Introduction Geodetic reference frames and deformation models

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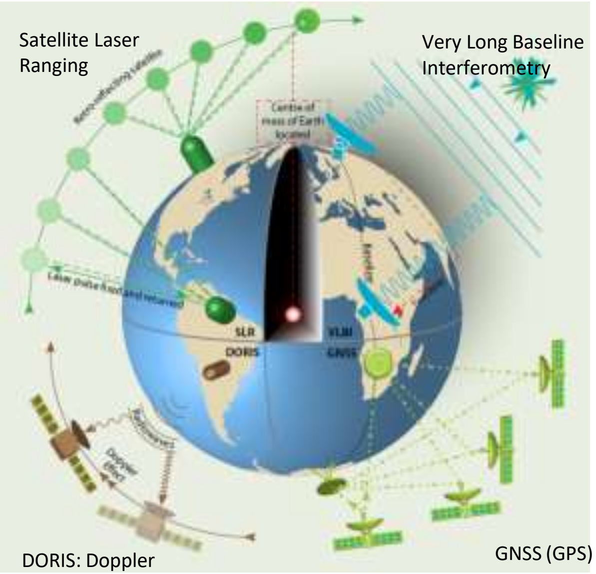
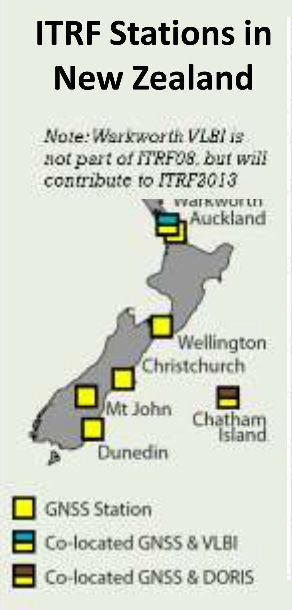
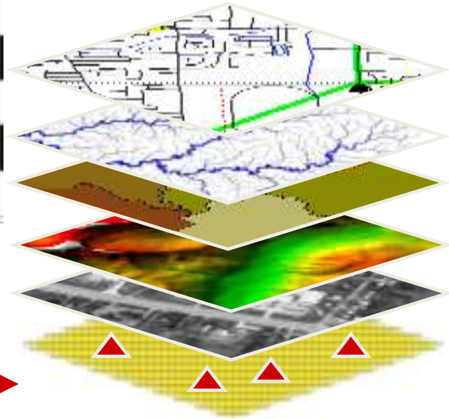
Chris Crook Land Information New Zealand







ITRF2014 **NDM**

ITRF

Required by GPS

No deformation model

Datums and epoch dates change frequently

National datum

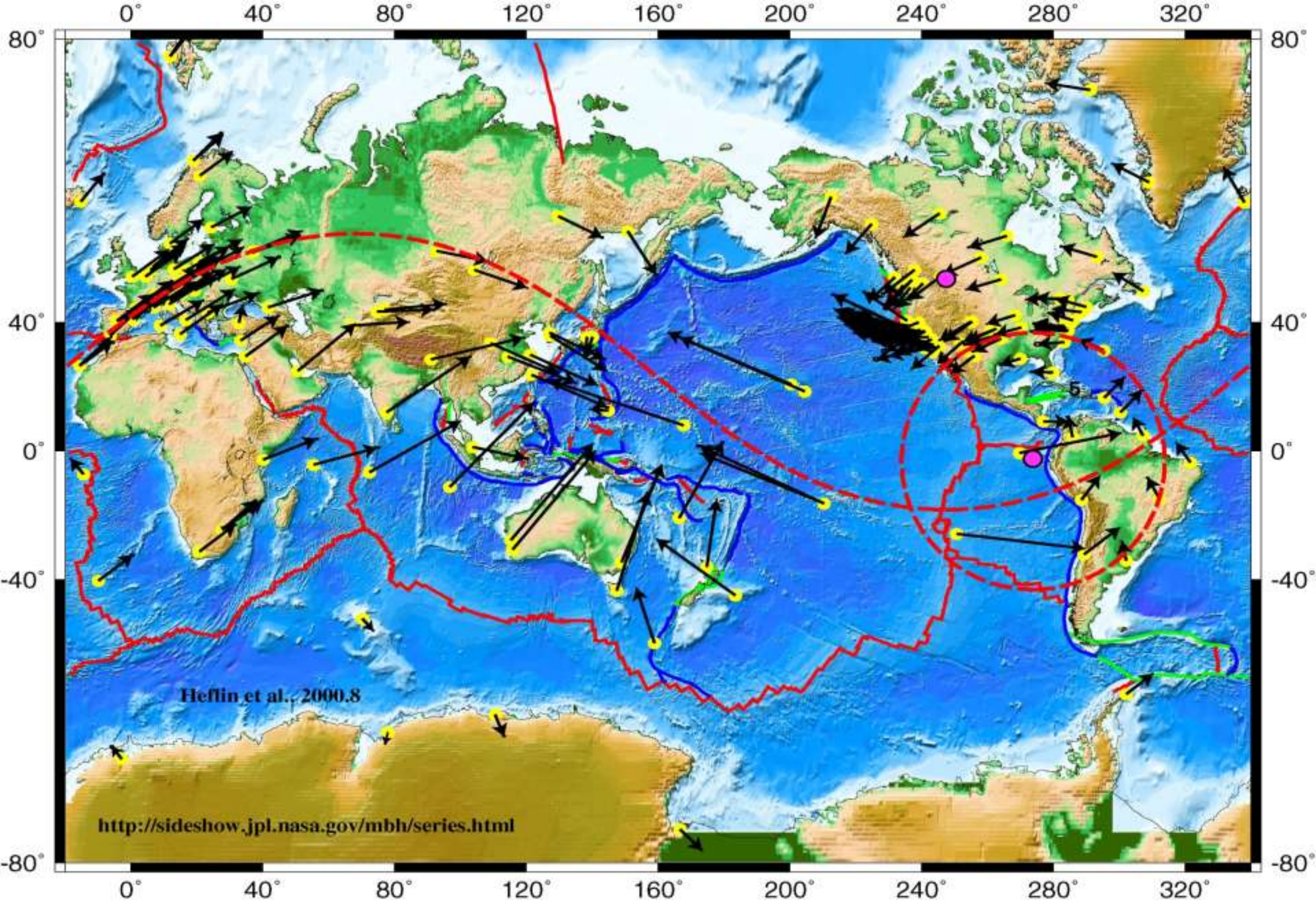
Stable coords

deformation model

Semi-dynamic datums

Coordinates transformed to common reference epoch using deformation model

cGNSS for active control



Small "circles" for N. American pole.

