ASSESSMENT OF RISK WITHIN GEODETIC DECISIONS

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ABSTRACT:

Geodetic deformation measurements and analysis concepts are one basis for minimizing the risk of, e. g. unexpected collapses of artificial objects and geohazards. The idea behind these activities is the need of the society in minimizing the negative environmental impacts. An optimal configuration for measurement setups and all other decisions shall therefore review and rate the risks of an individual project.

In this study, a methodology is introduced that allows considering consequences or costs within Geodetic decisions in order to meet the real requirements. The theoretical concept is based on the so called utility theory in decision making. This allows to introduce costs or consequences for type I and II errors. Finally, the most beneficial decision is chosen which leads to the minimum costs or consequences for the specific project. Additionally, this procedure allows identifying the most beneficial measurements in Geodetic monitoring to reduce the risk of an individual process. The paper shows the general concept of the decision making process with consideration of risk (cost, consequence) values. The theoretical concept is applied to two examples in landslide monitoring and in the quality assessment within an automatic inspection process.

1. INTRODUCTION TO DECISION MAKING

1.1 General introduction

Geodetic deformation measurements and analysis concepts are one basis for minimizing the risk of, e. g. unexpected collapses of artificial objects and hazards. The idea behind these activities is the need of the society in minimizing the negative environmental impacts. An optimal configuration for measurement setups and all other decisions shall therefore review and rate the risks of an individual project. But in the nowadays methodology in Applied Geodesy, the mathematically founded decisions are usually based on probabilities and significance levels but not on the risk (consequences or costs) itself. A typical example is more or less the intuitive choice of the type I error within hypothesis testing in order to detect, e. g. significant movements of slopes. These choices based on probabilities have no reference to practical applications, where wrong decisions (appearing with the same probability) may lead to fundamental different consequences.

In the classical case, hypothesis testing serves to check information, which is available on the (unknown) parameters (e. g. Koch 1999). In the classical case of hypothesis testing in linear models, two decisions are possible. The result of the test is the acceptance or the rejection of the predefined hypotheses. The two assumptions are typically called null hypothesis and alternative hypothesis, respectively.

A suitable way to consider consequences within the decision process is the so called "*utility theory*" in decision making. It allows to rate the possible decisions with cost functions. This verbally motivated procedure will be mathematically introduced in the Sections 2 and 3. Section 4 then transfers the general concept to typical Geodetic problems.

The practical relevance of the approach presented here is meaningful if the consequences or costs can be realistically evaluated. Two examples will be given in Section 5.

1.2 The three main decision cases

In classical hypothesis testing three important cases are of interest: In case (i) both the probability density functions (pdf) of the test objects under the null and alternative hypothesis are known (see Section 3.1). A second case (ii) is when the null hypothesis is much more probable and therefore the pdf of the alternative hypothesis is not (exactly) known. In case (iii) neither the pdf of the objects under the null hypothesis nor the pdf of the objects under the alternative hypothesis is known. The cases (ii) and (iii) are typically the standard in Geodetic Monitoring.

Whereas in case (i) no regions of acceptance or rejection are needed for the test decision (Luce and Raiffa, 1989), in the cases (ii) and (iii) acceptance and rejection regions have to be defined for classical hypothesis tests. In case (iii) there is no statistically founded method to define the region of acceptance. However, it can be constructed, e. g. based on expert knowledge which leads to regulatory thresholds (see Nguyen and Kreinovich 1996, Niwitpong et al. 2008 or Neumann 2009). The test decisions in case (ii) and (iii) are obtained by comparing a measured value with the regions of acceptance and rejection.

2. CLASSICAL CASE OF HYPOTHESIS TESTING

The classical cases of decision making in Geodesy are so called hypotheses tests with two possible alternatives (acceptance or rejection of the null hypothesis). The general case of decision making allows to judge any number of alternatives within a decision. For the sake of simplicity, only hypothesis testing with two alternatives will be treated in the following.

