XXVII FIG CONGRESS
11–15 SEPTEMBER 2022
Warsaw, Poland

Volunteering for the future – Geospatial excellence for a better living

Mathematical model and results of a new positioning algorithm

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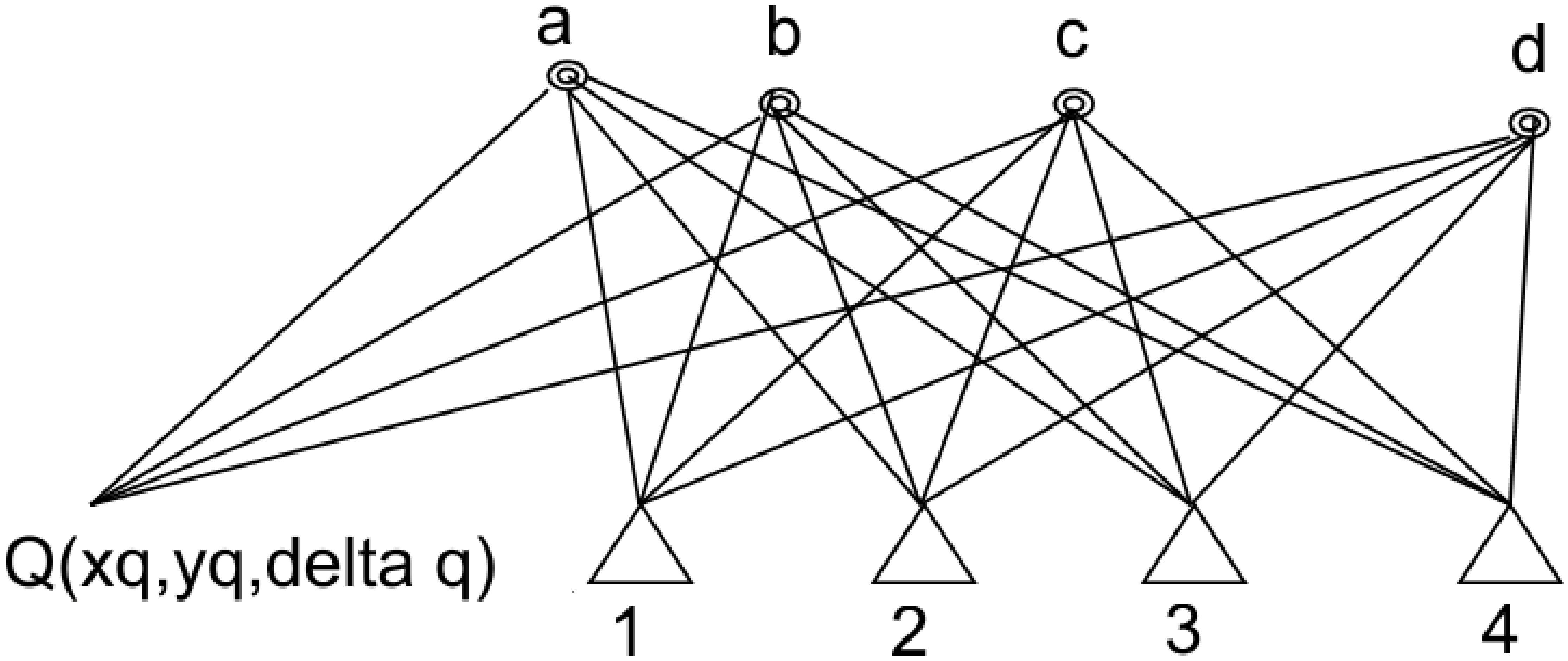
OUTLINE

- **A new mathematical model is briefly described, and a new positioning algorithm is announced.**
- **the basic principles of the methods for solving the positioning problem are presented, and the formulas and their derivation are given.**
- **The numerical example with simulated data and proof confirm the correct performance of the proposed positioning algorithm.**

- **New algorithm for determining of the point coordinates and a systematic errors in two-dimensional space in geodetic network solution is presented. In the proposed solution there is no need to know the initial approximate location of the determined point, nor the coordinates of the transition points.**


a
b
c
d
Q(xq,yq,delta q)
1
2
3
4

2.1 DETERMINATION OF THE COORDINATES OF A POINT $Q(x_0, y_0)$ AND A SYSTEMATIC ERROR δ_Q IN 2D ON THE BASIS OF REFERENCE AND TRANSITION POINTS

The example of determination of the coordinates (Fig. 1) of a point $Q(x_Q, y_Q)$ and a systematic error δ_Q on the basis of known network coordinates of four reference points $1(x_1, y_1), 2(x_2, y_2), 3(x_3, y_3), 4(x_4, y_4)$ is given.

