Introduction to Vertical Reference Frames and Vertical datums

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Outline

- Levelling Datums (Local; National; Global)
- Reference Surfaces; ellipsoids, geoids, quasigeoids
- Gravity and Gravity Potential; Geopotential Numbers
- Systems of Heights
- GNSS Heighting

Levelling Datums - Local

- As surveying practitioners we are all familiar with the concept of a Levelling Datum (from “dato” – latin “to Give”).
- “More broadly, a vertical datum is the entire system of the zero elevation surface and methods of determining heights relative to that surface”.

Levelling Datums - Local

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- "More broadly, a vertical datum is the entire system of the zero elevation surface and methods of determining heights relative to that surface".
- **Local Datums**: Used in stand-alone or sites of limited area (eg local government, mines; river valley water management …) and are defined to suit the task
- PLUSSES: No need to access other levelling control marks to relate to national datum
- MINUSSES: Is only useful/useable in the project(s) for which it is adopted
-

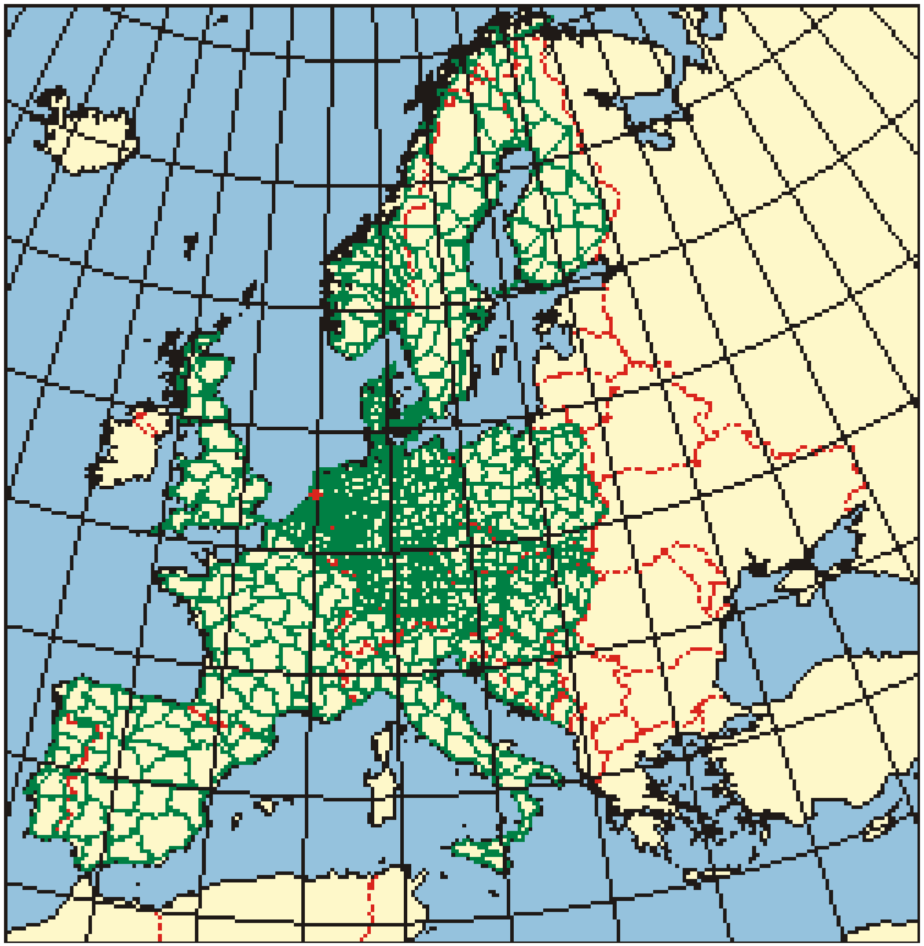
Levelling Datums - National

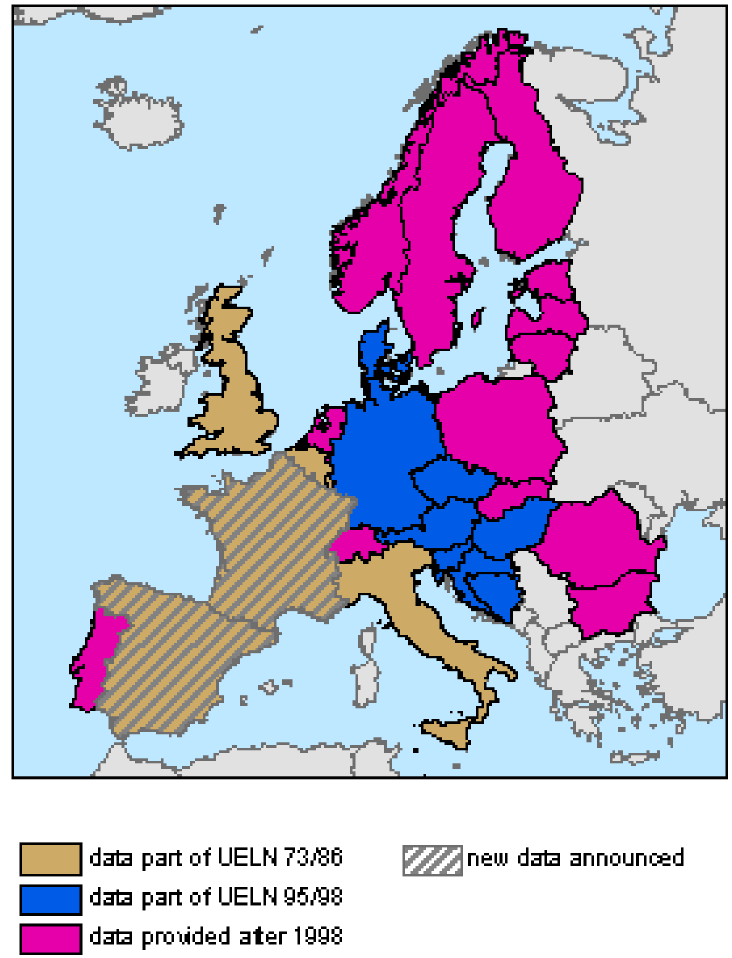
- National datums are defined to suit the ***national*** requirements
- Are historically ***defined physically by tide gauge*** observations to align with Mean Sea Level

Levelling Datums - National

- National datums are defined to suit the national requirements
- Are historically ***defined physically by tide gauge*** observations to align with Mean Sea Level
- PLUSSES: Is a ***unified*** height system; Is useful/useable in the project(s) for which it is adopted; can be used across a variety of (and all) projects
- MINUSSES: Need to access other levelling control marks to relate to national datum; may suffer distortions over extensive areas

United European Leveling Network (UELN)





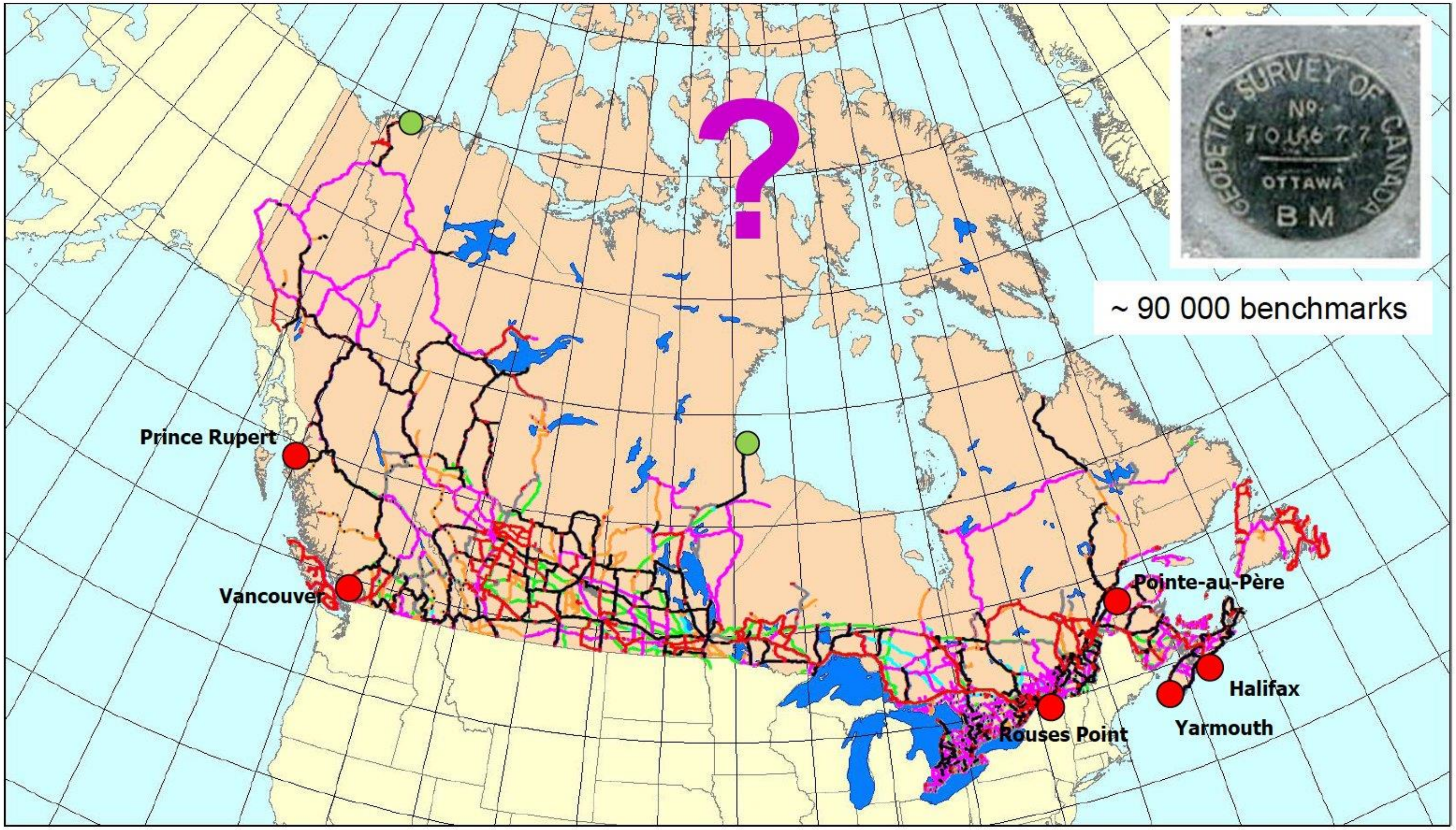
Levelling Datums – (Inter)-national

Europe: http://www.bkg.bund.de/geodIS/EVRS/

The objective of the United European leveling Network (UELN) project is to establish an unified vertical datum for Europe at the one-decimeter level with simultaneous enlargement of UELN as far as possible to the Eastern European countries.

- The results of the adjustment with status of end 1998 were handed over to each participating country under the name UELN95/98. One year later at the EUREF symposium 2000 in Tromsø a first definition of the European Vertical Reference System (EVRS) was adopted. The realization on the base of the UELN95/98 solution got the name EVRF2000.)

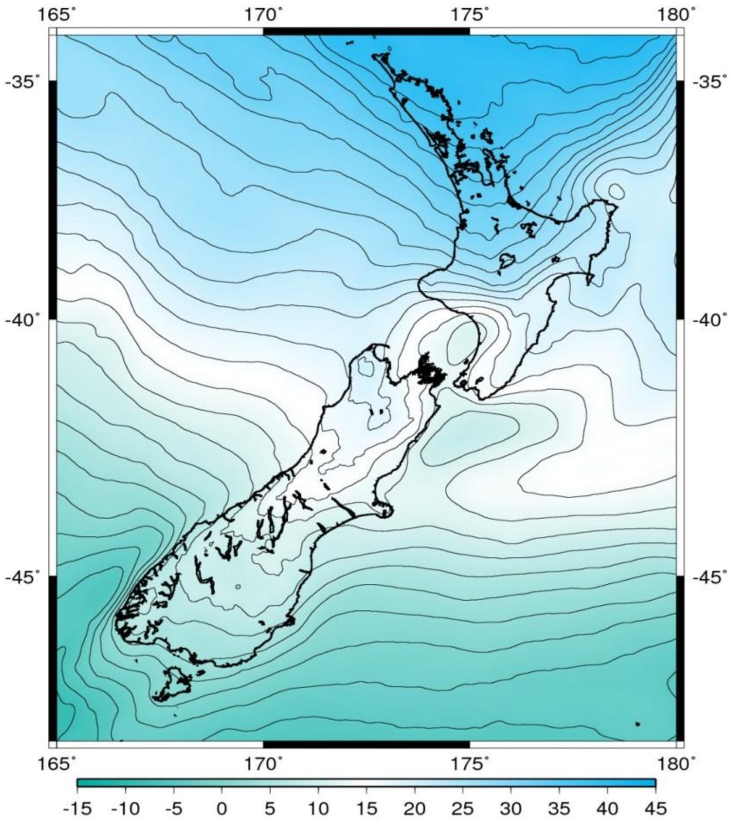
Canadian Geodetic Vertical Datum 1928 (CGVD1928)



