A Method of Building the Local Geoid Model in Vietnam

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Key words: GPS, Geoid, Vietnam, Leveling, Height.

SUMMARY

GPS points which were measured height by leveling (GPS/leveling points) are used to determine geoid undulation, following that, interpolating geoid undulation of another points to determine height by GPS. When using the global geoid model to determine geoid undulation, measured data at GPS/leveling points are used to assess the suitability of the geoid model with real surface geoid of measured area, not only that, these data can also be used to correct geoid model in the area exists GPS/leveling points. This paper presents a method to build local geoid models based on correction for the global geoid model from relative model of measurements at GPS/leveling points. The local geoid model GeoCP_MD.GGF has been set up to identify leveling in the area Cam Pha – Mong Duong with accuracy is higher than leveling values determined by the global geoid model EGM2008.

SUMMARY

Các điểm trong mạng lưới GPS có đo nối độ cao bằng thuỷ chuẩn hình học (các điểm song trùng) được sử dụng để xác định độ cao geoid (Undulation), từ đó nội suy độ cao geoid cho các điểm khác phục vụ chuyển độ cao bằng GPS. Khi sử dụng mô hình geoid toàn cầu để xác định độ cao geoid, số liệu đo tại các điểm song trùng được sử dụng để đánh giá mức độ phù hợp của mô hình geoid với mặt geoid thực tế tại khu đo, không những thế, các số liệu này còn có thể sử dụng để chính xác hoá mô hình geoid tại khu vực có các điểm song trùng. Bài báo này trình bày một phương pháp xây dựng mô hình geoid cục bộ dựa trên việc cải chính cho mô hình Geoid toàn cầu theo mô hình tương đối của trị đo tại các điểm song trùng. Mô hình Geoid cục bộ GeoCP_MD.GGF mới được lập cho phép xác định độ cao thủy chuẩn tại khu vực Cẩm Phả – Mông Dương có độ chính xác cao hơn so với giá trị độ cao thủy chuẩn xác định bằng mô hình Geoid toàn cầu EGM-2008.

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1. INTRODUCTION

GPS measurements help to determine the ellipsoidal height (height above a reference ellipsoid) with high accuracy, but this is only a theoretic system, in fact, we need to use the leveling height values (height above the geoid). Therefore, the question is need to transform the ellipsoidal height to leveling height. We can easily exploit the global geoid model, for example: OSU91A, EGM96 and EGM2008 from the GPS processing software to determine geoid undulation (N) for the conversion stage the ellipsoidal height (H) to leveling height (h) for points. If the GPS network has several points have been determined by leveling method with needed accuracy, we will have a basis for assessing the suitability of the geoid model used to the real geoid surface of the network area. Practising experience have shown that the global geoid models have low accuracy over Vietnam area, only ensures determining the leveling high with precision is commonly equivalent to technical leveling, in some case can reach the 4^{th} order leveling standard (standard of the 4^{th} order leveling network) [3,6], which is mainly for the plains and midlands, and more importantly can not predict certainty before implementing measurement. That is the limitation of global geoid model. At issue is need to have an accurate geoid model.

Currently, in Vietnam they have not built the precise geoid model for the whole country yet, so the building of local geoid model for each small area is necessary. If the GPS network has several GPS/leveling points are determined by leveling high with accuracy needed, we not only use GPS/leveling points to evaluate the accuracy of GPS leveling or add extra correction (interpolated) to improve the accuracy of GPS leveling in the current network, but also use them to upgrade the global geoid model over GPS network area by using them to determine the correction for geoid undulation (from the global geoid model) [2,5]. Geoid undulation values after correcting can be used to establish the local geoid model over GPS network area, we call this is correcting geoid model. Corrected geoid model will serve better for GPS leveling on this area in the next time.

2. THEORETICAL BASIS

2.1 The algorithm for determining the correction for geoid model by difference values

If points of GPS network have height determined by leveling, we will have the facility to determine the geoid undulation at points of GPS network by the formula:

$$N_i = H_i - h_i \quad (1)$$

Where H_i is the ellipsoidal height, h_i is the leveling height, N_i is the geoid undulation.

Geoid height is determined by equation (1) is called GPS-leveling geoid undulation. Characteristics of geoid undulation determined by equation (1) are that its value depends on the processing of GPS network and leveling networks, mainly depend on the original data in that network. If the GPS network has appropriate density, from geoid undulation values

determined by equation (1) we can build local geoid model over region covered by the GPS network.

From the global geoid model, we can get the geoid undulation at points of GPS network, geoid undulation is denoted by N_i^m. Make sure, geoid undulation N_i calculated in equation (1) and geoid undulation from the global geoid model are different.