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According to Koch (1999) the unknown parameters $\boldsymbol{\theta}$ span the parameter space B with $\boldsymbol{\theta} \in \mathrm{B}$. Then a statistical hypothesis test is defined as the assumption, that a parameter vector $\boldsymbol{\theta}$ belongs to the subset $\mathrm{b} \subset \mathrm{B}$ or to its complement B\b. The assumption $\mathrm{H}_0 : \boldsymbol{\theta} \in \mathrm{b} = \Theta_0$ is called the null hypothesis, whereas $\mathrm{H}_1 : \boldsymbol{\theta} \in \mathrm{B} \backslash \mathrm{b} = \Theta_1$ is the alternative hypothesis.

Based on the test situation, two correct and two incorrect decisions are possible. The incorrect decisions are well known as type I and type II error. The possibility weather to accept or reject the null hypothesis leads to four possible situations within the test decision:

- correct choice of the null hypothesis.
- correct choice of the alternative hypothesis.
- incorrect choice of the alternative hypothesis (type I error).
- incorrect choice of the null hypothesis (type II error).

Based on the observations $\boldsymbol{y}$ a test statistic $T = \boldsymbol{f}(\boldsymbol{y})$ is computed. The null hypothesis is accepted, if the test statistic T belongs to subspace S_A of the probability space of the observations, that can be assigned to the null hypothesis and it is rejected if it belongs to the subspace S_R that can be assigned to the alternative hypothesis.

The regions of acceptance A and rejection R are classical sets in $\boldsymbol{\theta} \in \Re^u$ which can be modeled by indicator functions:

$$\mathrm{i}_A(\boldsymbol{\theta}) = \begin{cases} 1, & \text{if } \boldsymbol{\theta} \in \mathrm{b} \\ 0, & \text{else} \end{cases}$$
$$\mathrm{i}_R(\boldsymbol{\theta}) = \begin{cases} 1, & \text{if } \boldsymbol{\theta} \in \mathrm{B}\backslash\mathrm{b} \\ 0, & \text{else.} \end{cases} \qquad (1)$$

The indicator function for the region of acceptance is $\mathrm{i}_A(\boldsymbol{\theta})$ and for the region of rejection $\mathrm{i}_R(\boldsymbol{\theta})$, respectively. Alternatively, the region of acceptance A for the null hypothesis and its indicator function i_A can be seen as a mapping of the parameter space $\mathrm{i}_A : \mathrm{B} \rightarrow \{0;1\}$:

$$A := \{x, \mathrm{i}_A(x) | x \in \Re\} \quad \text{with} \quad \mathrm{i}_A : \Re^u \rightarrow \{0;1\} \quad \text{and} \quad x \in \Re. \qquad (2)$$

It is clearly, that the two indicator functions are complementary to each other: $\mathrm{i}_A(\boldsymbol{\theta}) = 1 - \mathrm{i}_R(\boldsymbol{\theta})$:

$$R := \{x, \mathrm{i}_R(x) = 1 - \mathrm{i}_A(x) | x \in \Re\} \text{ with } \mathrm{i}_A : \Re^u \rightarrow \{0;1\} \text{ and } x \in \Re. \qquad (3)$$

The lower and upper bounds of the indicator functions can, e. g. be determined by a combined optimization of a type I and type II error for the subspace S_A and S_R of the probability space of the observations.

Within this paper we assume that the final test decision can be carried out in the one-dimensional space. This is typical within hypothesis testing in Geodesy, where multi-dimensional tests are mapped to the one-dimensional space. One example is the estimation of the posteriori variance factor for the global test in a Gauß-Markov-Model.

The test decision is finally based on the comparison of the test value $\mathrm{T} = \boldsymbol{f}(\boldsymbol{y})$ as realization of the test statistic with the region of acceptance and rejection. A graphical representation for such a hypothesis test is given in Figure 1, where the null hypothesis is accepted.

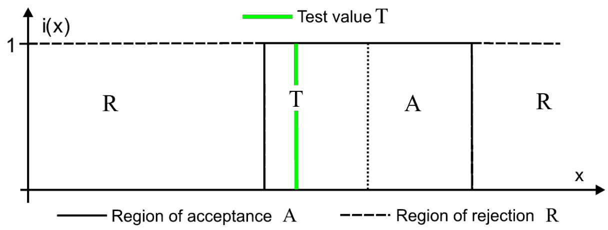

Figure 1. Classical test situation with regions of acceptance and rejection

3. TEST DECISION WITH COST FUNCTIONS

In this section, the general case of decisions making with the aid of cost functions for two known alternatives is introduced. The main idea is based on the utility theory in multi criteria decisions making (MCDM).