There are known four distances $d_{aQ}, d_{bQ}, d_{cQ}, d_{dQ}$ from the unknown transition points $a(x_a, y_a), b(x_b, y_b), c(x_c, y_c), d(x_d, y_d)$ to point $Q(x_Q, y_Q)$.

There are also known distances:

$d_{1a}, d_{1b}, d_{1c}, d_{1d}, d_{2a}, d_{2b}, d_{2c}, d_{2d}, d_{3a}, d_{3b}, d_{3c}, d_{3d}, d_{4a}, d_{4b}, d_{4c}, d_{4d}$ between four reference points $1(x_1, y_1), 2(x_2, y_2), 3(x_3, y_3), 4(x_4, y_4)$ and the transition points $a(x_a, y_a), b(x_b, y_b), c(x_c, y_c), d(x_d, y_d)$.

The coordinates of point $Q(x_Q, y_Q)$ and a systematic error δ_Q can be computed from the following author's formula [13]:

$$[x_Q \quad y_Q \quad \delta_Q] = \frac{1}{2}[\,T_{ab} \quad T_{ac} \quad T_{ad}] \begin{bmatrix} \Delta X_{ab} & \Delta Y_{ab} & d_{\mathrm{a}Q} - d_{\mathrm{b}Q} \\ \Delta X_{ac} & \Delta Y_{ac} & d_{\mathrm{a}Q} - d_{\mathrm{c}Q} \\ \Delta X_{ad} & \Delta Y_{ad} & d_{\mathrm{a}Q} - d_{\mathrm{d}Q} \end{bmatrix}^{-1} \quad (1)$$

T_{ab} – difference of two transition point indicators t_{bQ} and t_{aQ} in regard to point $Q(x_Q, y_Q)$,
T_{ac} – difference of two transition point indicators t_{cQ} and t_{aQ} in regard to point $Q(x_Q, y_Q)$,
T_{ad} – difference of two transition point indicators t_{dQ} and t_{aQ} in regard to point $Q(x_Q, y_Q)$,

$$K_a = \begin{bmatrix} \Delta x_{12} & \Delta y_{12} & \Delta d_{a21} \\ \Delta x_{13} & \Delta y_{13} & \Delta d_{a31} \\ \Delta x_{14} & \Delta y_{14} & \Delta d_{a41} \end{bmatrix}^{-1}$$

$$K_b = \begin{bmatrix} \Delta x_{12} & \Delta y_{12} & \Delta d_{b21} \\ \Delta x_{13} & \Delta y_{13} & \Delta d_{b31} \\ \Delta x_{14} & \Delta y_{14} & \Delta d_{b41} \end{bmatrix}^{-1}$$

$$K_c = \begin{bmatrix} \Delta x_{12} & \Delta y_{12} & \Delta d_{c21} \\ \Delta x_{13} & \Delta y_{13} & \Delta d_{c31} \\ \Delta x_{14} & \Delta y_{14} & \Delta d_{c41} \end{bmatrix}^{-1}$$

$$K_d = \begin{bmatrix} \Delta x_{12} & \Delta y_{12} & \Delta d_{d21} \\ \Delta x_{13} & \Delta y_{13} & \Delta d_{d31} \\ \Delta x_{14} & \Delta y_{14} & \Delta d_{d41} \end{bmatrix}^{-1}$$

Values of the transition point indicators $t_{aQ}, t_{bQ}, t_{cQ}, t_{dQ}$ can be computed [13]:

$$t_{aQ} = \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,1} + \frac{1}{2}\Delta t_{13_a} * K_{a2,1} + \frac{1}{2}\Delta t_{14_a} * K_{a3,1}\right)^2 + \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,2} + \frac{1}{2}\Delta t_{13_a} * \right.$$
$$\left. * K_{a2,2} + \frac{1}{2}\Delta t_{14_a} * K_{a3,2}\right)^2 - \mathrm{d}_{\mathrm{aQ}}^2;$$

$$t_{bQ} = \left(\frac{1}{2}\Delta t_{12_b} * K_{b1,1} + \frac{1}{2}\Delta t_{13_b} * K_{b2,1} + \frac{1}{2}\Delta t_{14_b} * K_{b3,1}\right)^2 + \left(\frac{1}{2}\Delta t_{12_b} * K_{b1,2} + \frac{1}{2}\Delta t_{13_b} * \right.$$
$$\left. * K_{b2,2} + \frac{1}{2}\Delta t_{14_b} * K_{b3,2}\right)^2 - \mathrm{d}_{\mathrm{bQ}}^2;$$