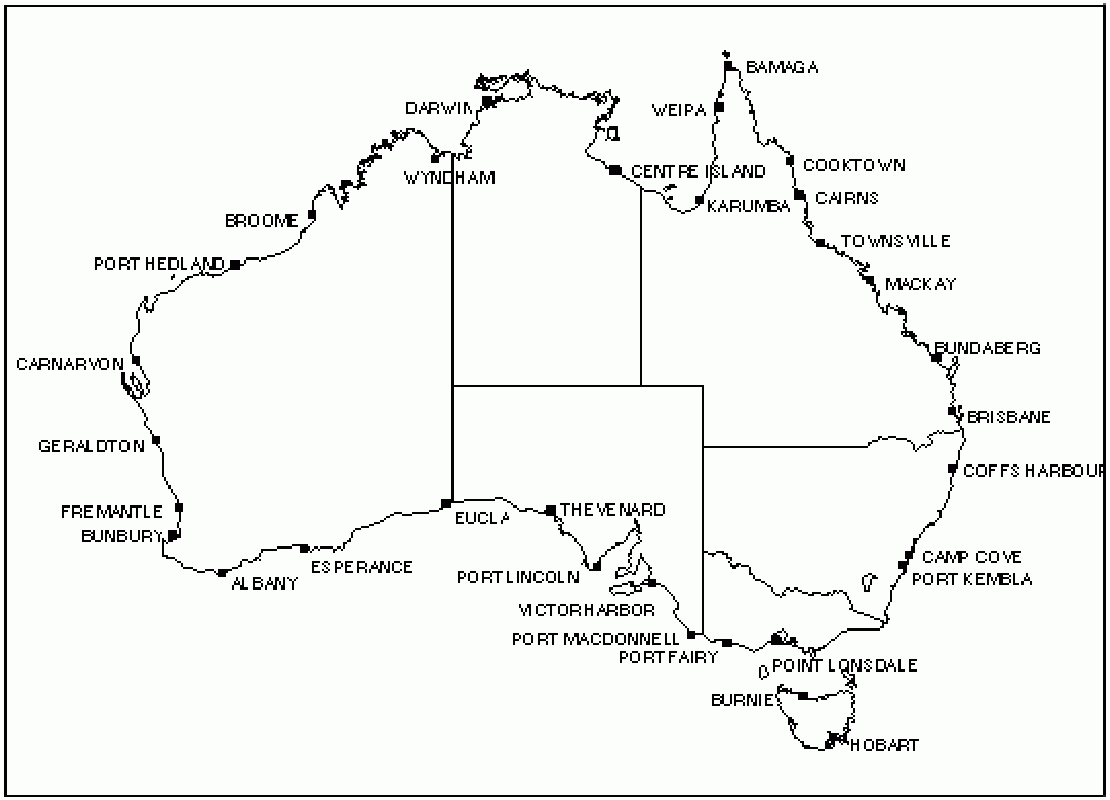
1906-1928 1929-1939 1940-1965 1966-1971 1972-1981 1982-1989 1990-2007

New Zealand Vertical Datum 2009

- Based on NZGeoid2009, a quasigeoid
- Normal-orthometric heights
- Includes offsets to 13 main levelling datums
- Primarily accessed via ellipsoidal heights from GNSS, transformed using NZGeoid2009
- 8cm nominal accuracy



Tide Gauges of the Australian Height Datum: AHD'71



The Australian Levelling Network (ANLN)

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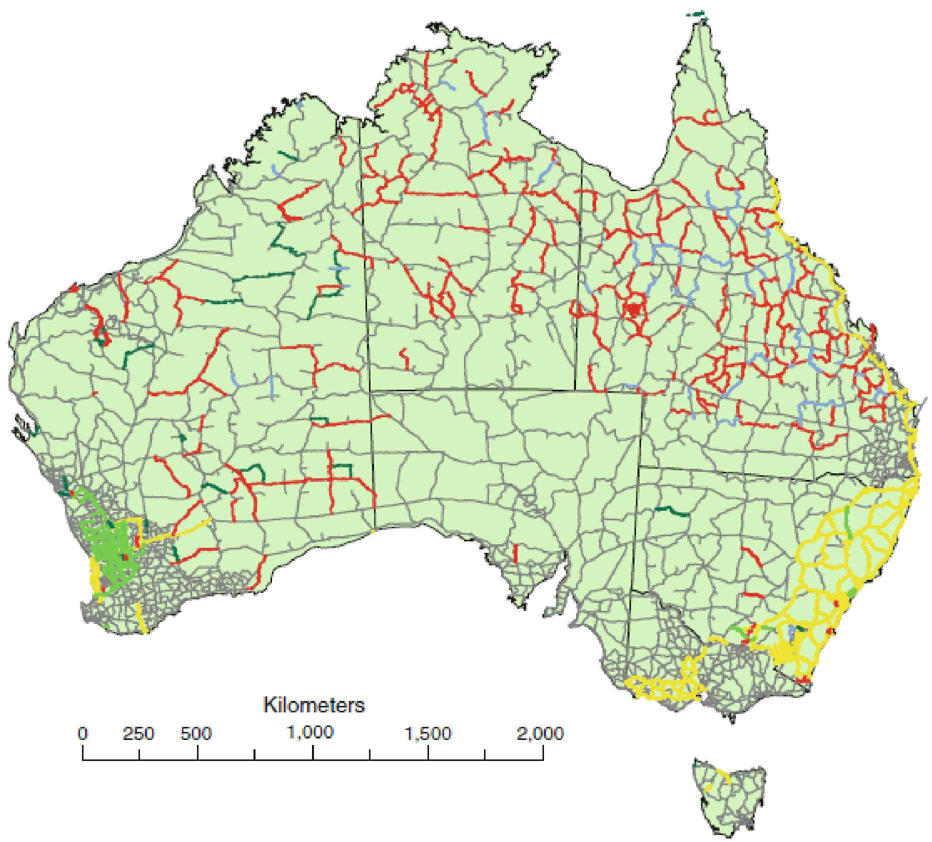
Fig. 1 The Australian National Levelling Network (ANLN). First-order sections are in *yellow*, second-order sections in *light green*, third-order in *thin grey*, fourth-order in *dark green*, one-way (third-order) in *red* and two-way (order undefined; Steed 2006, pers. comm.) in *blue*. Lambert projection. ANLN data courtesy of Geoscience Australia



Levelling Datums - National

- **USA**: See http://www.ngs.noaa.gov/datums/vertical/
- **Australia:** AHD: http://www.ga.gov.au/scientific-topics/positioning-navigation/geodesy/geodetic-datums/australian-height-datum-ahd

Levelling Datums - Global

- A Global datum is which aims to meet the needs of international geodetic science community (GGOS)
- Is ***defined*** by IAG Study Groups ***by analyses of*** Mean Sea Level as observed Satellite altimetry etc over the oceans

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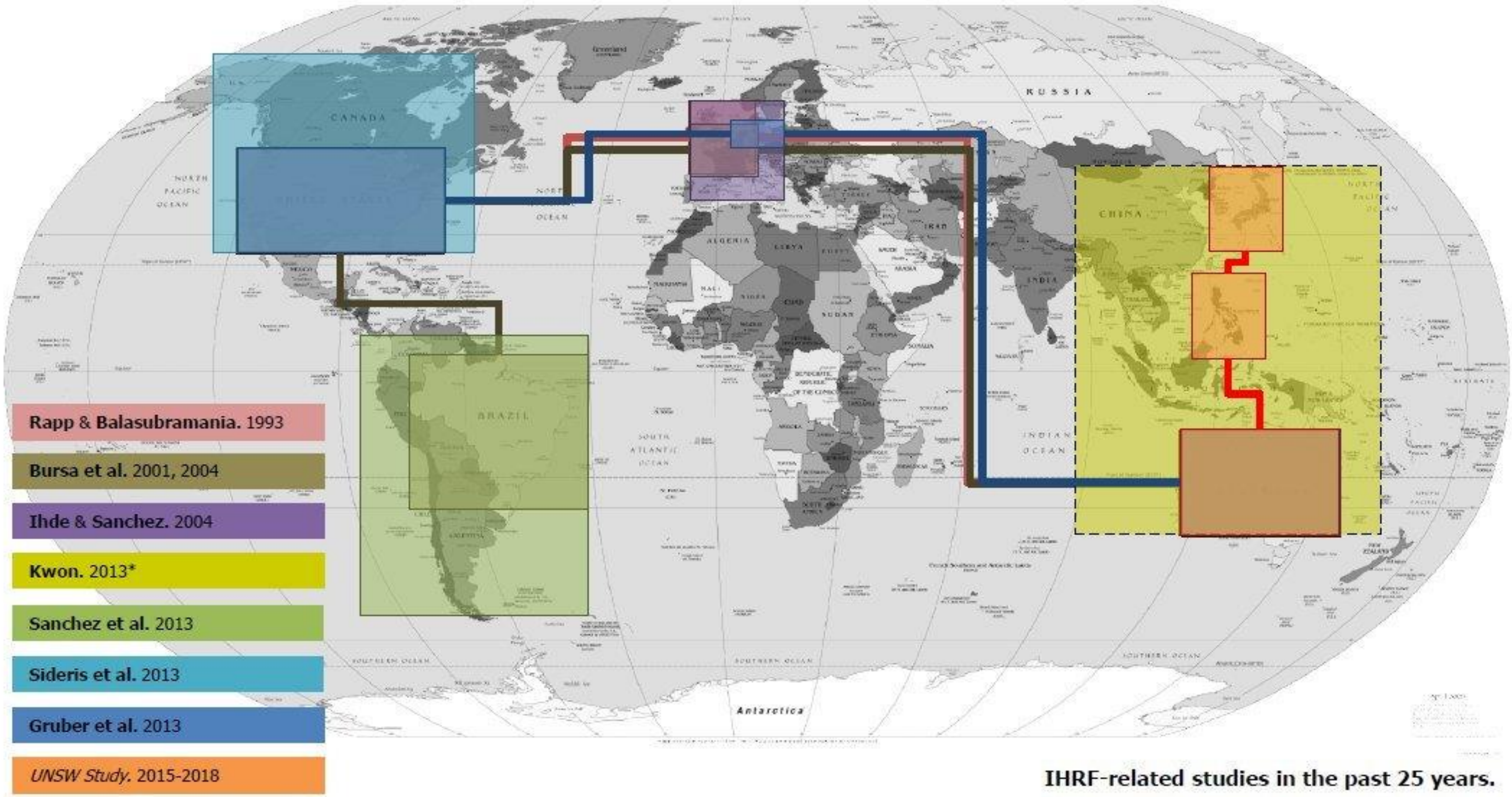
- Is ***defined*** by IAG Study Groups ***by analyses of*** Mean Sea Level as observed Satellite altimetry etc over the oceans
- This enables ***national datums to be tied together*** and
- Allows connections across oceans for global studies (eg Sea Level Changes).
- PLUSSES: Is a ***unified*** global height system; Is useful/useable in determining distortions in the National Height datums
- MINUSSES: Less intuitive than the local or national height datum

Levelling Datums: (b) National vs (c) Global)

Globalisation of Heights



IHRF-related studies in the past 25 years.

Levelling Datums: (a) Local; (b) National; (c) Global)

- So, who uses which of these datums??
- Most practitioners will be familiar with a) and b) as these have the most common usage.
- c) is more the province of the geodesists and other global scientists who wants to bring all heights around the Earth to a common reference.

Users of Levelling Datums:

- So, who uses these different datums??
- Results of a user survey (1988)

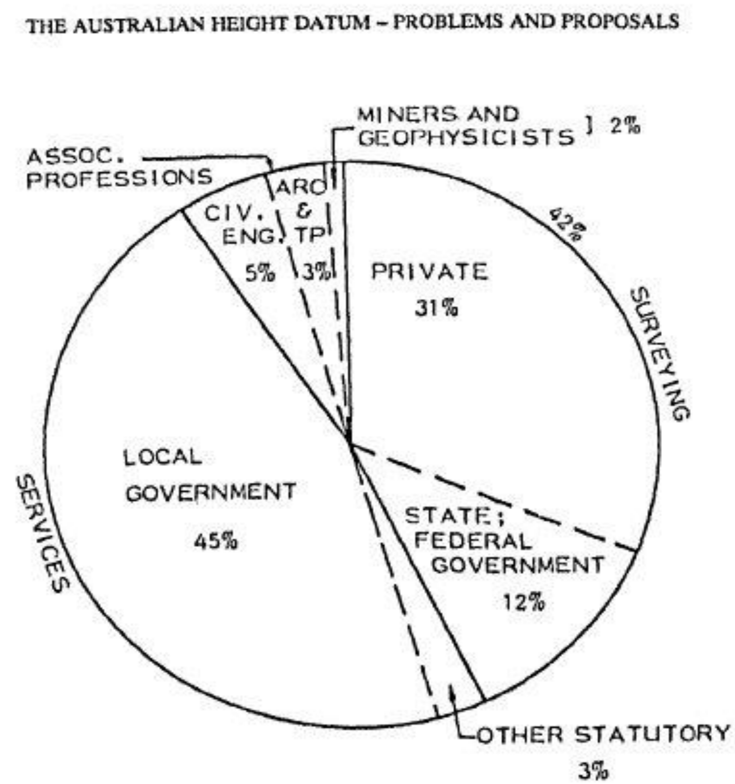


Figure 2: Organisations field of operation across Australia (as a % of total response) Total sample, 860.

Users of Levelling Datums:

- What do they use them for??

Note: Because of the limited (in area) scope of the tasks, in most cases only a local datum is required by most of these users!