Transformation Parameters ITRF2008 --> NAD 83

Translations: $T_x = 0.99343+0.00079 \bullet (t - 1997)$ m

in m $T_y = -1.90331-0.0006 \bullet (t - 1997)$ m

$T_z = -0.52655-0.00134 \bullet (t - 1997)$ m

Rotations: $R_x = [0.12467+0.01347 \bullet (t - 1997)] \bullet k$

in mas $R_y = [-0.22355-0.01514 \bullet (t - 1997)] \bullet k$

$R_z = [0.00027-0.05133 \bullet (t - 1997)] \bullet k$

Scale change: $S = -0.93496-0.10201 \bullet (t - 1997) \bullet 10^{-9}$

$k = 4.848 \bullet 10^{-9}$ Rotation rates NOAM DeMets EP

ITRF2014 --> ETRF2014

Translations: $T_x = 0$ meters$+0\bullet(t - 1989)$

$T_y = 0$ meters$+0\bullet(t - 1989)$

$T_z = 0$ meters$+0\bullet(t - 1989)$

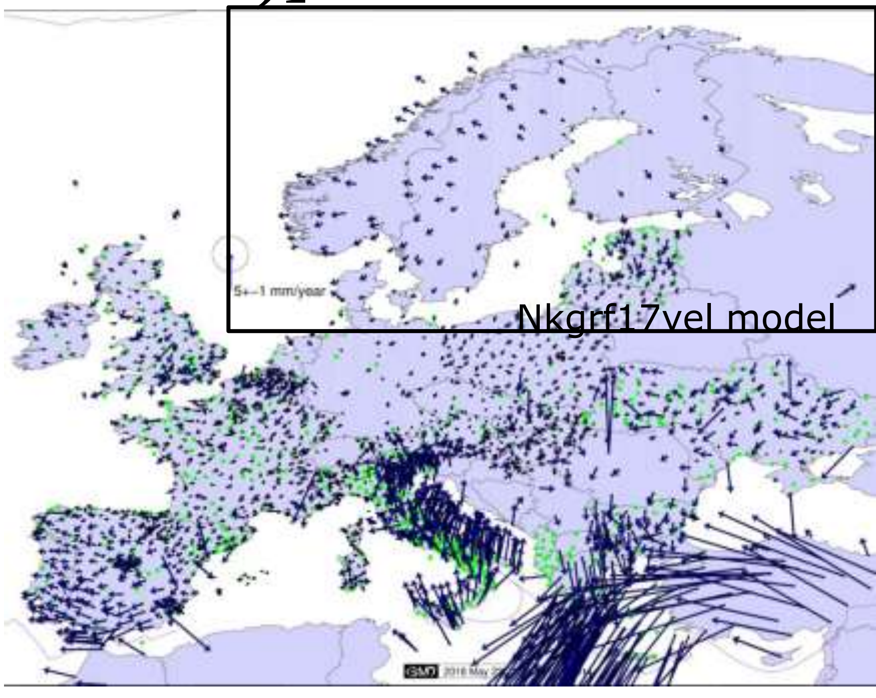
Rotations: $R_x = [0 + 0.085 \bullet(t - 1989)] \bullet k$

in radians $R_y = [0 + 0.531 \bullet(t - 1989)] \bullet k$

$R_z = [0 - 0.\ 770 \bullet(t - 1989)] \bullet k$

Scale change: $S = 0$

Rotation rates –EURA EP from Altamimi 2017



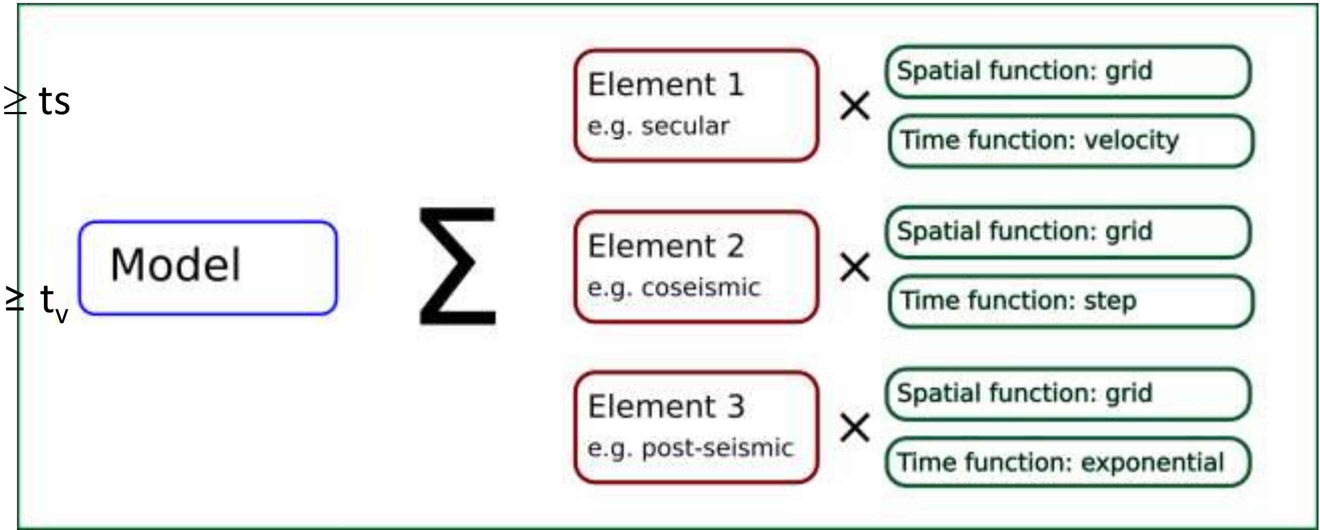
- EUREF velocities generally minimized for tectonically stable parts of the Eurasian plate
- (EUREF 2018, Stanaway 2019)