$$t_{cQ} = \left(\frac{1}{2}\Delta t_{12_c} * K_{c1,1} + \frac{1}{2}\Delta t_{13_c} * K_{c2,1} + \frac{1}{2}\Delta t_{14_c} * K_{c3,1}\right)^2 + \left(\frac{1}{2}\Delta t_{12_c} * K_{c1,2} + \frac{1}{2}\Delta t_{13_c} * \right.$$
$$\left. * K_{c2,2} + \frac{1}{2}\Delta t_{14_c} * K_{c3,2}\right)^2 - \mathrm{d}_{\mathrm{cQ}}^2;$$

$$t_{dQ} = \left(\frac{1}{2}\Delta t_{12_d} * K_{d1,1} + \frac{1}{2}\Delta t_{13_d} * K_{d2,1} + \frac{1}{2}\Delta t_{14_d} * K_{d3,1}\right)^2 + \left(\frac{1}{2}\Delta t_{12_d} * K_{d1,2} + \frac{1}{2}\Delta t_{13_d} * \right.$$
$$\left. * K_{d2,2} + \frac{1}{2}\Delta t_{14_d} * K_{d3,2}\right)^2 - \mathrm{d}_{\mathrm{dQ}}^2;$$

$$X_{ab} = x_b - x_a = \left(\frac{1}{2}\Delta t_{12_b} * K_{b1,1} + \frac{1}{2}\Delta t_{13_b} * K_{b2,1} + \frac{1}{2}\Delta t_{14_b} * K_{b3,1}\right) - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,1} + \right.$$
$$\left. + \frac{1}{2}\Delta t_{13_a} * K_{a2,1} + \frac{1}{2}\Delta t_{14_a} * K_{a3,1}\right);$$

$$Y_{ab} = y_b - y_a = \left(\frac{1}{2}\Delta t_{12_b} * K_{b1,2} + \frac{1}{2}\Delta t_{13_b} * K_{b2,2} + \frac{1}{2}\Delta t_{14_b} * K_{b3,2}\right) - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,2} + \right.$$
$$\left. + \frac{1}{2}\Delta t_{13_a} * K_{a2,2} + \frac{1}{2}\Delta t_{14_a} * K_{a3,2}\right);$$

$$X_{ac} = x_c - x_a = \left(\frac{1}{2}\Delta t_{12_c} * K_{c1,1} + \frac{1}{2}\Delta t_{13_c} * K_{c2,1} + \frac{1}{2}\Delta t_{14_c} * K_{c3,1}\right) - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,1} + \right.$$
$$\left. + \frac{1}{2}\Delta t_{13_a} * K_{a2,1} + \frac{1}{2}\Delta t_{14_a} * K_{a3,1}\right);$$

$$Y_{ac} = y_c - y_a = \left(\frac{1}{2}\Delta t_{12_c} * K_{c1,2} + \frac{1}{2}\Delta t_{13_c} * K_{c2,2} + \frac{1}{2}\Delta t_{14_c} * K_{c3,2}\right) - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,2} + \right.$$
$$\left. + \frac{1}{2}\Delta t_{13_a} * K_{a2,2} + \frac{1}{2}\Delta t_{14_a} * K_{a3,2}\right);$$

$$X_{ad} = x_d - x_a = \left(\frac{1}{2}\Delta t_{12_d} * K_{d1,1} + \frac{1}{2}\Delta t_{13_d} * K_{d2,1} + \frac{1}{2}\Delta t_{14_d} * K_{d3,1}\right) - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,1} + \right.$$
$$\left. + \frac{1}{2}\Delta t_{13_a} * K_{a2,1} + \frac{1}{2}\Delta t_{14_a} * K_{a3,1}\right);$$

$$Y_{ad} = y_d - y_a = \left(\frac{1}{2}\Delta t_{12_d} * K_{d1,2} + \frac{1}{2}\Delta t_{13_d} * K_{d2,2} + \frac{1}{2}\Delta t_{14_d} * K_{d3,2}\right) - \left(\frac{1}{2}\Delta t_{12_a} * K_{a1,2} + \right.$$
$$\left. + \frac{1}{2}\Delta t_{13_a} * K_{a2,2} + \frac{1}{2}\Delta t_{14_a} * K_{a3,2}\right);$$

The coordinates of point $Q(x_Q, y_Q)$ and a systematic error δ_Q can be computed from the following author's formula [13]:

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T_{ab} – difference of two transition point indicators t_{bQ} and t_{aQ} in regard to point $Q(x_Q, y_Q)$,
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T_{ad} – difference of two transition point indicators t_{dQ} and t_{aQ} in regard to point $Q(x_Q, y_Q)$,

Thank you for your attention

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