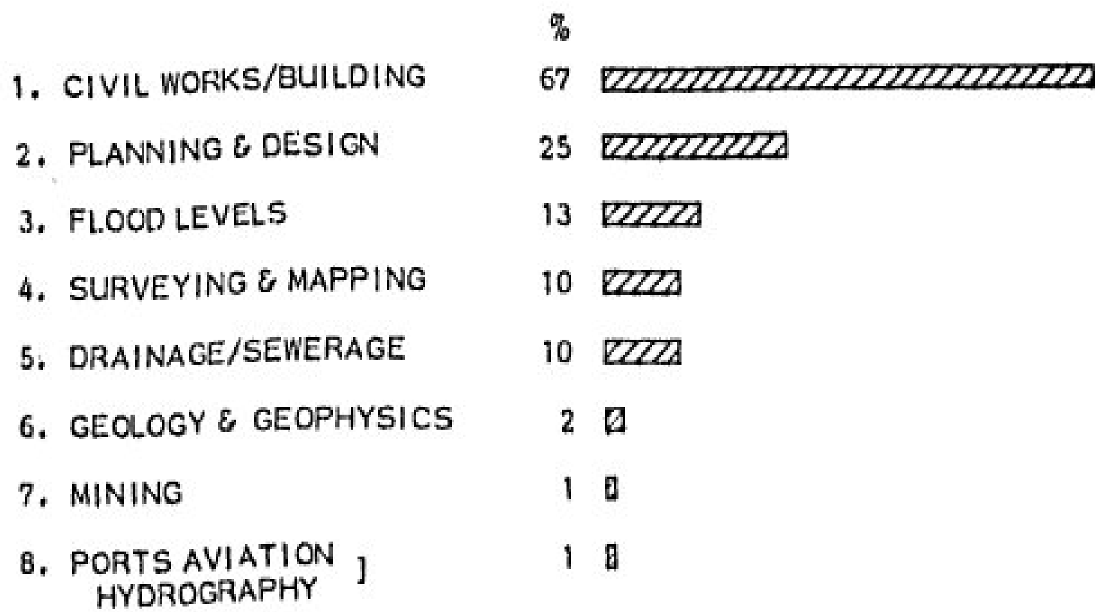
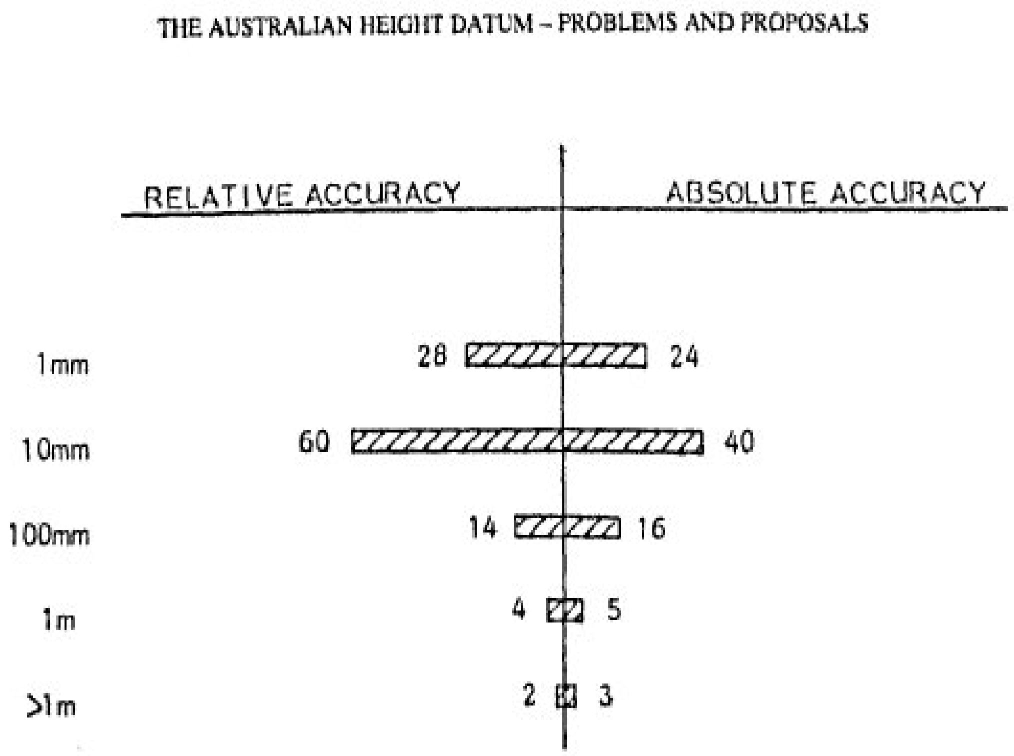


Figure 3: Purposes for which heights are required (as a % of 585, total number of respondents to the question – Queensland provided no summary).

The Australian Surveyor, December, 1988, Vol. 34 No. 4 365

Users of Levelling Datums:

- What precision /accuracy do they require??



Users of Levelling Datums:

- Techniques used

THE AUSTRALIAN HEIGHT DATUM – PROBLEMS AND PROPOSALS

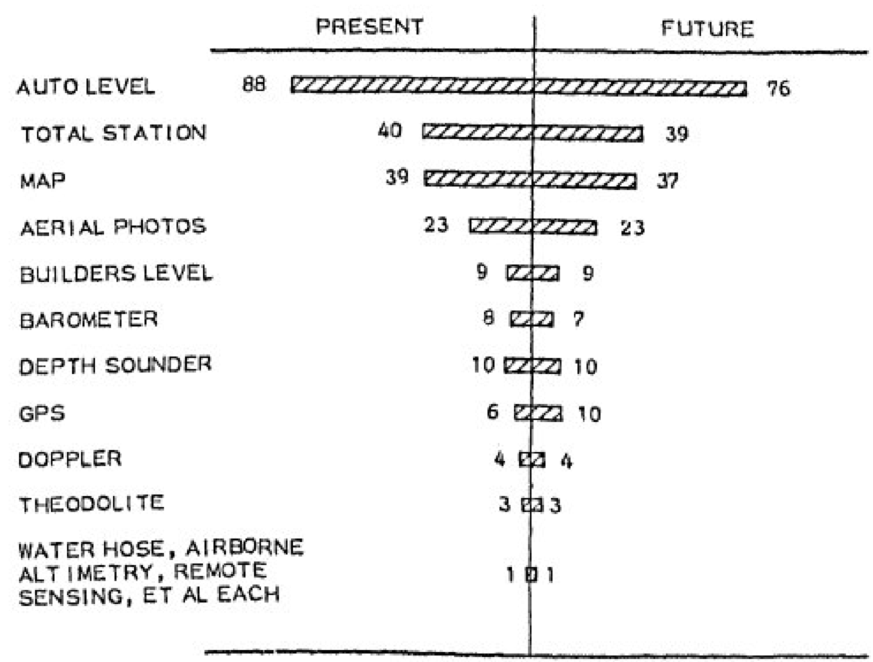


Figure 6: Methods for establishing/propagating heights (as a % of total number of respondents).

Users of Levelling Datums

- One important user group – the Hydrographic Surveyors – has investigated the use of a purely geometric datum – i.e. a global ellipsoidal model of the Earth (e.g.WGS84) as the Datum for Hydro Surveying, and
- measuring all heights (and depths) by GNSS, moving completely to a geometric form of determining height *(Greenland and Higgins, IAG Publication #37: FIG Guide on the Development of a Vertical Reference Surface for Hydrography, Sept 2006)*

Users of Levelling Datums

- MAIN MESSAGE:
- *Most users traditionally only need a national, or even local, Height Datum*
- *As the GNSS and aassociated technology becomes more mature, the need for a well defined and maintained Global Datum will increase.*

Section 2: Reference Surfaces; geoids, ellipsoids, quasigeoids

Let us expand upon our original definition of the vertical or height datum.

Recalling: “A vertical datum is the entire system of ***the zero elevation surface*** and methods of determining heights relative to that surface”.

Section 2: Reference Surfaces; geoids, ellipsoids, quasigeoids

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Recalling: "A vertical datum is the entire system of ***the zero elevation surface*** and methods of determining heights relative to that surface".

One absolute need of our height datum (for most purposes) is that a fluid ***acting only under the influence of gravity*** travels from a higher point relative to the datum to a lower point. This infers that on the datum itself all points must be/are at the same height (and zero!)

Vertical Reference Surfaces; geoids

This is, of course, why Mean Sea Level as determined by Tide Gauge (TG) observations has been conventionally used - for centuries - to determine a National Vertical Datum.

It is a close, and observable, approximation to a surface of zero height.

Vertical Reference Surfaces; geoids

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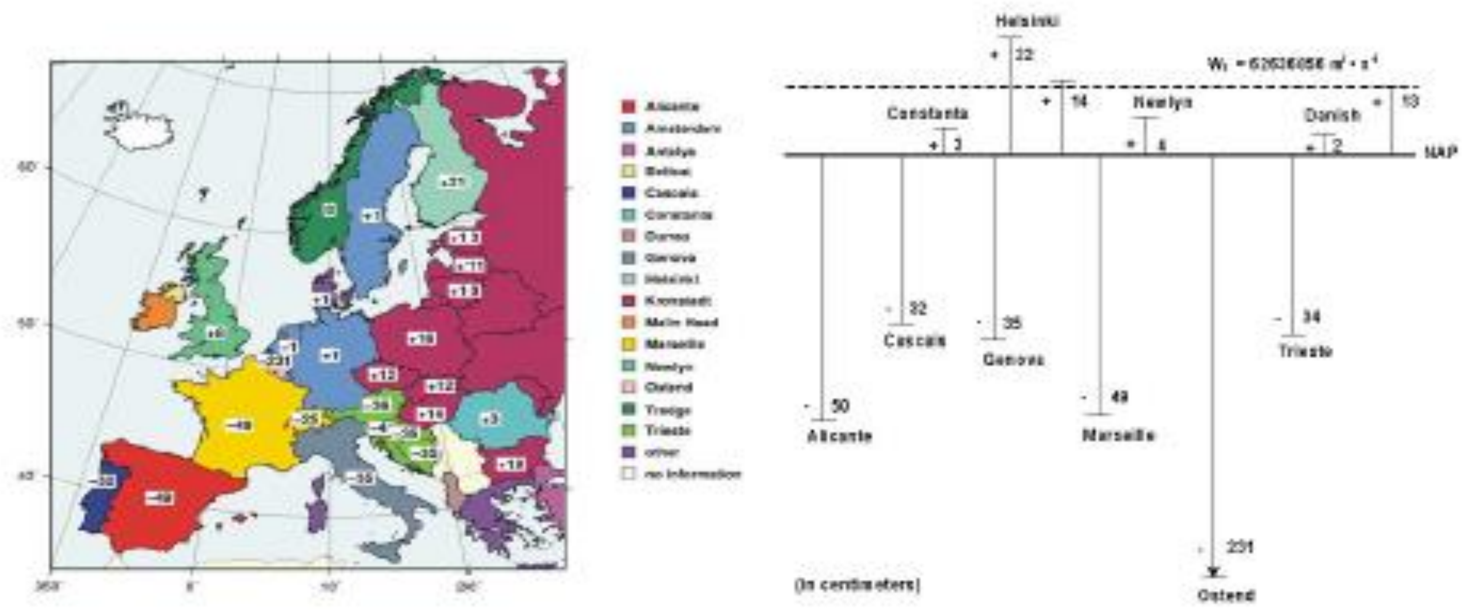
As our knowledge of the Earth's gravity field has improved, we have discovered (what we have long suspected) that there are distortions of our MSL-defined datum from a truly level surface

Distortions in the Vertical Reference Systems in Europe, and their relationships to a World Height System



Relationship between national Vertical Reference Systems in Europe and the European Vertical Reference System (NAP - Normaal Amsterdams Peil) derived by connecting levellings

and its relationship to World Height System (WHS, W_0 = 62636856,0 m^2s^{-2}) derived tide gauge and satellite altimeter observations.