Functional models

- Time functions are usually defined using two values.
  - $f_t = f_1 (t_s)$ when $t >= t_s$ usually 0 or -1 for rev patches
  - ft = f(t) when for ts <= t

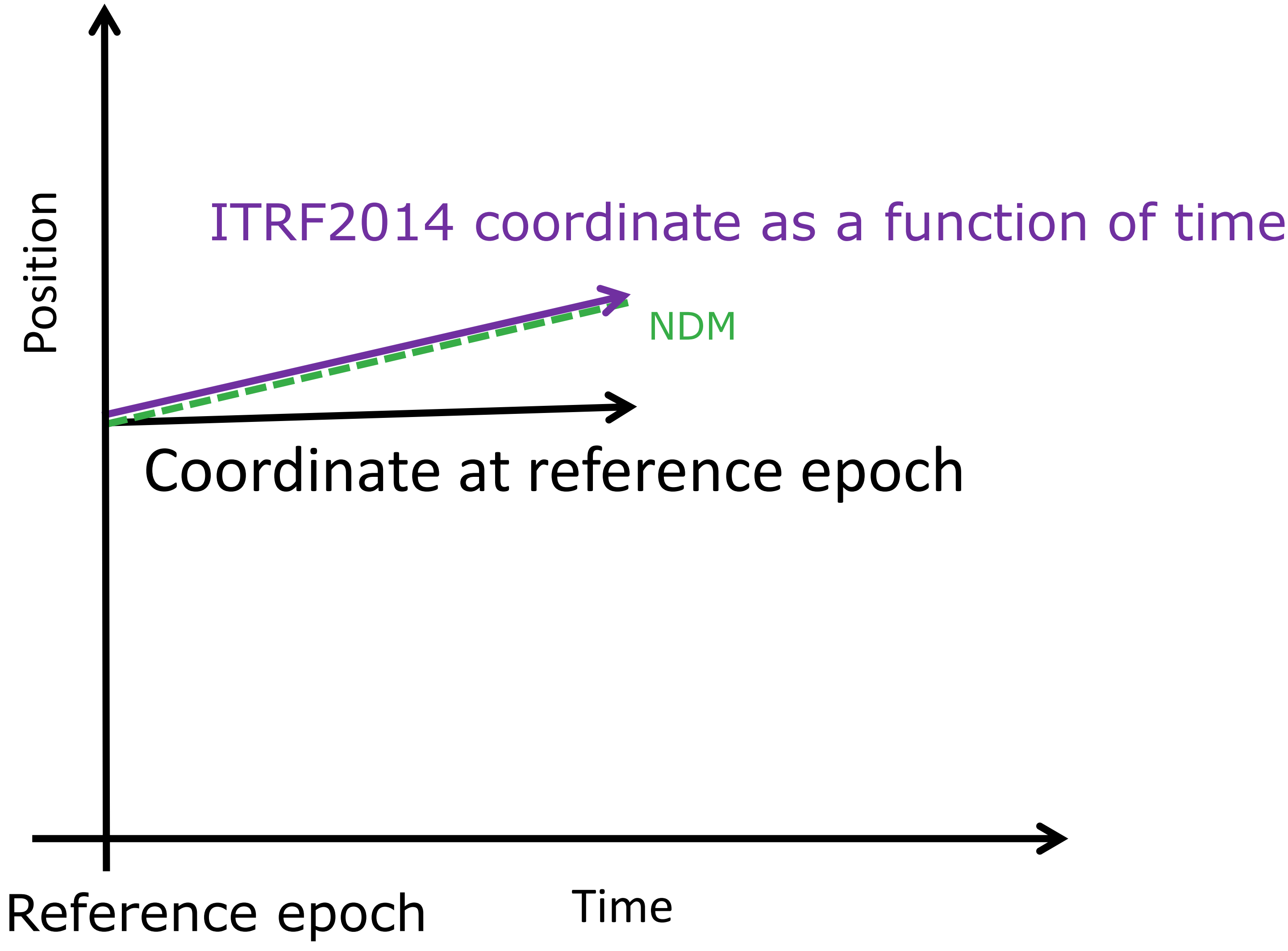
- linear velocity $f_t = v(t - t_0)$ Currently used for the secular velocity
- step $f_t = 0$ for $t < t_s$ Used for earthquake displacements
  - $f_t = 1$ for $t >= t_e$
- linear ramp $f_t = 0$ for $t < t_s$ For temporary velocity changes
  - $f_t = (f_0 \cdot (t_1 - t) + f_1 \cdot (t - t_0))/(t_1 - t_0)$ for $t_s <= t < t_e$

- Post-seismic deformation
- Exponential
  - $f_t = 0$ for $t < t_s$
  - $f_t = f_1\left(1 - e^{\varepsilon(t_0 - t)}\right)$ for $t \geq ts$
- or Logarithmic
  - $f_r(t) = 0$ for $t < t_v$
  - $f_r(t) = \ln\left(1 + \frac{t - tv}{\tau}\right)$ for $t \geq t_v$