3.1 Test decisions with the aid of the utility theory

In the most general case, both probability subspaces S_A and S_R are known. This is equal with the knowledge of the pdfs ρ_0 and ρ_1 of the objects under the null- and alternative hypothesis. The following introduction is based on Kreinovich et al. (2008) and Figueira (2005).

The probability that a randomly chosen object belongs to the null hypothesis is given by $\mathrm{P}(\mathrm{T}|\mathrm{H}_0)$ and for the alternative hypothesis $\mathrm{P}(\mathrm{T}|\mathrm{H}_1)$, respectively. In case of a realization of the test statistics (test value T) the probability $\mathrm{p}_0(\mathrm{T}) = \mathrm{P}(\mathrm{H}_0|\mathrm{T})$ that T belongs to the null hypothesis, can be computed based on the Bayes' Theorem (Klir, 2006, S. 67):

$$\begin{aligned} \mathrm{p}_0(\mathrm{T}) &= \frac{\mathrm{P}(\mathrm{T}|\mathrm{H}_0) \cdot \mathrm{P}(\mathrm{H}_0)}{\mathrm{P}(\mathrm{T}|\mathrm{H}_0) \cdot \mathrm{P}(\mathrm{H}_0) + \mathrm{P}(\mathrm{T}|\mathrm{H}_1) \cdot \mathrm{P}(\mathrm{H}_1)} \\ &= \frac{\rho_0(\mathrm{T}) \cdot \mathrm{P}(\mathrm{H}_0)}{\rho_0(\mathrm{T}) \cdot \mathrm{P}(\mathrm{H}_0) + \rho_1(\mathrm{T}) \cdot \mathrm{P}(\mathrm{H}_1)}. \end{aligned} \qquad (4)$$

Where $\mathrm{P}(\mathrm{H}_0)$ and $\mathrm{P}(\mathrm{H}_1)$ are the probabilities, that a randomly chosen object satisfies the null or alternative hypothesis. The probability that the test value belongs to the alternative hypothesis is:

$$\mathrm{p}_1(\mathrm{T}) = 1 - \frac{\rho_0(\mathrm{T}) \cdot \mathrm{P}(\mathrm{H}_0)}{\rho_0(\mathrm{T}) \cdot \mathrm{P}(\mathrm{H}_0) + \rho_1(\mathrm{T}) \cdot \mathrm{P}(\mathrm{H}_1)} = 1 - \mathrm{p}_0(\mathrm{T}). \qquad (5)$$

According to Section 2, in hypothesis testing four situations are possible in the final test decision. The main idea behind utility theory is to judge each possible situation with a utility value:

- $U_{0,0}$: utility for a correct choice of the null hypothesis.
- $U_{1,0}$: utility for an incorrect choice of the alternative hypothesis (type I error).
- $U_{1,1}$: utility for a correct choice of the alternative hypothesis.
- $U_{0,1}$: utility for an incorrect choice of the null hypothesis (type II error).

In the next step, the expected total utilities K_0 and K_1 for the null and alternative hypotheses are computed as

$$K_0 = p_0(T)U_{00} + p_1(T)U_{01} = p_0(T)(U_{00} - U_{01}) + U_{01}, \quad (6)$$

and

$$K_1 = p_0(T)U_{10} + p_1(T)U_{11} = p_0(T)(U_{10} - U_{11}) + U_{11}. \quad (7)$$

For the final decision, the hypothesis with the largest expected utility (the minimum costs) is chosen. The null hypothesis is selected, if

$$p_0(T)U_{00} + p_1(T)U_{01} \geq p_0(T)U_{10} + p_1(T)U_{11} \quad (8)$$

holds. Since a correct decision is always better than the incorrect one, with $U_{00} > U_{10}$ and $U_{11} > U_{01}$ Equation (8) can be rewritten as:

$$\frac{p_0(T)}{p_1(T)} \geq \frac{U_{11} - U_{01}}{U_{00} - U_{10}}. \quad (9)$$