Transformation Parameters between European Height Reference Frame and pan-European EVRS
(see also crs.bkg.bund.de/crs-eu)

Country	Identical Points		Parameters			RMS	Residual Deviations	
	Number	Kind	Vertical Translation in cm	Slope in Latitude in cm / 100km	Slope in Longitude in cm / 100km	in cm	min in cm	max in cm
Austria	114	UELN	- 35.6	- 2.8	- 2.8	3.1	- 6.1	+ 6.1
Bosnia and Herzegovina/ Croatia	40	UELN	- 34.5	- 0.3	- 0.9	0.7	- 1.0	+ 1.4
Belgium	4	EUVN	- 231.1	- 0.8		0.2	- 0.2	+ 0.2
Bulgaria	36	UELN	+ 18.2	+ 0.1	- 0.2	0.2	- 0.6	+ 0.4
Switzerland (LN02)	225	UELN	- 24.5	- 10.2	- 1.6	3.3	- 8.6	+ 9.4
Czech Republic	53	UELN	+ 11.6	+ 1.7		1.4	- 3.5	+ 2.8
Germany (DHHN92)	443	UELN	+ 1.4	- 0.1		0.2	- 0.7	+ 0.6
Denmark	707	UELN	+ 1.1	+ 0.1	+ 0.5	0.3	- 0.9	+ 0.8
Estonia	36	UELN	+ 13.3	- 0.7	+ 0.2	0.3	- 0.5	+ 0.5
Spain	70	UELN	- 48.6	- 0.2	+ 0.3	1.0	?	?
Finland	66	UELN	+ 21.3			0.3	- 0.7	+ 0.9
France	8	EUVN	- 48.6			0.5	- 0.4	+ 1.0
Great Britain	5	EUVN	+ 8.1	- 2.7	- 1.1	1.9	- 1.2	+ 2.2
Croatia	40	UELN	- 34.5	- 0.3	- 0.9	0.7	- 1.0	+ 1.4
Hungary	35	UELN	+ 14.0	+ 0.4	- 0.1	0.3	- 0.7	+ 0.6
Italy	9	EUVN	- 35.3	+ 0.2	+ 0.3	0.7	- 0.6	+ 1.1
Lithuania	46	UELN	+ 10.2		+ 0.1	0.2	- 0.2	+ 0.3
Latvia	123	UELN	+ 10.5		+ 0.2	0.7	- 2.0	+ 2.2
Netherlands	757	UELN	- 0.5			0.2	- 2.1	+ 0.4
Norway	117	UELN	- 0.1	- 0.5	+ 1.7	3.7	- 7.6	+ 7.0
Poland	98	UELN	+ 16.0	+ 0.5		0.5	- 2.0	+ 0.9
Portugal	5	EUVN	- 31.5			1.3	- 1.4	+ 2.1
Romania	46	UELN	+ 2.8	+ 0.1	+ 0.1	0.2	- 0.5	+ 0.9
Sweden	21	EUVN+Tide G	+ 1.0	- 0.6		1.1	- 2.3	+ 2.0
Slovenia	9	UELN	- 41.1	- 1.6	+ 0.4	0.3	- 0.4	+ 0.4
Slovakia	3	EUVN	+ 12.2	+ 1.0		0.2	- 0.1	+ 0.1

Vertical Reference Surfaces; geoids

- So, for a variety of reasons, this TG approach turns out to be flawed: MSL as defined by ***a network of tide gauges*** does ***not*** represent ***a truly level surface***.

- To find this level surface we have to resort to first principles of Physics, and specifically of the force of Gravity, and its associated phenomena.

Vertical Reference Surfaces; geoids

- So, for a variety of reasons, this TG approach turns out to be flawed: MSL as defined by ***a network of tide gauges*** does ***not*** represent ***a truly level surface***.
- To find this level surface we have to resort to first principles of Physics, and specifically of the force of Gravity, and its associated phenomena.
- And the term used to define this "level" surface – the surface of ***equal gravitational potential*** – is a **"Geop".**
- One of these Geops – the one "at" MSL, we choose to be our reference for heights, is "***The Geoid***".

Vertical Reference Surfaces; geoids, quasigeoids

- Wikipedia : *The **geoid** is the shape that the surface of the oceans would take under the influence of Earth's gravitation and rotation alone, in the absence of other influences such as winds and tides. This surface is extended through the continents (such as with very narrow hypothetical canals).*

Vertical Reference Surfaces; geoids, quasigeoids

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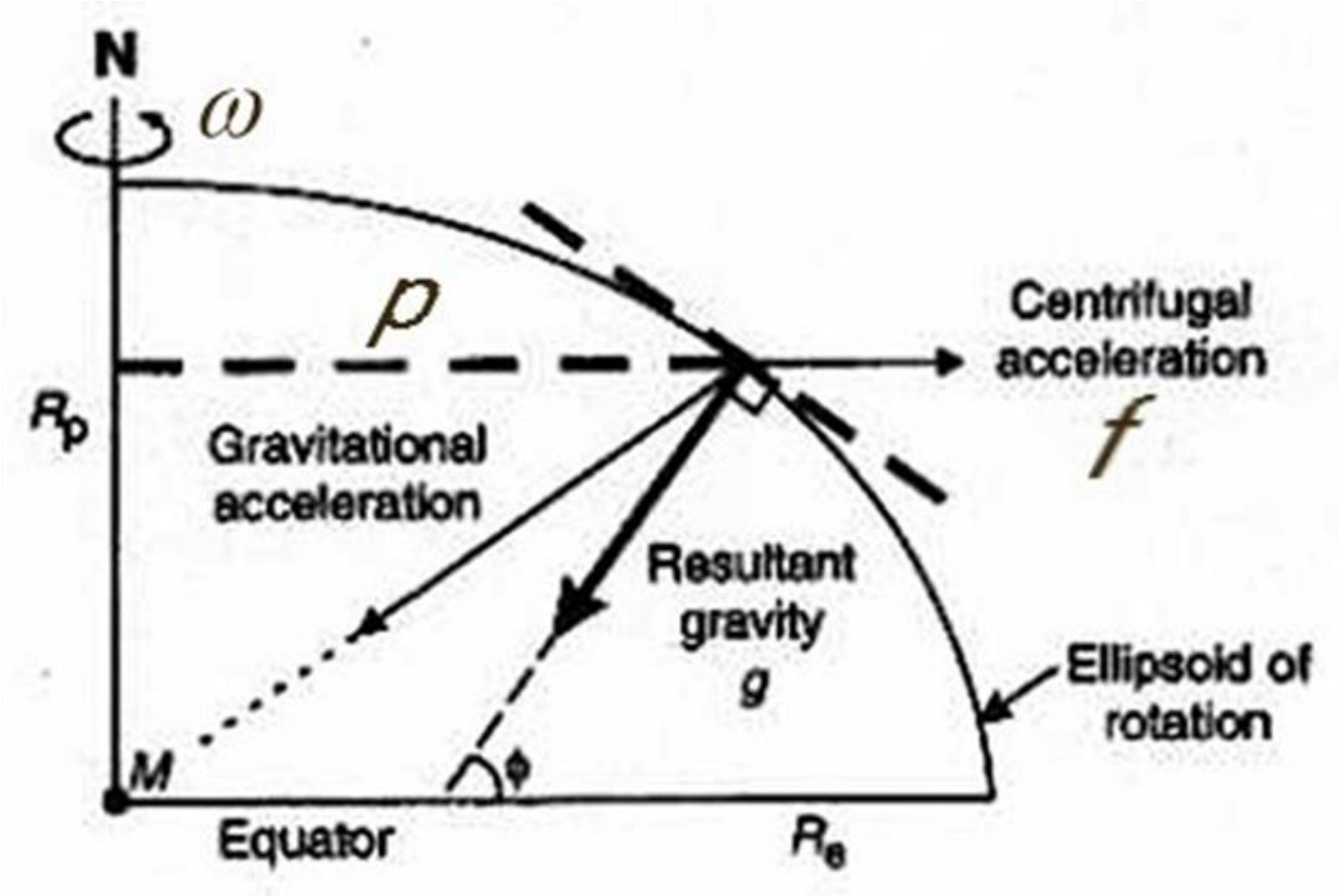
- *All points on the geoid have the same gravity potential energy (the sum of gravitational potential energy and centrifugal potential energy).*
- *The force of gravity acts everywhere perpendicular to the geoid, meaning that plumb lines point perpendicular and water levels are parallel to the geoid.*

Vertical Reference Surfaces; geoids, quasigeoids



Vertical Reference Surfaces; geoids, quasigeoids

- The QuasiGeoid is a surface a bit like a geoid – except it isn't one!
- *It is however a useful concept that we invoke to get around some of the problems associated with defining the Geoid*
- *More on this later.*

Vertical Reference Surfaces; geoids, quasigeoids

- To explore geoids etc. further we have to understand the concepts of Gravity, and Gravitational potential

First, some background:

- We recall the concept of a **scalar quantity** (value of some phenomenon at a point, e.g. temperature T, height H, gravity potential W)

Vertical Reference Surfaces; geoids, quasigeoids

- Compare this **scalar** to a ***vector*** (which be could thought of as a gradient or change of a scalar quantity with space or time, e.g. temperature gradient, ground slope, & gravitational acceleration or force)
-

Vertical Reference Surfaces; geoids, quasigeoids

- Compare this **scalar** to a ***vector*** (which be could thought of as a gradient or change of a scalar quantity with space or time, e.g. temperature gradient, ground slope, & gravitational acceleration or force)
- relationship between scalar and vectors at a point in space:

$T_{grad} = dT/ds;$

Slope = $dH/ds;$ and

$\mathbf{g} = dW/ds$

where **g** is the gravitational acceleration or force

W is the gravitational potential, and

ds is the distance over which dW is measured.

Vertical Reference Surfaces; geoids, …

- Compare this **scalar** to a ***vector*** (which be could thought of as a gradient or change wrt distance or time, e.g. temp gradient, ground slope, gravitational acceleration or force)
- relationship between scalar and vectors at a point in space:
- dT/ds; dH/ds; and ***g*** = dW/ds) (1)
- Take temperature, for example - we can measure temperature T at two points , T_i and T_j with a thermometer, and measure or find their separation s_{ij} from geometry,
- and thus compute their gradient by knowing the ΔT_{ij};
- Similarly, could find the change of temperature with Time,
- Or ΔH separated by distance s_{ij} (give examples).

Vertical Reference Surfaces; geoids…

- A line or surface of equal *temperature* (an isotherm) is defined as one where there is NO change in temperature between two points in space, separated by distance "r".
- This surface can be located in space by measurement – simply by moving a thermometer around the space and locating in X,Y,Z the points of equal T (say 20^{o} C).

Vertical Reference Surfaces; geoids, quasigeoids

- Similarly a *contour line* is drawn between two points of *equal height* $H = constant$; no matter the distance *ds* between points along this line, the slope along it is zero because the ΔH between these points is zero.

Vertical Reference Surfaces; geoids, quasigeoids

- Similarly a *contour line* is drawn between two points of *equal height H*; no matter the distance between points along this line, the slope along it is zero because the ΔH between these points is zero.
- But since we cannot measure W, how can we locate a geop, a surface of constant geopotential (or, specifically, a geoid)??
- We need to be able to do this so we can locate this “level” ***reference surface*** – the surface of ***equal gravitational potential*** – or “***The Geoid***” – our reference surface for vertical elements.

Vertical Reference Surfaces; geoids, quasigeoids

- However, we ***cannot*** measure the (scalar) geopotential W; we can measure (the vector) gravity ***g*** however – with a gravimeter, or very sensitive balance - and from this infer gravitational potential.
- Thus, by rearranging (1), if we know **g**, we can deduce ΔW

  viz **g** = dW/ds

Vertical Reference Surfaces; geoids, quasigeoids

- Obviously the value of this gradient is dependent upon direction. If we measured T at two points of equal temperature then the gradient would be zero, or “an isotherm”
- And when the direction between the two points is normal to the above level surface, the gradient would be at a maximum.

Vertical Reference Surfaces; geoids, quasigeoids

- Similarly a *contour line* is drawn between two points of *equal height H*; no matter the distance between points along this line, the *slope* along it is *zero* because the ΔH between these points is zero.

Earth's Gravity and its Potential

- MAIN MESSAGE
- The reference surface we desire for elevations - or height datum - is one of constant gravitational; potential, or a Geop.
- The specific Geop for our datum is one which aligns with MSL – the Geoid
- A Network defined by Tide Gauges is at best, an approximation to the Geoid

Gravity and Geopotential; Geopotential Numbers

Sometimes, the subject of gravity can be too heavy for some people

Gravity and Geopotential; Geopotential Numbers

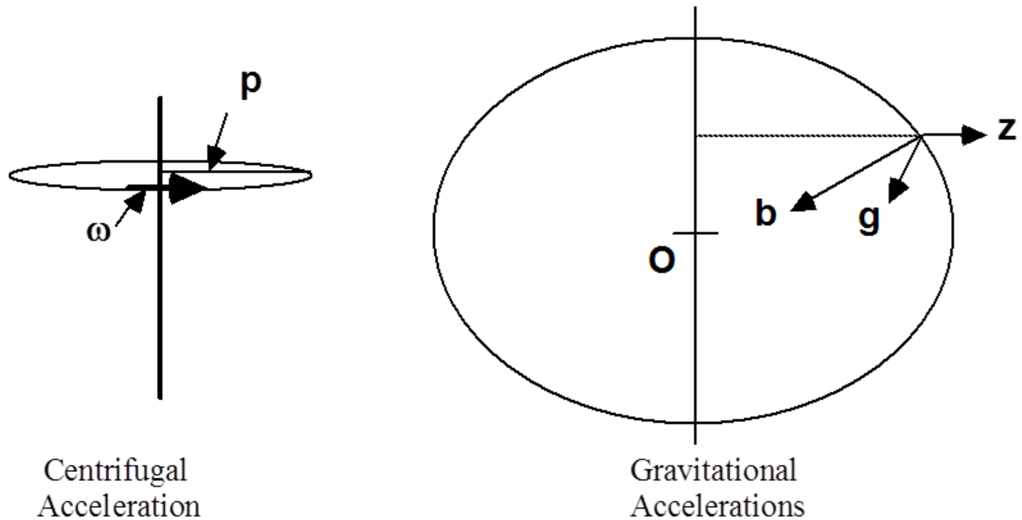
- However, ***we*** will proceed
- Now, further drill down into these concepts of scalar and vector quantities to the world of Gravity, and its associated Gravitational Potential, or for the Earth, Geopotential .
- As mentioned above, the Earth's gravity is a product of 2 forces – that a result of the Earth's Mass (**M**), and the other as a result of its rotational spin (ω)