Velocity only deformation model

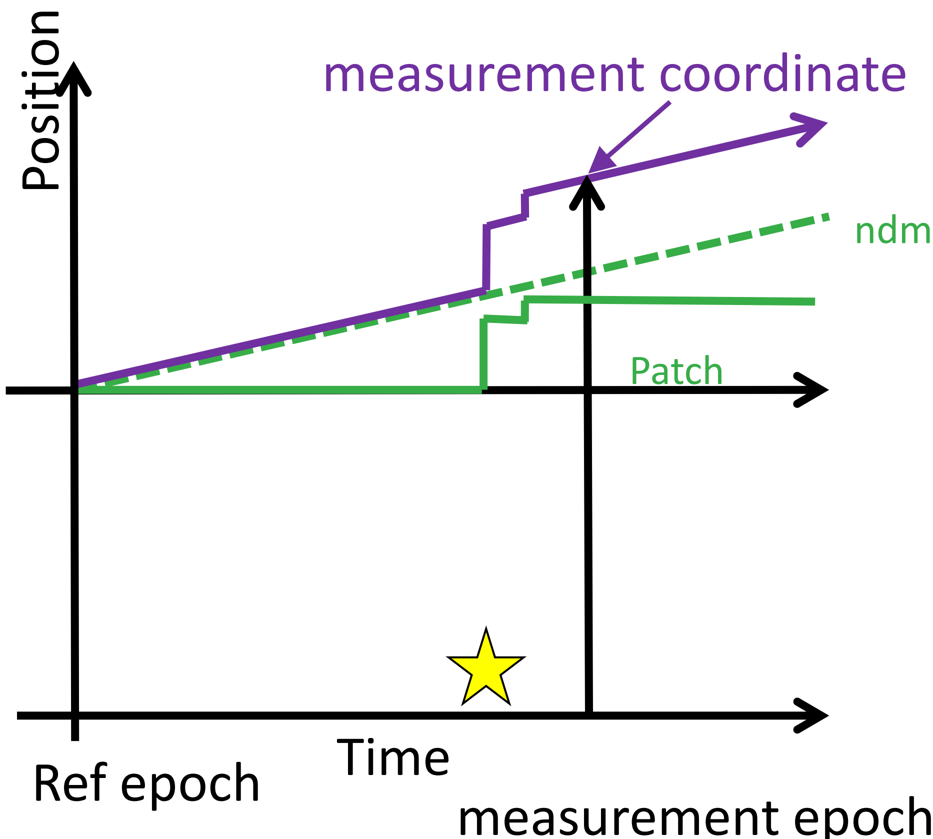
$m_k(t, \theta, \lambda) = v_k(\theta, \lambda)\mathrm{t}$



Velocity and earthquakes

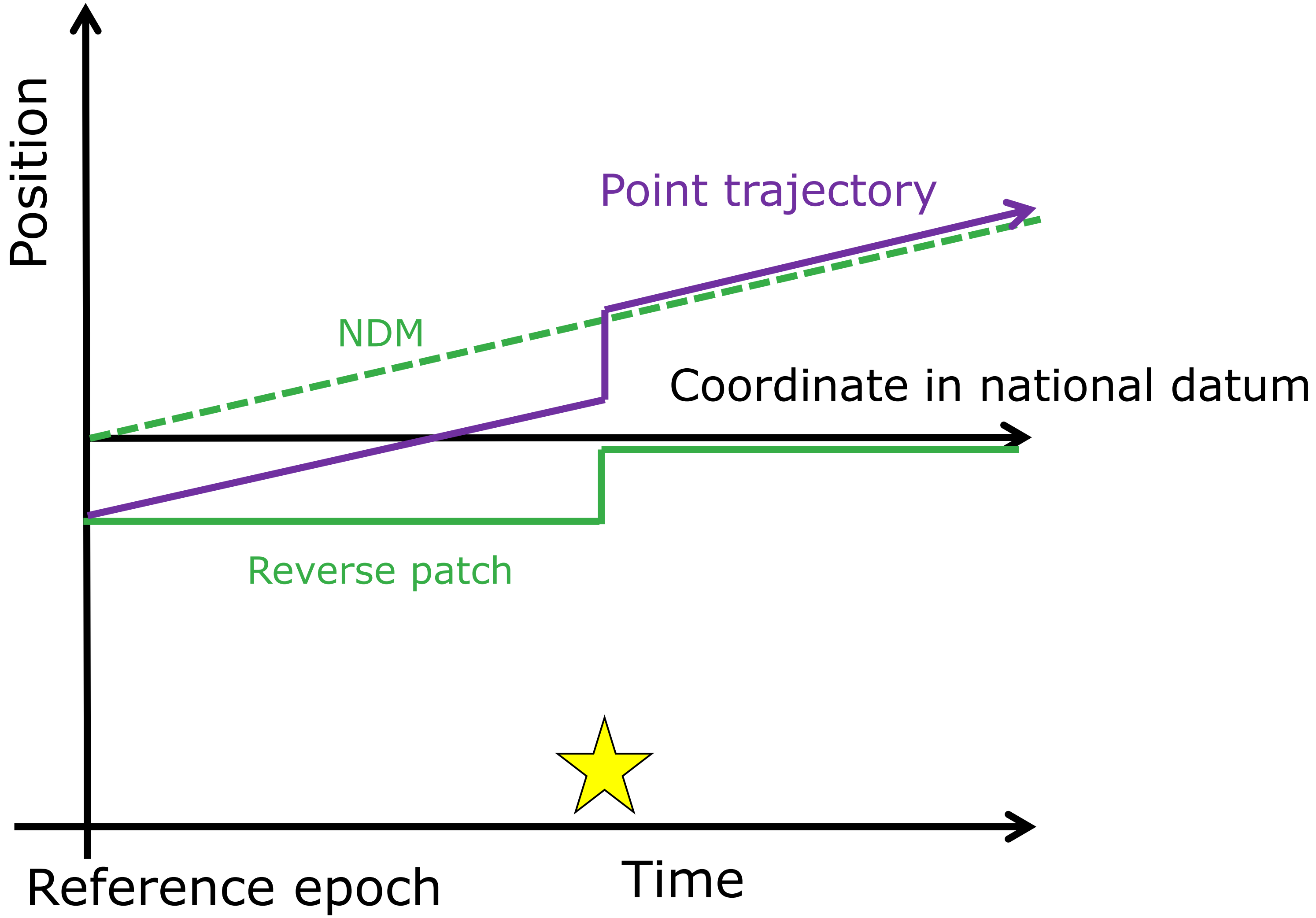
$$m_k(t,\theta,\lambda) = v_k(\theta,\lambda)\text{t}+E_{ki}(\theta,\lambda)H(t-t_i) + P_{ki}(\theta,\lambda)H(t-t_i)\left(1-e^{\frac{t-t_i}{tc_i}}\right)$$