If one considers the relationship from Equation (4), the final test decision can then be introduced as:

$$\frac{\rho_0(T)}{\rho_1(T)} \geq r_0 = \frac{(U_{11} - U_{01})p_1(T)}{(U_{00} - U_{10})p_0(T)} \Rightarrow \text{accept } H_0. \quad (10)$$

The right hand side of Equation (10) is known, so

- the null hypothesis will be accepted if for the quotient $\rho_0(T)/\rho_1(T) \geq r_0$ holds,
- and the alternative hypothesis will be chosen if $\rho_0(T)/\rho_1(T) < r_0$.

The above introduced criterion is known as ***Neyman-Pearson criterion***.

4. TRANSFERRING THE IDEAS TO GEODETIC APPLICATIONS

In Geodesy, the distribution of objects (see Section 2 and 3) is usually not known, since there are not enough, e. g. monitoring objects like slide slopes to construct a reasonable pdf. Therefore, the hypothesis is formulated based on the observations, and according to Section 2, the test statistic itself is computed based on the observations $T = \boldsymbol{f}(\boldsymbol{y})$.

The distribution of the test statistics under the null hypothesis is known and the pdf of the test statistics under the alternative hypothesis cannot be (exactly) specified. Therefore we should firstly judge the consequences of decision and then ask what kind of magnitude, e. g. of a deformation is reasonably detectable with the measurements when considering utility values.

4.1 Test decisions for a known H_0

Let us assume, that the pdf of the test statistic under the null hypothesis is $T_{H_0} \sim \rho_{T,H_0}(x, \lambda = 0)$ with a non-centrality parameter λ equal to zero. This pdf can be computed based on the observations because the parameters are estimated based on the mathematical model and the observations itself. This is of great importance, because the hypotheses are formulated based on the parameter space (see Section 2).

If the assumptions of the null hypothesis are prevailing, the alternative hypothesis can be formulated as the negation of the null hypothesis. Due to the computation of the test statistic based on the observations, we can also assume that the test static under the alternative hypothesis follows the same pdf $T_{H_1} \sim \rho_{T,H_1}(x, \lambda \neq 0)$ but with a non-centrality parameter which is different from zero. The concept described here is, e. g. applied to the case of a general linear hypothesis within a Gauss-Markoff model. An example will be given in Section 5.2 in order to explain the theoretical concept which is now introduced in the following.

The hypotheses based on the parameter can therefore (in the linear models) be formulated as a linear combination of the parameters. With the matrix **C** and the vector **w**, we obtain for the null hypothesis (e. g. Koch 1999, pp. 279):

$$H_0 : \mathbf{C}\boldsymbol{\theta} = \mathbf{w}, \quad (11)$$

and the alternative hypothesis as negation:

$$H_1 : \mathbf{C}\boldsymbol{\theta} = \overline{\mathbf{w}} \neq \mathbf{w}. \quad (12)$$

The non-centrality parameter of the pdf $\rho_{T,H_1}(x, \lambda \neq 0)$ can be computed with the aid of the expected changes of the parameter under the null hypothesis:

$$\lambda = \boldsymbol{\varphi}(\overline{\mathbf{w}} - \mathbf{w}). \quad (13)$$

In case of linear hypothesis, we obtain the test value T according to Koch (1999, pp. 279f) :

$$T = \frac{1}{r\hat{\sigma}^2}(\mathbf{C}\boldsymbol{\beta} - \mathbf{w})^T \left[\mathbf{C}\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{C}^T\right]^+ (\mathbf{C}\boldsymbol{\beta} - \mathbf{w}) \quad (14)$$

with **A** the Jacobi (design) matrix of the estimation, $\hat{\sigma}^2$ the a-posteriori variance factor and $[...]^+$ the pseudo-inverse due to rank deficiencies. The test statistic *T* itself follows a central ($\lambda = 0$) Fisher distribution F for the null hypothesis:

$$T_{H_0} \sim F(r, f, \lambda = 0) \quad (15)$$

where $r = rg\left[\mathbf{C}\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{C}^T\right]$ is the number of linear independent combinations of the tested parameters and f is the degree of freedom of the estimation. Hence, under the alternative hypothesis, the test static follows a non-central ($\lambda \neq 0$) Fisher distribution F':

$$T_{H_1} \sim F'(r, f, \lambda) \quad (16)$$

with

$$\lambda = \frac{1}{\sigma^2}(\overline{\mathbf{w}} - \mathbf{w})^T \left[\mathbf{C}\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{C}^T\right]^+ (\overline{\mathbf{w}} - \mathbf{w}). \quad (17)$$

The consideration of cost functions is then based on the following loop:

i) Choose changes in the parameters $\mathbf{dw} = \overline{\mathbf{w}} - \mathbf{w}$.
ii) Compute the non-centrality parameter for the test statistic under the alternative hypothesis with (13) or (17).
iii) Compute the probabilities $p_0(T)$ and $p_1(T)$ with Equation (4) and (5), assuming $P(H_0) = P(H_1)$.
iv) Compute the expected total utilities K_0 and K_1 for both hypotheses according to (6) and (7).
v) Carry out the final test decision based on (8) or (10).

In the next steps, the changes in the parameters $\mathbf{dw} = \overline{\mathbf{w}} - \mathbf{w}$ are increased and every time the steps ii) to v) are carried out in a loop.

This leads to a graph which shows the function of the test decision depending on the changes in the parameter with respect to the utility values of each decision. The more expensive a type II error in comparison to the type I error, the earlier the null hypothesis is rejected. This is equivalent to the conclusion, that the more risk (costs) may appear, e. g. from a slide slope, the more accurate the measurements have to be in order to detect specific changes in the parameters. This is reasonable but with the above shown procedure now strictly measureable and therefore considerable within automatic monitoring and decision concepts. Section 5.2 shows an example for the methodology.

The strategy from this section takes into account, that the magnitude of critical changes in the parameters is not exactly known. The strategy of considering cost and consequences in a decision, if lower and upper bounds on the critical movements are available, is given in the following.

4.2 Test decisions for regulatory thresholds

Regulatory thresholds are important, if given tolerances, e. g. within a production or inspection process have to be checked by measurements. The aim is to detect deviations between the actual dimension and the nominal dimension of an object. The nominal dimension is defined by lower and upper bounds which define the regulatory threshold. A typical example is the inspection of the length of a machine axis. But also regulatory thresholds for critical movements of a slope play a key role in monitoring concepts.

Within hypothesis testing in case of regulatory thresholds, the uncertainty of the test statistic T is described by a pdf $T \sim \rho_T(x)$. The regions of acceptance A and rejection R are defined through the indicator functions $i_A(\boldsymbol{\theta})$ and $i_R(\boldsymbol{\theta})$, respectively. The probabilities $p_0(T)$ and $p_1(T)$ that the test value T belongs to A or R can be computed by:

$$p_0(T) = \int_A \rho_T(x)\, dx \quad \text{and} \quad p_1(T) = 1 - \int_A \rho_T(x)\, dx. \tag{18}$$

According to Section 3.1, the expected total utilities K_0 and K_1 for the null and alternative hypothesis are obtained by:

$$\begin{aligned} K_0 &= p_0(T)U_{00} + p_1(T)U_{01} = p_0(T)\left(U_{00} - U_{01}\right) + U_{01}, \\ K_1 &= p_0(T)U_{10} + p_1(T)U_{11} = p_0(T)\left(U_{10} - U_{11}\right) + U_{11}. \end{aligned} \tag{19}$$

Finally, the test decision is based on the selection of the most beneficial hypothesis:

$$p_0(T)\left(U_{00} - U_{01}\right) + U_{01} \geq p_0(T)\left(U_{10} - U_{11}\right) + U_{11}. \tag{20}$$

This equation can be rearranged as:

$$p_0(T) \geq p_{0,crit} = \frac{U_{11} - U_{01}}{U_{00} - U_{01} - U_{10} + U_{11}}. \tag{21}$$