Gravity and Geopotential; Geopotential Numbers

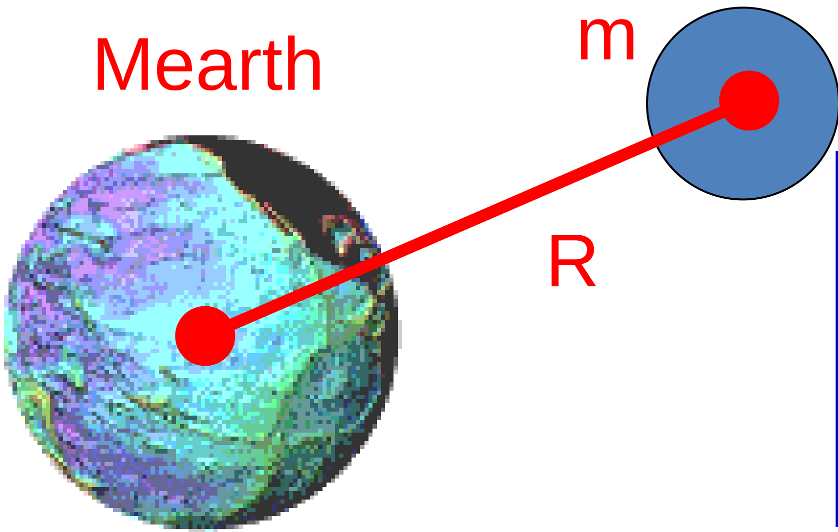
GRAVITATIONAL ACCELERATION + CENTRIFUGAL ACCELERATION = TOTAL ACCELERATION OF GRAVITY

b + **z** = **g**

Figure 2.1: Gravitational Acceleration Components



Newton’s Universal Law of Gravitation



$$\vec{F} = \frac{GM_{earth}m}{\vec{R}^2}$$

Where G = gravitational constant
$6.672 \times 10^{-11}\ m^3kg^{-1}\ s^{-2}$

Gravity and Geopotential; Geopotential Numbers

GRAVITATIONAL		CENTRIFUGAL	=	TOTAL ACCELERATION
ACCELERATION	+	ACCELERATION		OF GRAVITY
b	**+**	**z**	**=**	**g**

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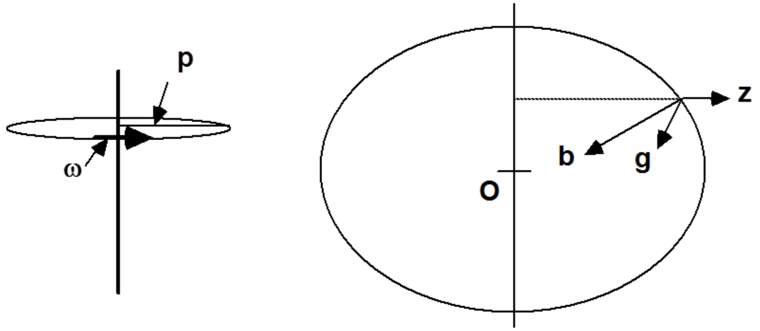
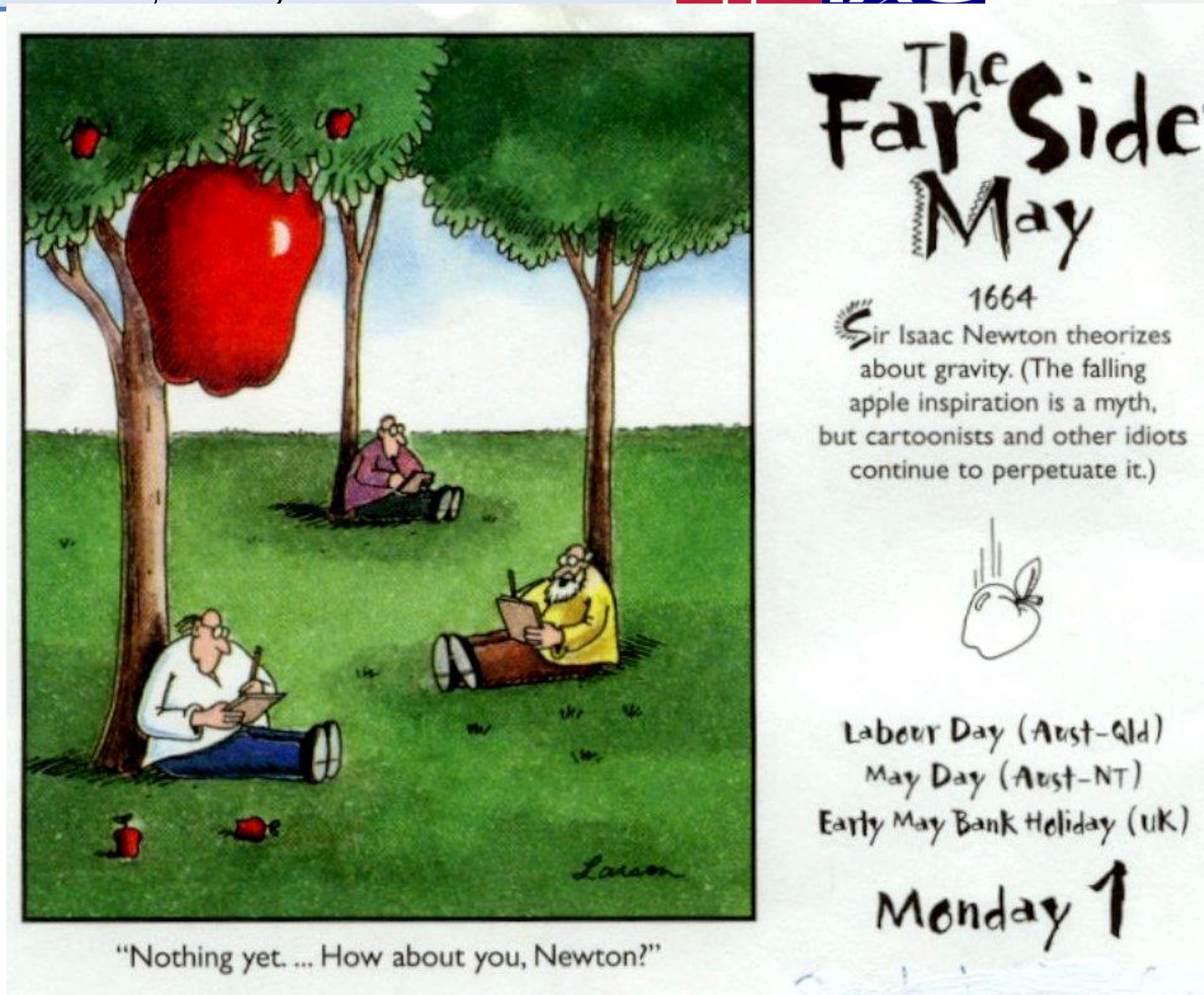


Note: a useful concept in this context is to think of
"**acceleration**" *as* "**force per unit mass**".


The Far Side®
May
1664
Sir Isaac Newton theorizes about gravity. (The falling apple inspiration is a myth, but cartoonists and other idiots continue to perpetuate it.)
Labour Day (Aust-Qld)
May Day (Aust-NT)
Early May Bank Holiday (UK)
Monday 1
Larson
"Nothing yet. ... How about you, Newton?"

Gravity and Geopotential; Geopotential Numbers

- A surface of constant gravity potential is called an **equipotential, level or geopotential** (geop) **surface**
- It is defined by W = W (<u>r</u>) = constant *(think isotherm)*
- The potential difference,
- dW = **<u>g</u>** • d<u>s</u> = g ds cos (**<u>g</u>**, ds)
- WHERE

ds =the element of distance “s” and its direction; ie a vector

(**g**, ds) = the angle between to direction of gravity and the direction of the line element

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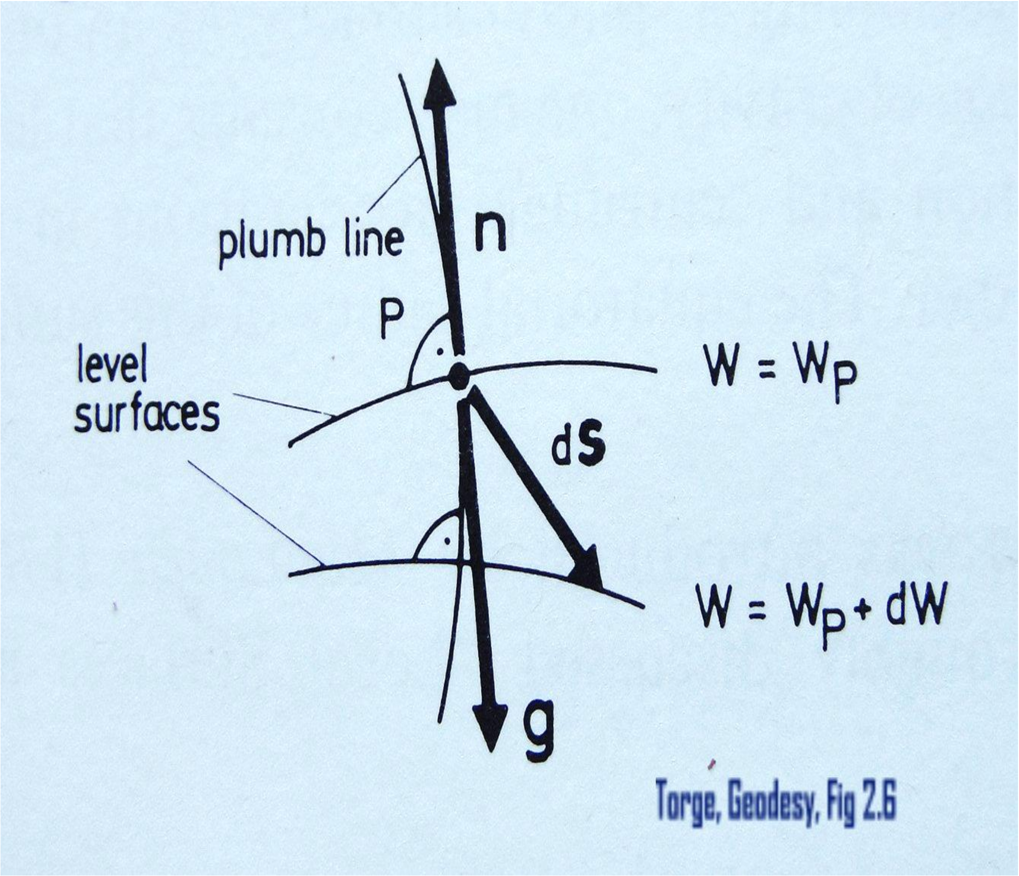


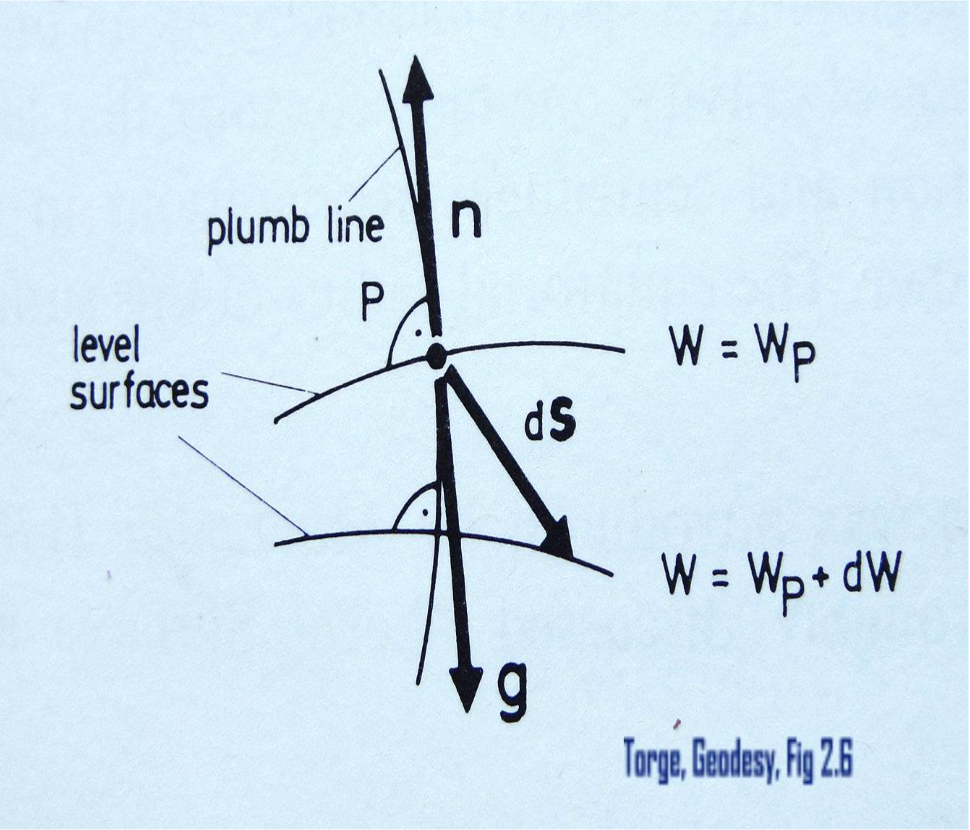
Gravity and Geopotential; Geopotential Numbers

If d$\underline{s}$ is taken along a ***level*** surface W = a constant (by definition),

Then dW = 0

& **g** is normal to W = Wp (as cos (**g**, ds) = 0)

Thus level surfaces are normal to plumblines - (the direction of gravity)