- *m* is the displacement
- *v* is the velocity (ndm)
- *E* is the earthquake shift (patch)
- *P* post-seismic decay coefficient
- Tc is the time constant
- H is the step function



- step $f_t = 0$ for $t < t_s$
- $f_t = 1$ for $t > t_s$

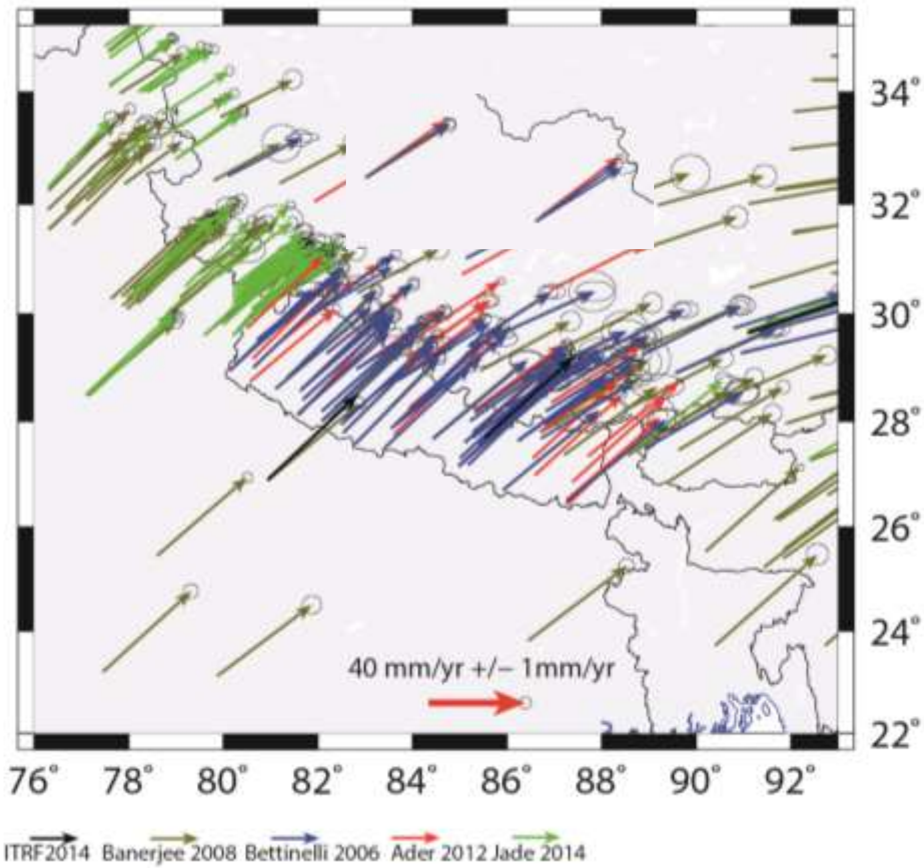
"Reverse" patch

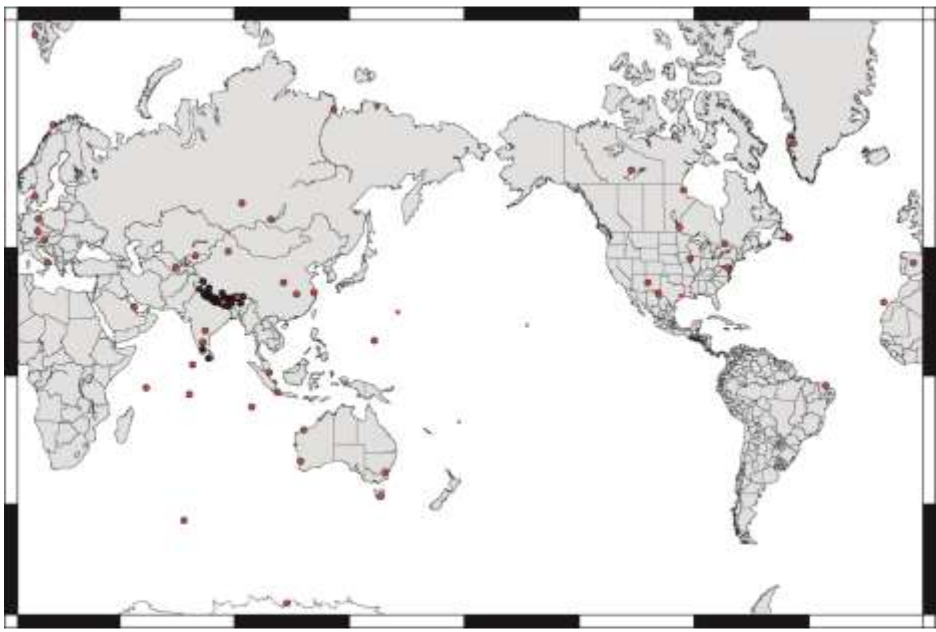