The null hypothesis is selected, if the probability $p_0(T)$ is larger or equal than the critical probability $p_{0,crit}$. The presented test strategy is based on the general theory of decision making with two possible alternatives. The here presented decision making procedure can also be extended due to the linguistic imprecision or fuzziness of the formulated hypotheses such as "The allowed length of a machine axis is approximately…". This leads to the definition of regions of transition between strict acceptance and rejection of a given hypothesis. Additionally, it is possible to consider non-stochastic uncertainties, like systematic measurement errors. The strategy with non-stochastic measurement uncertainties and linguistic uncertainty for regulatory thresholds is explained in Neumann and Kutterer (2011) and Niwitpong et al. (2008).

5. EXAMPLES

In practice, there are various possibilities for the application of the presented strategy. In the following, two examples are presented to show the theoretical procedure more in detail. The first example deals with the monitoring of the vertical displacements of a machine axis for an inspection process and the second with the monitoring of a slide slope with the aid of total station measurements.

5.1 Monitoring the vertical displacements of a machine axis

In the following, the survey of the vertical displacements Δh of a machine axis is of particular interest. The displacements are observed with two different total stations (*Leica* TCA 1101 and TM 30). The uncertainties of the displacements of the machine axis are computed based on the measurement uncertainties of the instruments. The results are given in Table 1. The distribution of the measurement uncertainty is Gaussian with zero mean. The uncertainty of the vertical displacements are given in Table 1.

Instrument	Standard deviation of the vertical displacements
TCA 1101	1.5 mm
TM 30	0.6 mm

Table 1. Uncertainty of the displacements of the machine axis

The machine axis is relevant to the milling process of automotive engines. The maximum allowed vertical displacement is 2 mm otherwise the milling process destroys the engines. The regions of acceptance and rejection are therefore defined through a regulatory threshold with the following indicator functions:

$$\begin{aligned} i_A(\Delta h) &= \begin{cases} 1, & \text{if } \Delta h \in b = 0...2 \text{ mm} \\ 0, & \text{else} \end{cases} \\ i_R(\Delta h) &= \begin{cases} 1, & \text{if } \Delta h \in B \backslash b \\ 0, & \text{else.} \end{cases} \end{aligned} \tag{22}$$

Probabilities	TM 30	TCA 1101
$p_0(T)$	0.878	0.486
$p_1(T)$	0.122	0.514

Table 2. The probabilities $p_0(T)$ and $p_1(T)$ for the test decision.

The test value T of the vertical displacement is obtained with 1.3 mm (for an easier understanding of the example for both instruments). The test value lies clearly inside the region of acceptance A. Due to the lower uncertainty of the TM 30, the probability $p_0(T)$ that the test value T belongs to A is larger than that of the TCA 1101 (see Equation (18) and Table 2).

The utility value $U_{0,0}$ for a correct choice of H_0 is defined through the cost (500 €) for the monitoring process. The utility values $U_{1,0}$ and $U_{1,1}$ lead to the installation of mechanical adaptations in order to stabilize the machine axis (2500 €). In case of an incorrect choice of the alternative hypothesis (worst case) the engines are destroyed and additionally, the installation of mechanical adaptations are necessary in order to stabilize the machine axis ($U_{0,1}$ = 12500 €).

The expected utilities for the two hypotheses are computed with the Equation (19) and the results are given in Table 3.

Utilities	TM 30	TCA 1101
K_0	**-1457.62 €**	-4094.02 €
K_1	-2500.00 €	**-2500.00 €**

Table 3. Expected utilities for the two hypotheses.

Although the uncertainty of the TM 30 is much smaller than that of the TCA 1101, the costs for the alternative hypothesis are equal for both instruments. The null hypothesis for the TCA 1101 is rejected and the measurements of the TM 30 are classified as belonging to the region of acceptance.

5.2 Geodetic Monitoring of a slide slope

The second example deals with the monitoring of a slide slope with a geodetic total station. The slide slope consists mainly of two critical areas. The left part of the slide slope lies above a street and the right part above some industrial facilities. If significant movements occur, the slope is stabilized with reinforcements. In case of a collapse of the slope, the street and/or the industrial facilities are destroyed.