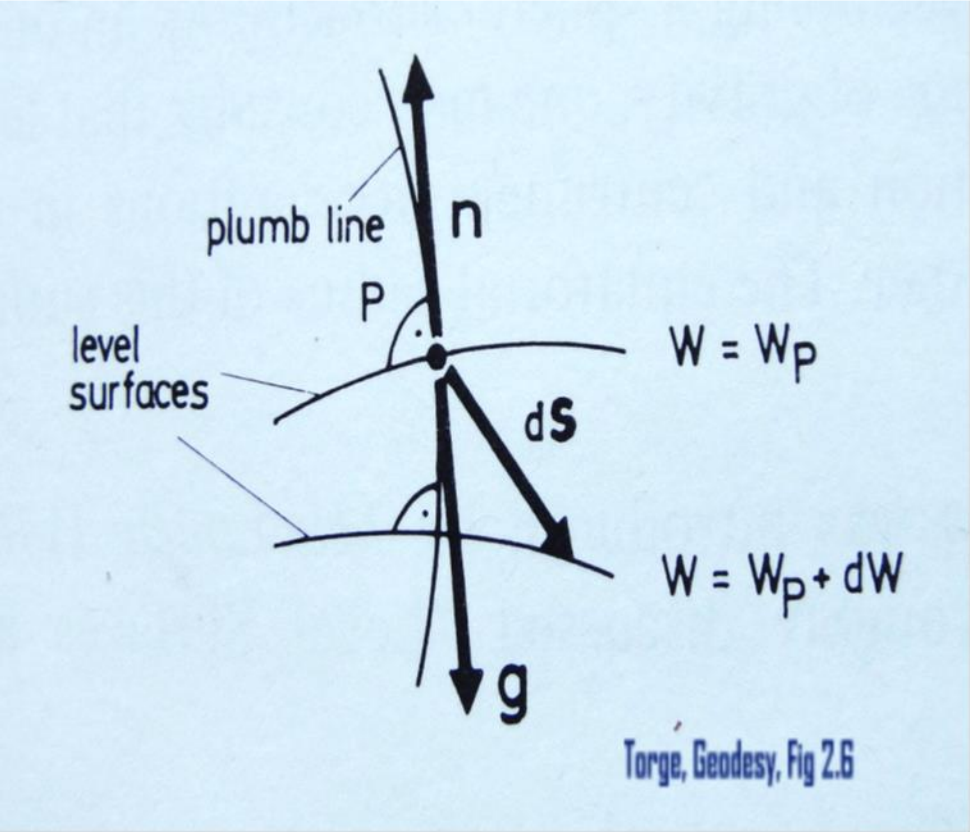


Gravity and Geopotential; Geopotential Numbers

If d$\underline{s}$ is along outer normal, $\underline{n}$, cos($\underline{\mathbf{g}}$, ds) = -1, (as cos -π= -1)

Then dW = - g dn

providing the **link between potential difference** (a physical quantity) **and differences in height** (a geometric quantity).

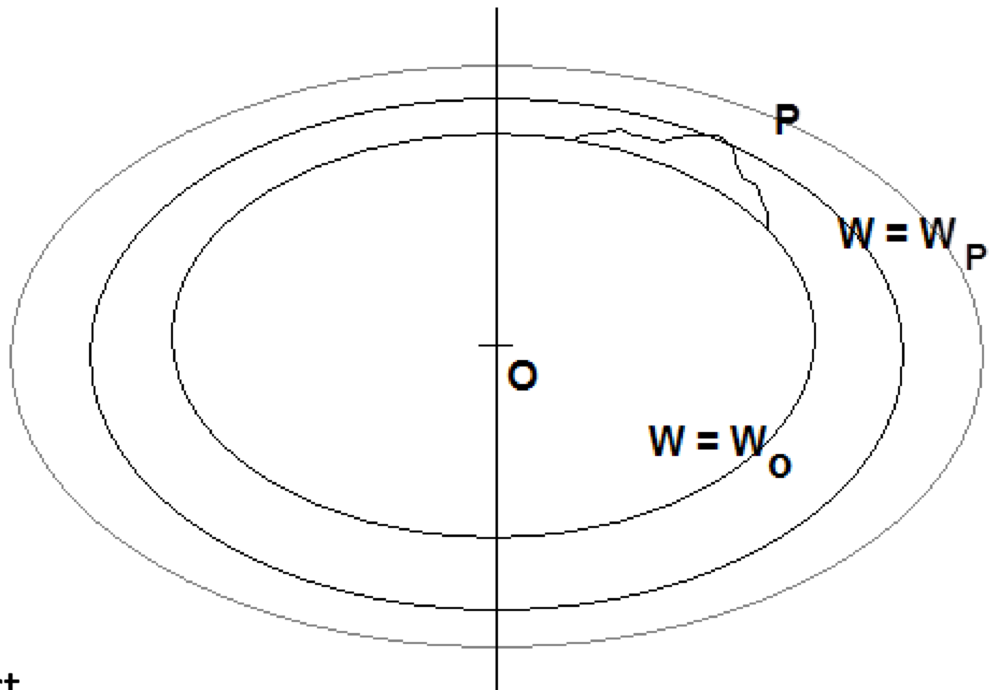


Gravity and Geopotential; Geopotential Numbers

We see that if we move along a level surface W= constant, no work is done.

***NB*. g *can vary on a level surface*.** That is, the level surfaces are not parallel and thus plumb lines curve in space (see figure). Because **g** increases 0.05 ms-2 from the equator to the poles, level surfaces converge towards the poles.

Thus, two level surfaces, 100 m apart at equator, will be 99.5 m apart at poles



Gravity and Geopotential; Ellipsoidal Model for the Earth's gravity field

b'   +   z̲   =   γ̲

A very useful approximation to the Earth's equipotential field is the ellipsoid. By ascribing to it the Earth's rotation, ω, and its mass, M, we can generate a model gravity field, γ, and associated gravitational potential, U.

*By comparing these "γ" values with the observed "**g**" values, we can compute the variations of the real equipotential field, W, from the model (or normal) gravity field, U*

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- On Geodetic Reference System 1980, (Torge, 1989, p. 51) where

$$a = 6\,278\,137\text{m}, \quad f^{-1} = 298.2572...$$

$$\gamma = 978032.7\,(1 + 0.005\,3024 \sin^2\phi - 5.8e^{-6} \sin^2 2\phi)$$

where γ is in mGal

Gravity and Geopotential; Ellipsoidal Model for the Earth's gravity field

$$\gamma = 978032.678\,(1 + 0.005\,3024 \sin^2 \phi \; - \; 5.8e^{-6} \sin^2 2\phi)$$

- where γ is in mGal

Thus, to find the normal (i. e. model-generated) gravity in Christchurch,

$\phi = -43^{o}33'$, so

$\gamma = 980535.164$ mGal

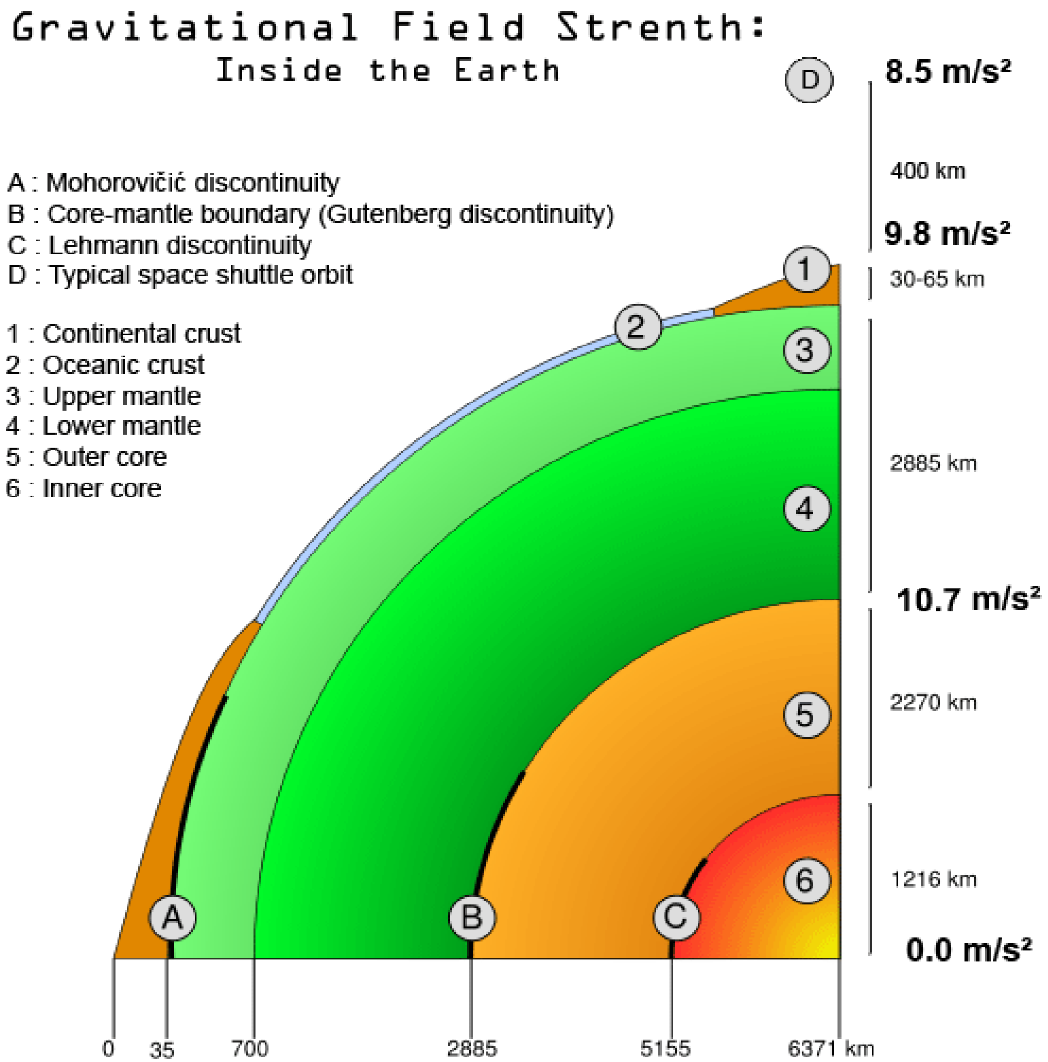
NB The ellipsoid is an ellipse rotated about the minor axis, so γ is latitude dependent only!

Gravity and Geopotential; Ellipsoidal Model for the Earth's gravity field

$$\gamma = 978032.7\,(1 + 0.005\,3024 \sin^2\phi - 5.8e^{-6} \sin^2 2\phi)$$

- where γ is in mGal
- ***g** and* γ are about 978000 mGal at the equator, and 983000 mGal at the poles
- i.e., there is an increase of about 5000 mGal from the equator to the pole – pretty big change!

Variation of Gravity in Earth Space

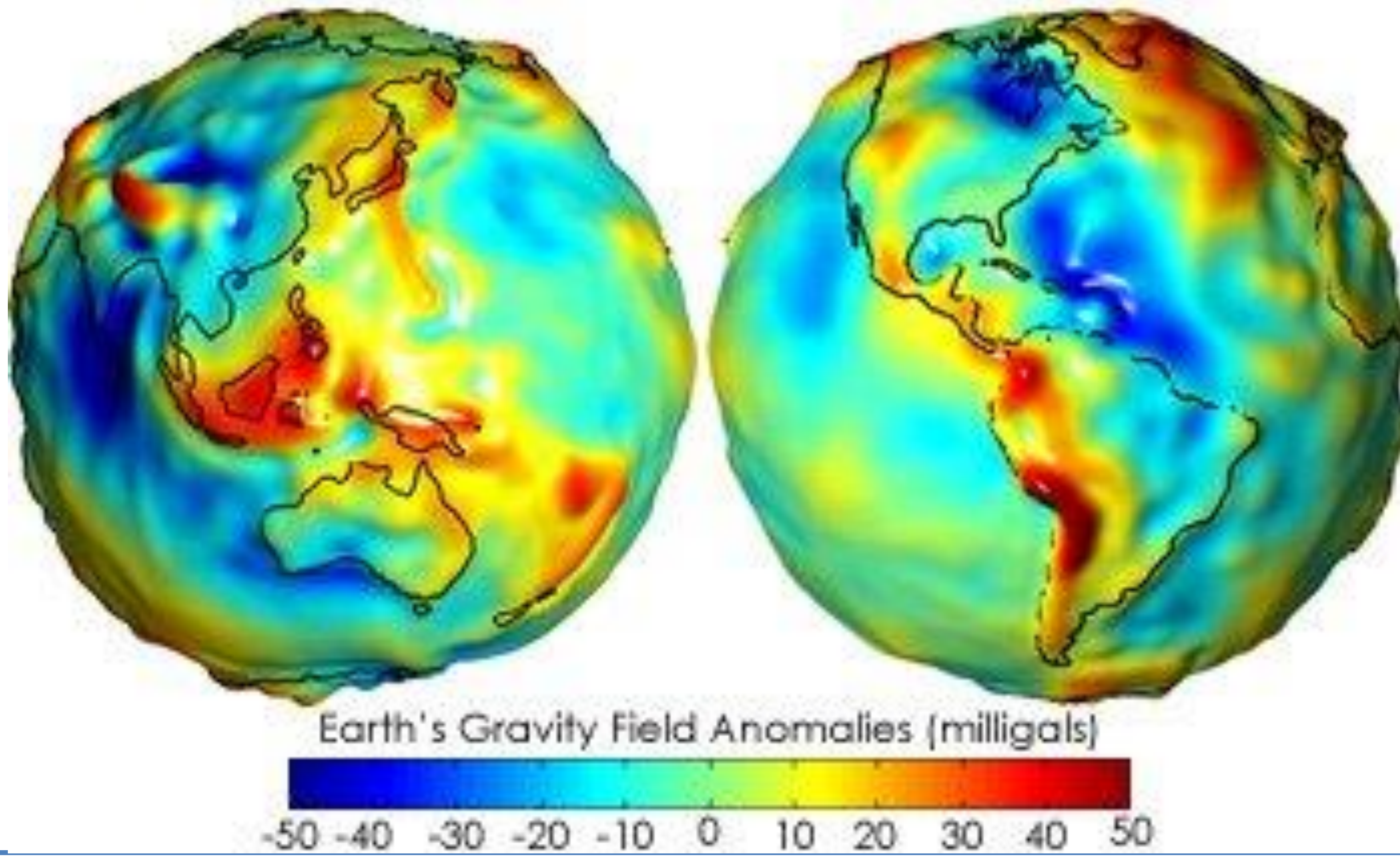


Gravity and Geopotential; Ellipsoidal Model for the Earth's gravity field

A note about units used in Gravity

- Gravity is an acceleration, & is thus measured in m/s^2 or "metres per second per second"
- In Gravimetric Geodesy, we use the term "gal" (after Galileo), where
- 1 gal = 1 m/s^2; 1 milligal is 10^{-3} gals; 1 kilogal is 10^3 gal etc

Departures of Real Earth Gravity from gravity computed from the Ellipsoidal Model



Gravity and Geopotential; Ellipsoidal Model for the Earth's gravity field

Note here, globally, there is only a range of 100 mGal (-50 to + 50mGal) when the ellipsoidal model is used as a reference to compare with model-generated gravity actual gravity.