- step f_t = -1 for $t < t_s$
- f_t = 0 for $t > t_s$

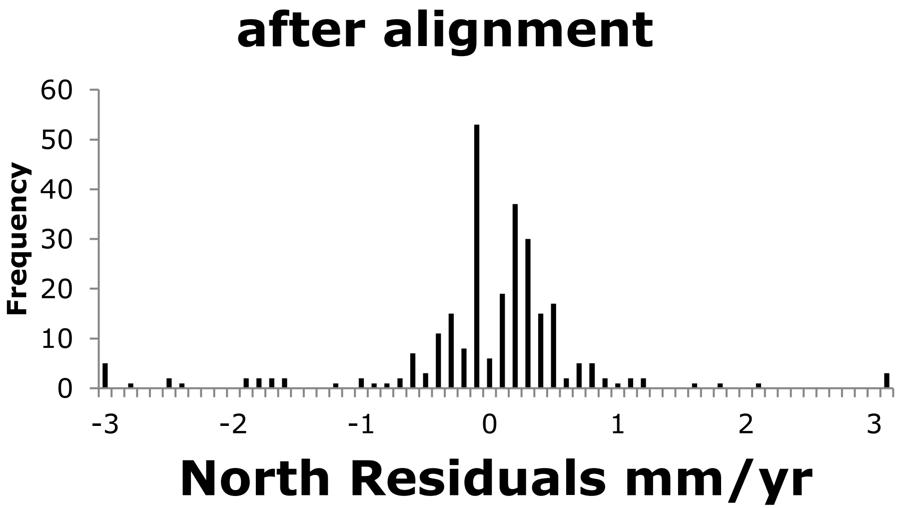
Secular velocity field

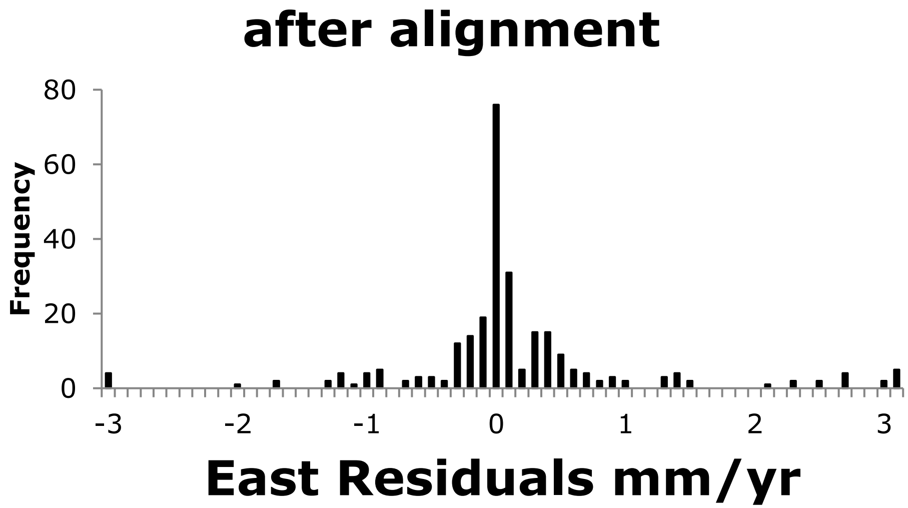
- Velocity from four recent studies were aligned with the ITRF2014 velocities
- The combined velocity field was used to produce a grid file with a density of 20 points/degree