If the GPS network was measured carefully and the leveling network was measured with high accuracy, the value N_i was calculated by equation (1) with accuracy is higher than the value obtained from the global geoid model. From these data, we can calculate to determine the correction to geoid undulation N_i^m at the right of the GPS network to improve the accuracy of global geoid model on the areas of GPS/leveling points [7,8]. In fact, we do not need to use entire global geoid model, that only need to draw from the global model a particular geoid model for a region, such as Vietnam region. When adjustment to correct it only performed for this particular geoid model. The determination of the correction on global geoid model based on measured GPS-leveling data is called geoid model matching method; matching methods can be applied for the geoid determined by gravity data [7].

To not consider the initial value the ellipsoidal height H and the leveling height h when adjust geoid model, principles of correction calculations for geoid model are only based on "the difference values" between pairs of points i, j. There are three types of difference set include:

- The levelling height difference: $\Delta h_{i,j} = h_j - h_i$
- The ellipsoidal height difference: $\Delta H_{i,j} = H_j - H_i$
- The geoid height difference determined from the global geoid model: $\Delta N_{i,j}^m = N_j^m - N_i^m$

From equation (1), we determine the geoid height difference GPS-leveling for two points i, j, as follows:

$$\Delta N_{i,j} = \Delta H_{i,j} - \Delta h_{i,j} \quad (2)$$

We set the adjustment value equation:

$$\Delta N_{i,j} + V_{i,j} = (N_j^m + dN_j) - (N_i^m + dN_i) \quad (3)$$

Where dN_i, dN_j is the correction to geoid undulation at point i and point j (called correction to geoid model), which take a role as unknowns in this problem.

In equation (3), geoid undulation is determined from geoid model serves as an approximate value of the unknowns.

From (3) have correction equation:

$$\boldsymbol{V_{i,j} = dN_j - dN_i + l_{i,j}} \quad (4)$$

Where: $$l_{i,j} = N_j^m - N_i^m - \Delta N_{i,j} = \Delta N_{i,j}^m - \Delta N_{i,j} \quad (5)$$

A set of correction equations (4) will be under conditions $V^TPV = \min$, which have the standard equations:

$$A^T PA.X + A^T PL = 0 \quad (6)$$

Where: A is the coefficient matrix of correction equations, including the coefficients of equation (4), P is the weighting matrix, X is the vector of unknowns, including the correction (dN) for geoid model, L is the vector of the free rank, with elements defined by equation (5).

Weighting of the measured value is determined by the formula:

$$P_i = \frac{1}{L_i} \quad (7)$$

Where L_i is the length of baseline is measured by GPS.

Do not consider geoid undulation of any point is the original, so that the coefficient matrix of the standard equation (6) is a singular matrix with defect d = 1. To solve the standard equation, we apply the method of adjustment of free networks.

$$X = -(A^T PA)^{\sim} A^T PL \quad (8)$$

Where $(A^T PA)^{\sim}$ is a pseudo-inverse matrix of coefficient matrix of the standard equation.

There are several methods to find pseudo-inverse matrix such as Moore–Penrose's method, Helmert–Wolf's method, method of adding extra conditions [4].

According to [2], adjusted geoid model at GPS/leveling points with the unusual variation which is due to uncorrected at the grid points (not smoothing). So to make the model be usable, the question is required a suitable interpolation algorithm to determine the correction at the grid points to make sure the principle remain corrections at the GPS/leveling points and the further grid points are located from the GPS/leveling points, the lesser absolute magnitude of the corrections are.

2.2 The algorithm of interpolation correction for geoid undulation at grid points

The interpolation algorithm in inverse distance weighting is appropriate to determine the model correction at grid points. Under this method, weighting is obtained by inverting distance from point needs determining geoid undulation values to the points have already had geoid undulation values around it. Geoid undulation value at point i is determined by the following expression:

$$N_i = \frac{\sum_{j=1}^{n} N_j P_j}{\sum_{j=1}^{n} P_j} \quad (9)$$

In which, the relations are given to calculate the geoid undulation point i as related to the surrounding reference points j = 1, 2, 3, ..., n.

Weighting is calculated using the formula:

$$P_j = \frac{1}{d_j^{\alpha}} \quad (10)$$

d_j = distance between interpolation point i and j^{th} reference point in kilometer.

α = power of the distance, it can be 1, 2, 3 or 4. In the case study, α has chosen 2 empirically.

3. THE EXPERIMENTAL CALCULATIONS

Experimental data is GPS network with accuracy the 4^{th} order GPS standard was measured on the field of Cam Pha – Mong Duong, Quang Ninh province. Network of 26 points with 24 baselines, which have nine GPS/leveling points (leveling measurements with

accuracy the 4th order leveling standard). Combination charts of nine GPS/leveling point links are shown in figure 1.