This makes the computations for geoid solutions a lot more manageable!

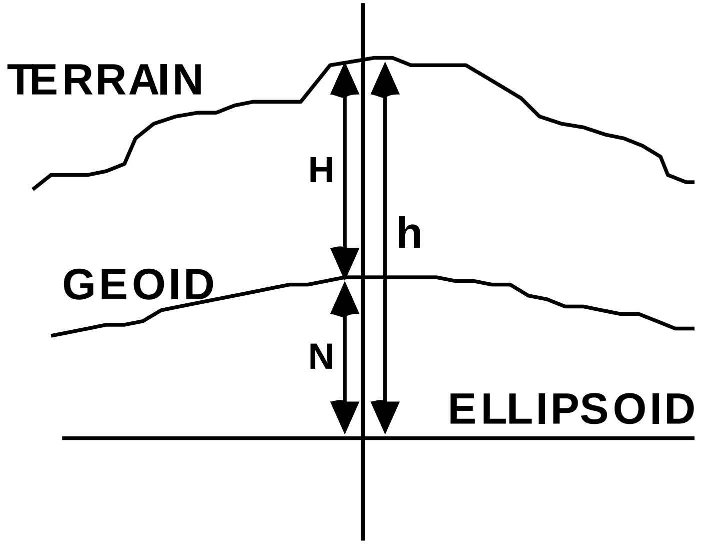
Gravity and Geopotential; Ellipsoidal Model for the Earth's gravity field

Real Earth

Earth's surface: land masses & ocean masses
Subtract topography
¬
Geoid

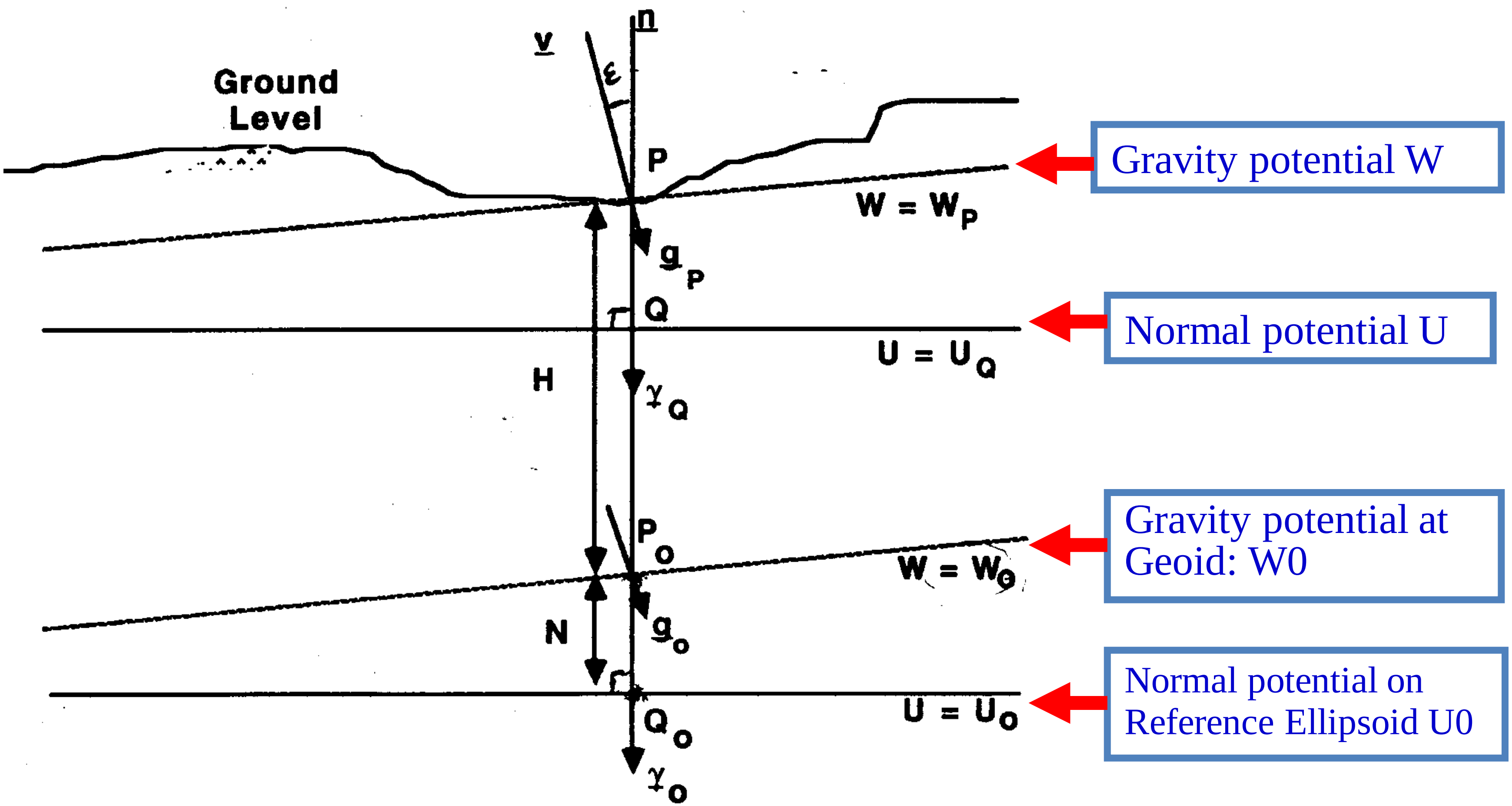
which is an equipotential (level) surface and the reference for terrestrial observations and datum for heights

Model Earth



The geoid is modelled by the ellipsoid, with a, f, - (geometry) and GM, ω - (physical properties)

Figure of Equipotential



Gravity and Geopotential; Ellipsoidal Model for the Earth's gravity field

Real Earth		Model Earth	
Gravity potential or geopotential	W	Normal or model potential	U
Geopotential at geoid	- W_0	Normal potential on the reference ellipsoid	- U_0
		$U_0 = \text{fn}(a, f, GM, \omega)$	
NOTE:	$U_0 = W_0$	*DESIRABLE*	
Observed / Terrestrial	gravity g	Normal / Model	gravity γ
$g = \frac{dw}{dh}$		$\gamma = \frac{dU}{dh} = \text{fn}(a, f, GM, \omega)$	

$$T = W - U \quad \text{(disturbing potential)}$$

$$\Delta g = \text{fn}(g, \gamma, \varphi, H) = g_{P_0} - \gamma_{Q_0}$$

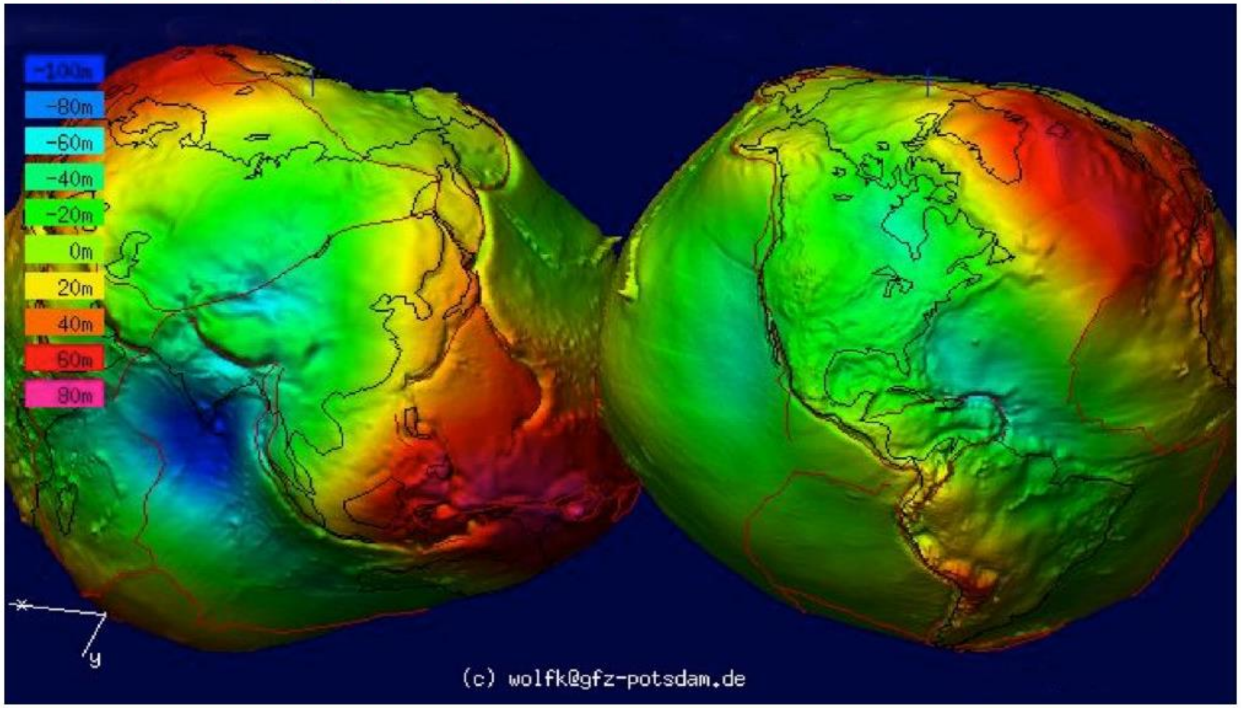
$$N = \frac{T}{\gamma} = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g \, S(\psi) \, d\sigma$$

Gravity and Geopotential;

Ellipsoidal Model for the Earth's gravity field

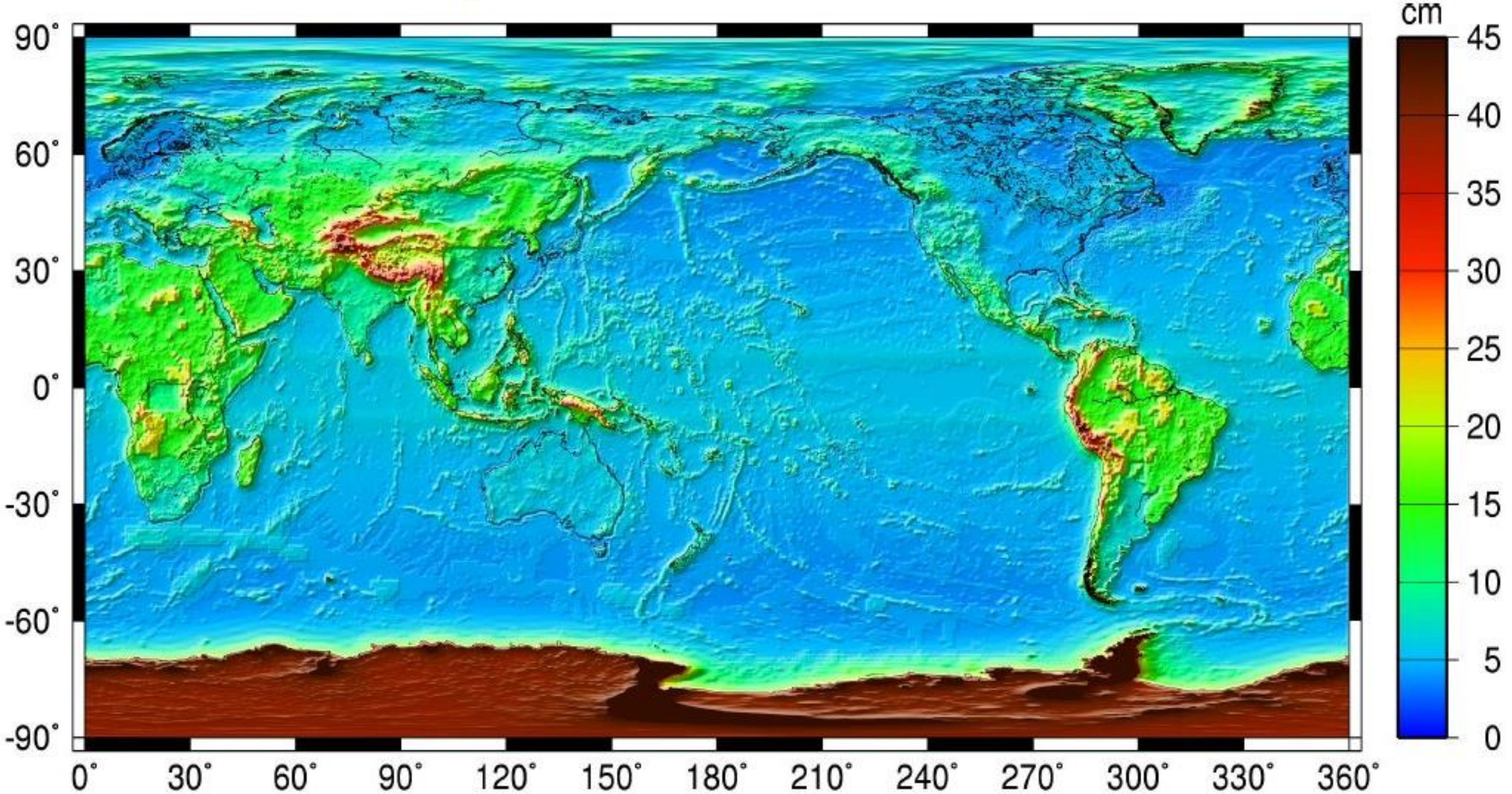
T = the differences between W and U can be converted to a linear value to provide the Geoid Heights – or Geoid undulations N