In order to reduce the risk and the negative environmental impacts a monitoring of the slope is carried out. It consists of a total station observing two object points. One object point is located on the left part (P_1) and the other on the right part (P_2) of the slope. Although in reality there are many points that have to be observed within the monitoring process, for a better understanding of the methodology, only 2 points are considered in this example. Additionally, only theoretical uncertainties are taken into account.

Hence, the question occurs what magnitude of the movements of the slope can be detected, when the risk or consequences of the damage of the street and the industrial facilities shall (optimally) be considered.

The movements $\Delta def^i_{P_1/P_2}$ of the slide slope for each point are defined by the point-to-point distance between two measured epochs:

$$\Delta def^i_{P_1/P_2} = \sqrt{\left(X^0_{P_1/P_2} - X^i_{P_1/P_2}\right)^2 + \left(Y^0_{P_1/P_2} - Y^i_{P_1/P_2}\right)^2 + \left(Z^0_{P_1/P_2} - Z^i_{P_1/P_2}\right)^2},$$

with

$$\begin{aligned} X^i_{P_1/P_2} &= \cos\left(Hz^i_{P_1/P_2}\right)\cdot\cos\left(V^i_{P_1/P_2}\right)\cdot d^i_{P_1/P_2}, \\ Y^i_{P_1/P_2} &= \sin\left(Hz^i_{P_1/P_2}\right)\cdot\cos\left(V^i_{P_1/P_2}\right)\cdot d^i_{P_1/P_2}, \\ X^i_{P_1/P_2} &= \cos\left(V^i_{P_1/P_2}\right)\cdot d^i_{P_1/P_2}. \end{aligned} \quad (23)$$

Hz is the horizontal direction, V the zenith angle and d the slope distance. The index i denotes the measured epoch. The measurement uncertainties of the total stations are assumed as Gaussian with zero mean under the null hypothesis. The theoretical standard deviations σ are given for the

- Distance: $\sigma_d = 1\,\text{mm} + 0.6\,\text{ppm}$
- Horizontal direction: $\sigma_{Hz} = 0.15\ \text{mgon}/\sqrt{2}$
- Zenith angle: $\sigma_V = 0.15\ \text{mgon}/\sqrt{2}$

Under the null hypothesis, the slope is stable: $H_0 : \Delta def^i_{P_1/P_2} = 0$

The alternative hypothesis is the negation of H_0. The utility values for this example are given in Table 4. In case of a type I error or the correct choice of the alternative hypothesis, the slope is stabilized with some reinforcements. In case of a type II error, the street (P1) or the industrial facilities (P2) are destroyed. The execution of the monitoring process costs 1000 € ($U_{0,0}$).

Utilities	P1 above street	P2 above industrial facilities
$U_{0,0}$	-1.000 €	-1.000 €
$U_{1,0}$	-25.000 €	-25.000 €
$U_{1,1}$	-25.000 €	-25.000 €
$U_{0,1}$	-100.000 €	-1.000.000 €

Table 4. Utility values for the four possible situations.

In this example, the pdf of the movements is computed by a Monte Carlo simulation by the above given uncertainties of the measurements (see, e. g. Koch 2008). According to the concept from Section 4.1, the detectable changes in the movements of the points are of interest. Due to the one-dimensional space of the movements, the non-centrality parameter λ of the pdf $\rho_{\Delta def}(\lambda)$ of the deformations under the alternative hypothesis is equal to the movements under the alternative hypothesis.

For a given non-centrality parameter, the pdfs under the null and alternative hypotheses are computed. The figure 2 shows an example with the non-centrality parameter $\lambda = 4.2$ for the alternative hypothesis.