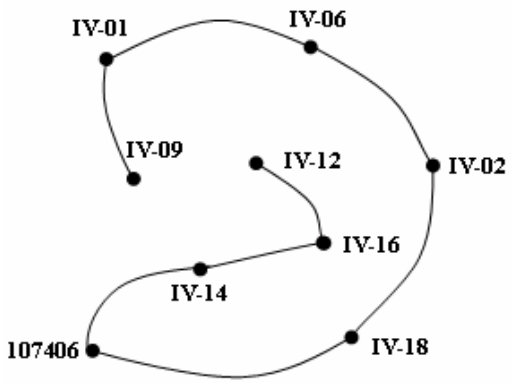


Figure 1. Combination chart of 9 GPS/leveling points

3.1 Calculating the model correction at the GPS/leveling points

With actual data of the network above and geoid model EGM2008, according to the formula (2) and (5), calculate the ellipsoidal height difference ΔH, the leveling height difference Δh, the geoid height difference ΔN^m, then calculate the free rank of the correction equation, the results calculated in table 1.

Table 1. The calculated values ΔH, Δh, ΔN^m , and the free rank l

Name of the high difference	ΔH (m)	Δh (m)	ΔN^m (m)	$L = \Delta h + \Delta N^m - \Delta H$
IV-01→IV-06	-6.824	-6.921	0.077	-0.020
IV-06→IV-02	-1.002	-1.040	0.083	0.045
IV-02→IV-18	-18.693	-18.711	0.008	-0.010
IV-18→107406	-0.956	-0.783	-0.164	0.009
107406→IV-14	45.607	45.515	0.065	-0.027
IV-14→IV-16	55.118	55.130	0.063	0.075
IV-16→IV-12	38.556	38.453	-0.045	-0.148
IV-09→IV-01	-72.830	-72.715	-0.029	0.086

After making and solving a system of standard equations (6), will determine the value of the unknowns, they are corrections of geoid undulation (dN) at the GPS/leveling points. Since then, we will get the geoid height adjustment.

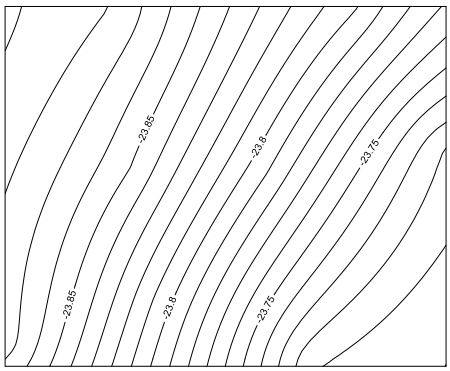
Table 2. The model corrections and the geoid undulation adjustment

Name of points	Geoid undulation from the global geoid models (m)	Correction dN (m)	Geoid undulation adjustment (m)
IV-01	-23.879	-0.005	-23.884
IV-06	-23.802	0.015	-23.787
IV-02	-23.719	-0.030	-23.749
IV-18	-23.711	-0.020	-23.731
107406	-23.875	-0.029	-23.904
IV-14	-23.810	-0.002	-23.812
IV-16	-23.747	-0.077	-23.824

IV-12	-23.792	0.071	-23.721
IV-09	-23.850	0.081	-23.769

After calculating the corrected geoid undulation at the GPS/leveling points can render geoid model for Cam Pha – Mong Duong area. To be able to visually see the change of the geoid surface after adjustment, there is combined the uncorrected geoid model also.

In figure 2a and 2b are charts in 2D and 3D of the geoid model EGM2008 of the Cam Pha - Mong Duong area without adjustment.



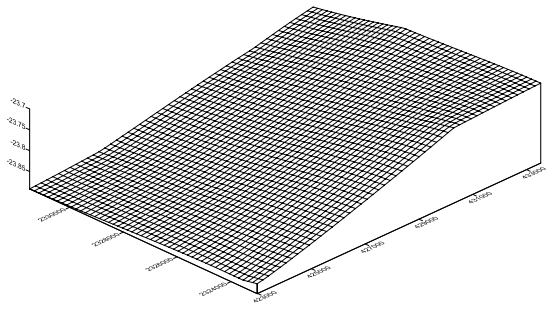
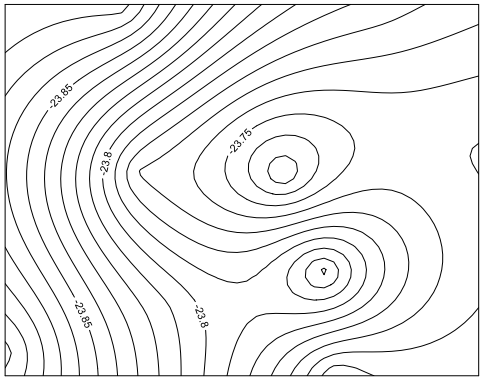


Figure 2a. Chart in 2D without adjustment Figure 2b. Chart in 3D without adjustment

In figure 3a and 3b is chart in 2D and 3D of the geoid model has been adjusted.