◆ **What does the geoid look like?**



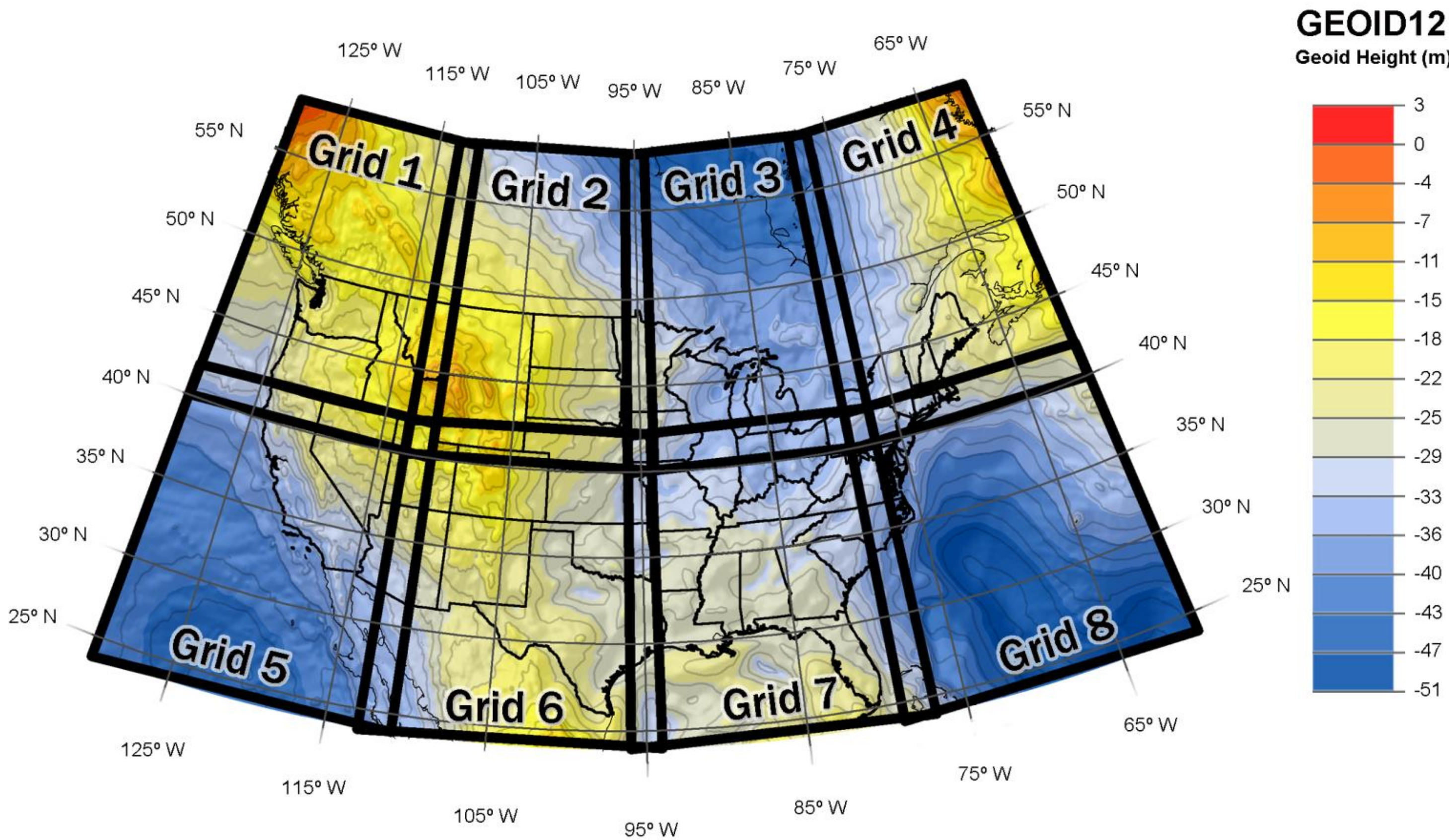
Gravity and Geopotential; Ellipsoidal Model for the Earth's gravity field

❖ EGM2008 accuracy

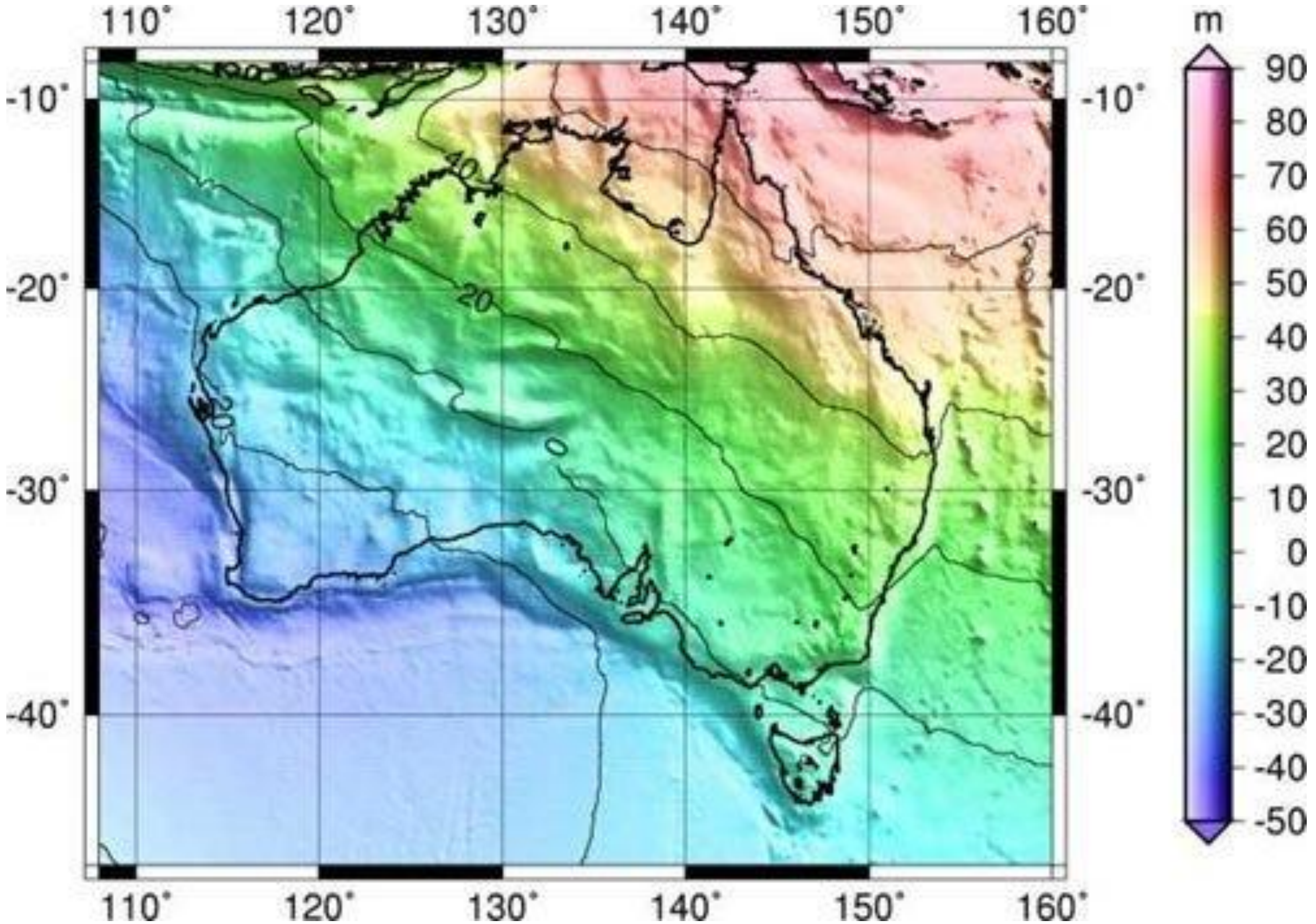


credit: N.Pavlis via M Sideris

The Geoid across Coterminous USA



The Geoid across Australia



Height Systems

- There have been a variety of techniques developed to transform our measured "H" (or "DH)" value to something which reflects geopotential/ geopotential difference. These can be generally defined by converting C_P, the geopotential number at point P, to a linear height, by scaling it by some estimate of the gravity vector.
- The type of resulting height is determined by which estimate of the Gravity vector is used in this scaling process.

Height Systems

- *DYNAMIC HEIGHT*
- As we have just shown, the height as determined by levelling (very often, but not strictly correctly, termed "Orthometric Height"), is not in itself a measure of geopotential – or by extension, geopotential difference.
- The only height system that guarantees flow of unrestricted fluid from a greater to a lower height, and has units of length, are "dynamic heights" (H&M, 1967, p 163), where
- $H^D = C/\gamma_0$
- and γ_0 is normal gravity at some standard latitude, usually 45^0.

Height Systems

- *ORTHOMETRIC HEIGHT*
- To be rigorously determined, orthometric height needs g_{mean} - the mean value of gravity along the plumbline between the subject point and its projection onto the geoid.
- That is
- $H^o = C/g_{mean}$
- Often g at point P is not observed, and it is not possible to measure "g" under the surface, so some modelling of gravity, or some alternative to estimating its value, needs to be found.

Height Systems

- *NORMAL-ORTHOMETRIC or NORMAL HEIGHT*
- To avoid the need for a value of g_{mean}, another estimate of mean gravity along the plumbline was invented (Molodensky et al, 1962) which used γ_{mean}, the mean of the ***normal*** gravity along the plumbline to scale C.
- Such values are called “normal-orthometric” heights.
- Thus
- $H^{O\text{-}N} = C/\gamma_{mean}$

Height Systems

- In Summary
- Height H = C/G, where
- C = geopotential number;
  C's units are in kgal metres (1 gpu = 10 $m.ms^{-2}$ or 10 m^2s^{-2}).
- The C value of a point is very similar to its H value above MSL (C = 0.98H; V&K, p. 366)

SUMMARY

- dynamic height; $G = \gamma_0 =$ constant
- orthometric height, $G = g_{mean}$
- normal height: $G = \gamma_{mean}$

Height Systems

- In Summary
- Height H = C/G, where
- C = geopotential number;

G is chosen according to which height system is adopted for the network, eg

- dynamic height; $G = \gamma_0 =$ constant
- orthometric height, $G = g_{mean}$
- Normal-orthometric height: $G = \gamma_{mean}$

C's units are kgal metres. The C value of a point is very similar to its H value above MSL (C = 0.98H; V&K, p. 366)

Height Systems

- In Summary
- Height H = C/G, where
- C = geopotential number;

G is chosen according to which height system is adopted for the network, eg

- A number of researchers have proposed their own method to establish **G**, and there is a nice review and comparison of these in Filmer, M. S., 2010, "An examination of the AHD", PhD Thesis, Curtin U, WA.

Height Systems

- dynamic height; $G = \gamma_0 =$ constant
- orthometric height, $G = g_{mean}$
- normal-orthometric height: $G = \gamma_{mean}$
- *Normal Orthometric heights*: New Zealand, Australia,
- *Normal Heights*: European Vertical Network (geopotential numbers); SIRGAS *(www.sirgas.org/index.php?id=56)*

- A very nice study comparing the impact using different height systems on the ANLN can be found in Filmer, M., Featherstone, W. and Kuhn, M. (2010), *"The effect of EGM2008-based normal, normal-orthometric and Helmert orthometric Height Systems on the ANLN"*, j. Geod, 84: pp. 501-513