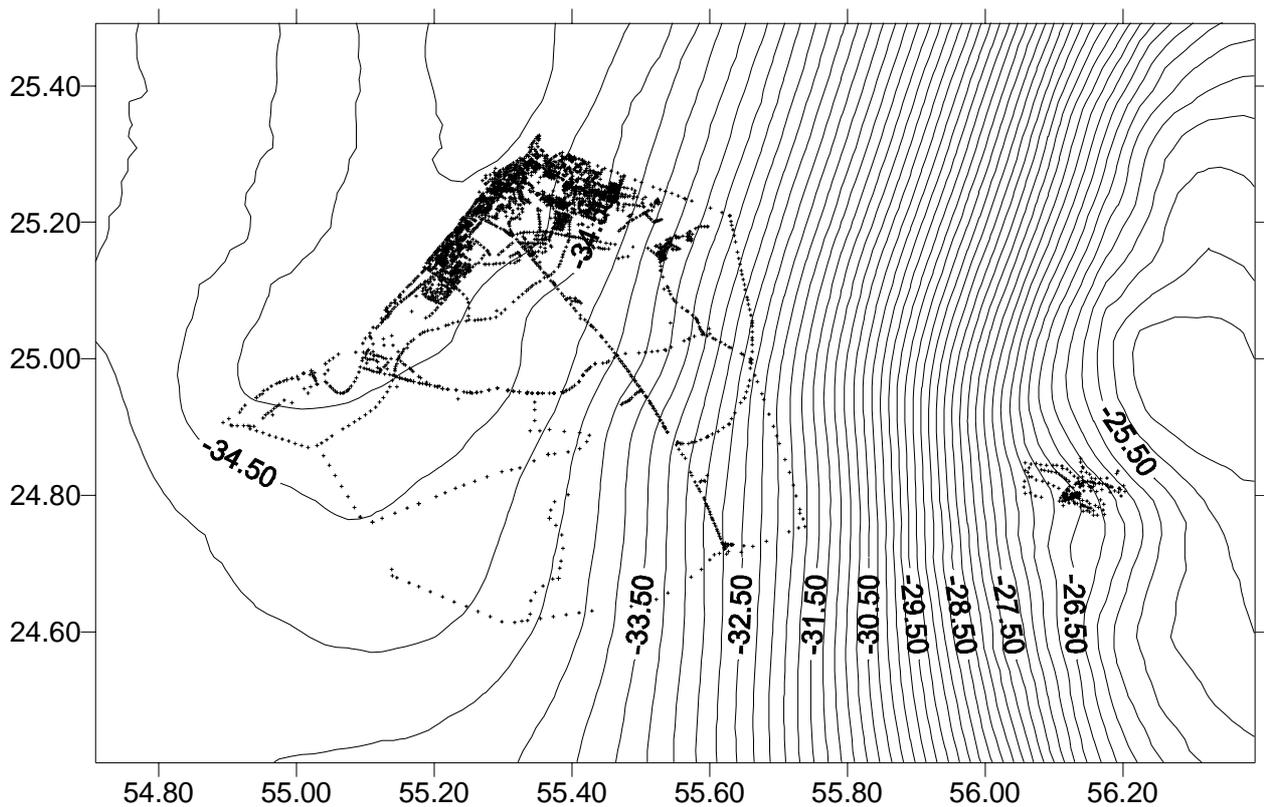


Fig. 4. Dubai geoid computed from gravity, GPS and leveling.
Contour interval 20 cm.

5. CONCLUSIONS

The outcome of this investigation is a detailed centimetres gravimetric geoid for Dubai Emirate. The geoid is recovered using 1km x 1 km gravity data and 20m X 20m Digital Terrain Elevation Model as well as GPS and leveling data combined with EGM96 (360,360) spherical harmonics potential coefficient set. To compute the gravimetric geoid in Dubai Emirate, the modified Stokes's kernel is being used instead of the original ellipsoidal Stokes's kernel, to reduce truncation errors as the former tapers off more rapidly than the latter (the influence of distant gravity anomalies on local geoid heights is reduced). The reduction is proportional to the degree L of the satellite model being used to recover the long wavelength component of the geoid.

To recover the long wavelength contribution of the gravimetric geoid, the EGM96 (360,360) model coefficient set complete to degree and order 360, has been chosen out of the variety of potential field models published so far, as probably the most recent solution available at the time of our computations. This implies that the smallest gravity field features represented in EGM96 (360,360) model have spatial extent of 0.5^0 spherical distance or 55 km. The short

wavelength component of the geoid is recovered by means of mean gravity anomalies after subtracting the corresponding EGM96 (360,360) model values from gravity anomalies.

The total gravimetric geoid was obtained from the summation of the long wavelength, geoid component and the short wavelength geoid components. This solution was referred to WGS-84 datum. The comparison of GPS/levelling geoid heights with the corresponding gravimetric values showed a reasonable agreement with RMS of 3-4 cm rms.

Finally, to meet the 1cm geoid, which has been the goal of geodesists and geophysicists, the effect of the atmosphere, the topography and the ellipticity of the reference surface on the gravity as well as the indirect effect on the computed geoid has been taken into account.

To verify our gravimetric solution, an independent geoid was derived from Global Positioning System (GPS) network established throughout the Emirate with its stations located at known benchmark heights. The derived Geoid model is precise enough to replace conventional levelling, particularly for third order levelling in large areas. In the main land Dubai the accuracy could be of the order of 1-3 cm on average. In Hatta region the accuracy ranges between of 5-10 cm. Still more information and data is necessary to improve the model in this part of the Emirate. We believe this Geoid model could meet the requirement of many potential users who would intend to convert GPS heights into their corresponding Mean Sea Level heights.

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