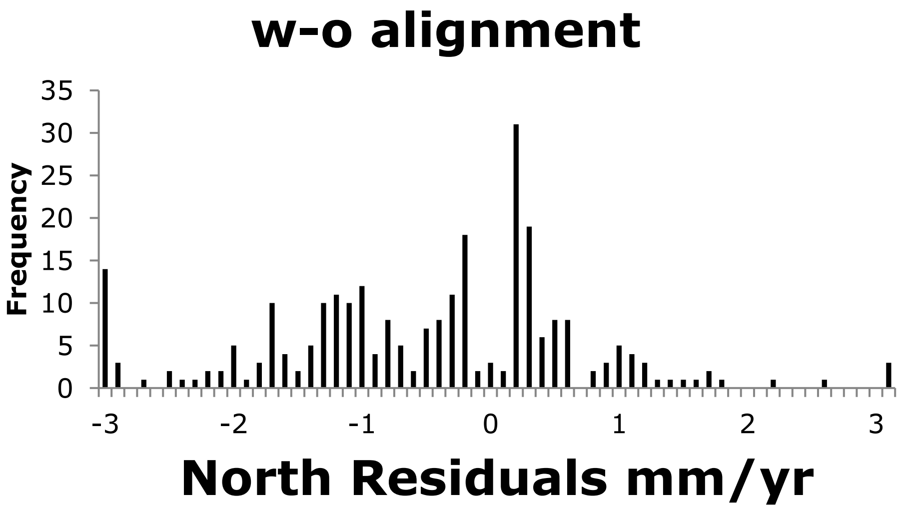


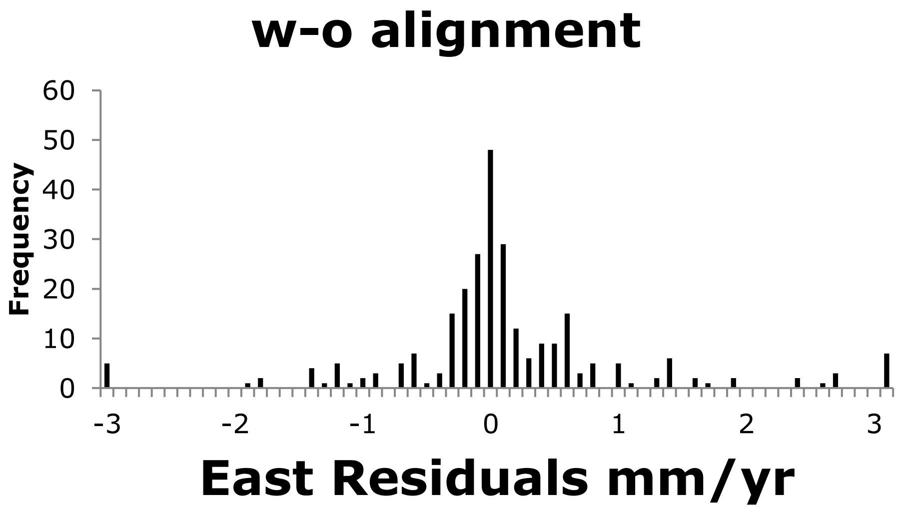


Residuals from alignment



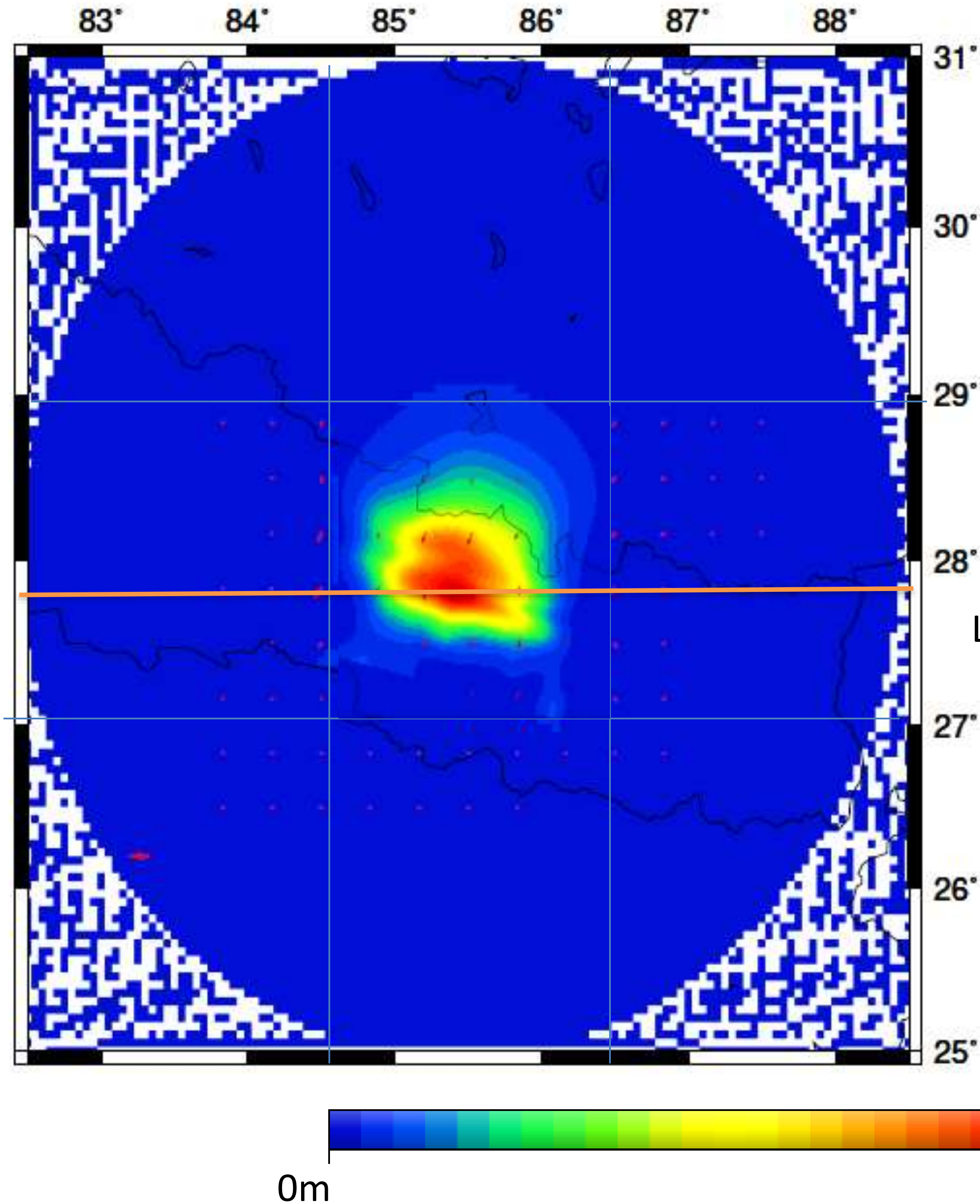






Earthquake patches

- earthquake patches are
  - Normally based on geophysical models
  - finite in extent
    - Introduce a taper where the amplitude of the shifts is lineally reduced to 0 to prevent a discontinuity at the edge of patches
    - Two critical distances:
      - » Taper starts at a particular distance from the epicenter when the signal is reduced to a low value
      - » Taper ends when the amplitude is reduced to 0
      - » At greater distances the amplitude of the shifts are 0 everywhere.
  - high amplitude signal is concentrated near the epicenter
    - Nested grids with higher resolution epicentral region