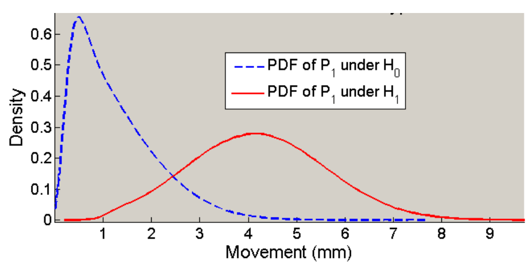

Figure 2. PDFs for the movements of P1 under the null and alternative hypothesis (non-centrality parameter $\lambda = 4.2$)

The test decision for a test value is numerically computed for differential small values of observed movements. This leads for a specific non-centrality parameter and with the Equations (6) and (7) to the expected total utilities K_0 and K_1. Figure 3 shows the utilities for P_1 depending on the test value T with the non-centrality parameter $\lambda = 4.2$ for the alternative hypothesis. The intersection between K_0 and K_1 defines the magnitude of the test value for which the test decision changes between the selection of the null and alternative hypothesis.

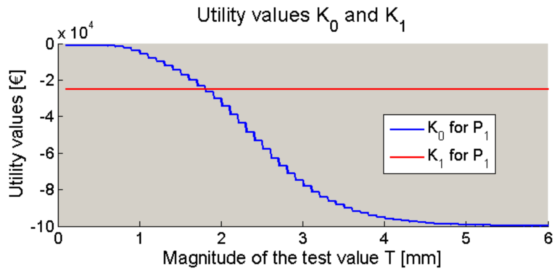

Figure 3. The expected total utilities K_0 and K_1 for the null and alternative hypothesis (non-centrality parameter $\lambda = 4.2$)

Finally, the Figure 4 shows the maximum test value for which the null hypothesis is accepted as a function of the critical movements. Above these values, the alternative hypothesis is selected. Due to the higher costs of the type II error of point P_2 the alternative hypothesis is chosen earlier in comparison to point P_1. For movements below 1.1 mm for P_1 and below 2.7 mm for P_2 it is not possible to distinguish between null and alternative hypothesis. In these cases, the alternative hypothesis is chosen in any case (see Figure 4).

Speaking from a view of measurement optimization, the point above the industrial facilities has to monitored more frequently and more accurate. The mathematical optimization problem to detect the measurements with the most beneficial information to reduce the risk for the hold monitoring project will be developed in near future.

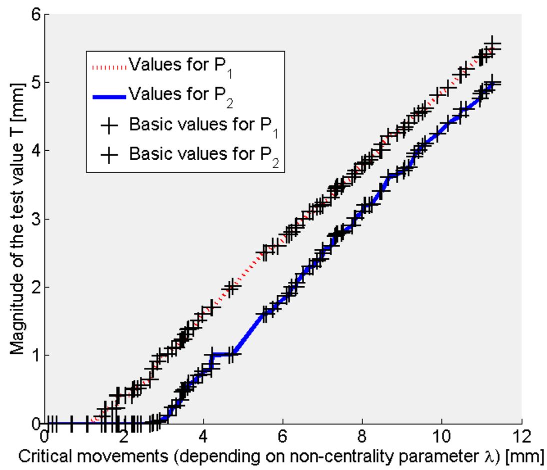

Figure 4. Maximum magnitude of the test value to accept H_0 as a function of critical movements

6. CONCLUSIONS AND OUTLOOK

The more risks or costs act on the assumption of a moving slide slope or on a collapse of a construction, the more accurate and reliable the geodetic monitoring concepts have to be. Currently, geodetic monitoring concepts and decisions do not consider the costs, consequences or risks of decisions and events, and they are therefore not optimal for hazard control. The measurement process, e. g. is in general carried out in a static and strict scheme. It does not allow detecting non-optimal regions with respect to the required quality or accordingly accuracy.

This paper shows a concept for the consideration of costs or consequences within the decision process. It therefore allows computing optimal decisions with respect to occurring consequences or risks. The final decision is based on the choice of the situation with the minimum occurring costs or consequences for the specific project. The main idea is based on the so called utility theory and multi criteria decision making. It is extended to typical geodetic decision within statistical hypothesis tests.

This presented strategy allows, e. g. an optimal steering for the measurement process. Within an optimization process, the most beneficial measurement for the hold monitoring project can be identified and carried out. This is in particular of importance, if measurements will take a longer time (e. g. for total stations).

In future, the strategy presented here shall be extended to multiple criteria decisions with more than two alternatives. Additionally, it is meaningful to numerically analyze the verbally motivated steering and optimization of the measurement process with respect to consequences.

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