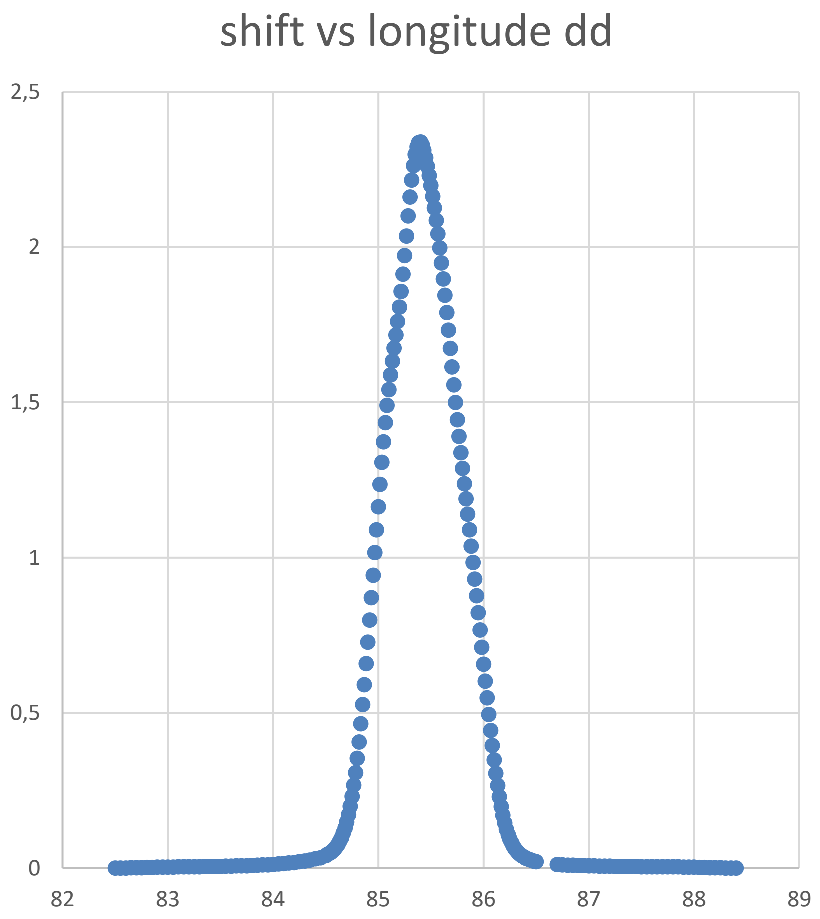


High res grid is used inside inner rectangle

Corse tapered grid is used everywhere else

Line of cross-section

Cross sections



Cross section along 27.75 ° Lat

Error propagation

- Deformation Equation

$$\begin{bmatrix} \emptyset(t) \\ \lambda(t) \\ h(t) \end{bmatrix} = \begin{bmatrix} \emptyset_0 + (H(t - t_i)E_{ci} + v_E * t) \\ \lambda_0 + (H(t - t_i)N_{ci} + v_N * t) \\ h_0 \quad + H(t - t_i)h_{ci} + v_h * t \end{bmatrix}$$

- Variance

$$\begin{bmatrix} S^2{}_{\emptyset(t)} \\ S^2{}_{\lambda(t)} \\ S^2{}_{h(t)} \end{bmatrix} = \begin{bmatrix} S^2{}_{\emptyset_0} + \left(H(t - t_i)S^2{}_{E_{ci}} \quad t + S^2_{v\,E} * t\right) \\ S^2{}_{\lambda_0} + \left(H(t - t_i)S^2{}_{N_{ci}} \quad t + S^2_{v\,N} * t\right) \\ S^2{}_{h_0} + H(t - t_i)S^2{}_{h_{ci}} \quad t + S^2_{v\,h} * t \end{bmatrix}$$

- Velocity contribution to coordinate errors increase with square root of time
    - the shorter the time since the reference epoch the better
- Errors from earthquakes are not time dependent
    - Hard to quantify, largest where deformation spatially variable

The Woodchester Wall

Leader Fault Kaikoura Earthquake 2016
Left lateral and reverse of ~1.5-2m
Motion concentrated on fault plane that broke surface

Darfield earthquake

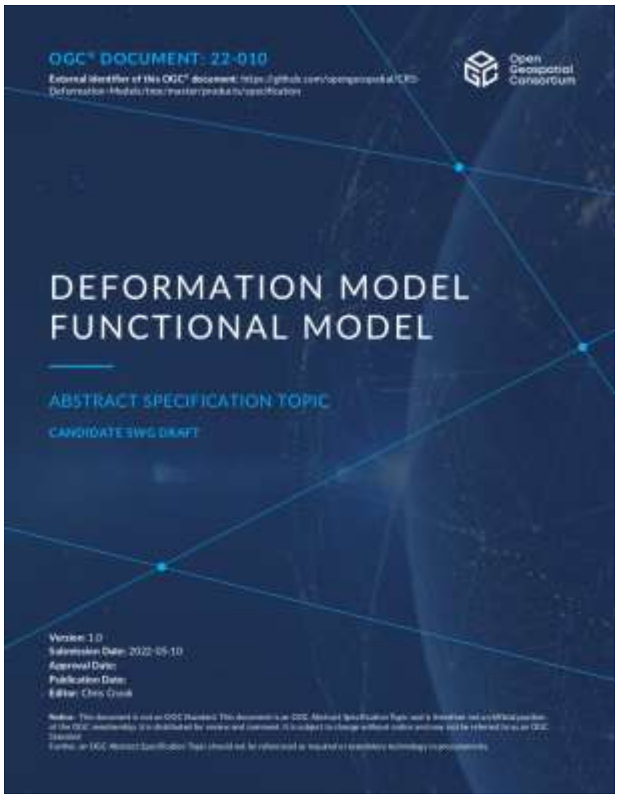
Strike-slip earthquake
Deformation distributed over 10-15 m wide zone

- Errors from earthquakes are largest near fault trace
  - Particularly if there is surface faulting
  - Consider changing reference epoch after an earthquake

Conclusions

- Modern datums aligned to a realization of the ITRF
- Maintaining a low distortion datum requires a mechanism to correct for crustal motion
- Can be an Euler Pole for low distortion areas
- Or deformation model for plate boundaries
    - Velocity model
    - Earthquake displacements and post seismic
    - Also supports time dependent least square adjustments
- Errors associated with velocity models increase with Squair root of time errors. Errors associated with earthquake models are largest near surface traces
- common reference epoch reset after major earthquakes
    - Draft Abstract Specification for deformation models has been released for comment by the OGC

draft Abstract Specification for deformation models



- The project team has endorsed this draft specification
- Seek promotion to SWG

Errors

A simplified approach to propagate positional errors for these functions neglecting most correlations is:

- $e \quad = \sqrt{{f_{e1}}^2 \cdot {e_1}^2 + {f_{e2}}^2 \cdot {e_2}^2 \ldots + {f_{en}}^2 \cdot {e_n}^2}$
- For each component the factor can be calculated using simple identities for error propagation:
- $if\ f(x) = c \cdot x\ then\ e_{fx} = c \cdot e_x$
- $if\ f(x) = x + y\ then\ e_{fx} = \sqrt{{e_x}^2 + {e_y}^2}$
- $f_{e1,t_1-t_2} = \sqrt{abs(f_{e1}(t_1)^2 - f_{e1}(t_o)^2)}$
-