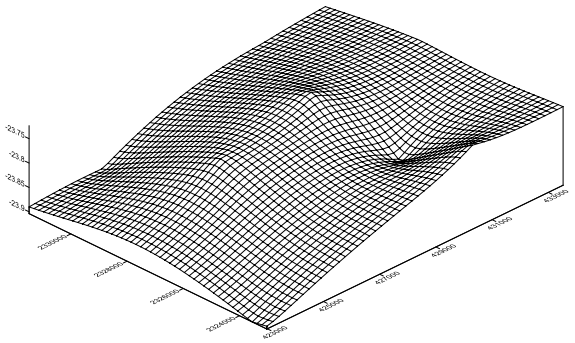
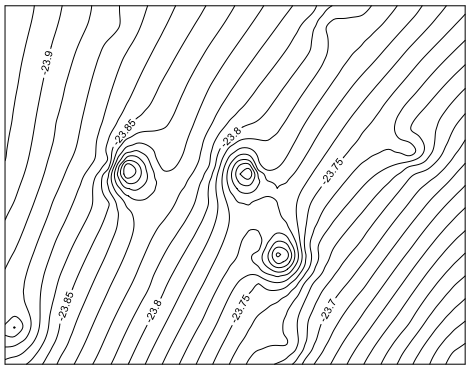


Figure 3a. Chart in 2D corrected Figure 3b. Chart in 3D corrected

In addition, we can also use combination data of grid points in the region to draw contour with 10 × 10 grid. Geoid undulation data at the grid points are taken from the global geoid model EGM2008.



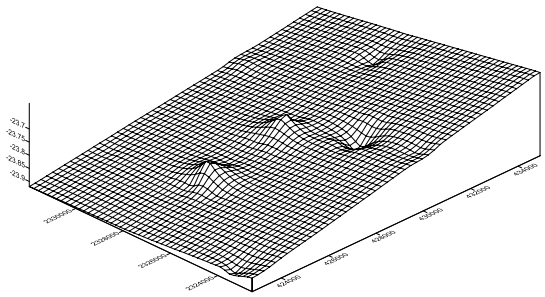


Figure 4a. Chart in 2D uncorrected grid Figure 4b. Chart in 3D uncorrected grid

It could be shown that, after data processing, geoid undulation at the GPS/leveling points received correction, hence, the geoid surface is altered matching GPS measurements and leveling measurements. However, the model is only changed suddenly at the GPS/leveling points, but the grid points have not changed. Therefore requires interpolation method to determine the correction factor for the grid points, this method is called smoothing geoid model after adjusting.

3.2 Interpolating geoid undulation correction at the grid points

Using geoid undulation correction at the GPS/leveling points conduct interpolation geoid undulation correction for the grid points by inverse distance weighting interpolation methods with coefficient $\alpha = 2$. Geoid undulation correction and geoid undulation after correction at the grid points is shown in the following table:

Table 3. Model correction and geoid undulation after correction of grid points

Name of points	Geoid undulation from the global geoid models (m)	Correction dN (m)	Geoid undulation adjustment (m)
1	-23.872	-0.025	-23.897
2	-23.843	-0.018	-23.861
3	-23.815	-0.006	-23.821
4	-23.785	-0.005	-23.790
5	-23.755	-0.012	-23.767
6	-23.725	-0.019	-23.744
…	…	…	…
95	-23.847	0.012	-23.835
96	-23.829	0.014	-23.815
97	-23.811	0.015	-23.796
98	-23.791	0.012	-23.779
99	-23.771	0.004	-23.767
100	-23.750	-0.001	-23.751

Using corrected geoid height at the grid points to draw a geoid model for Cam Pha – Mong Duong area.

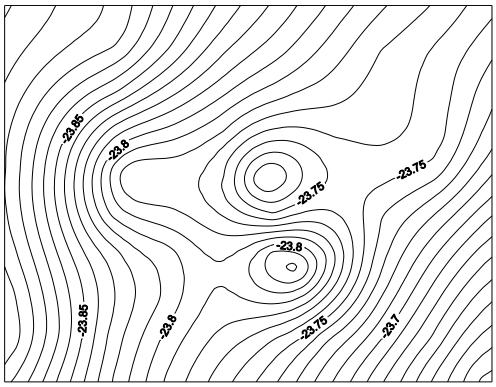


Figure 5a. Chart 2D grid corrected

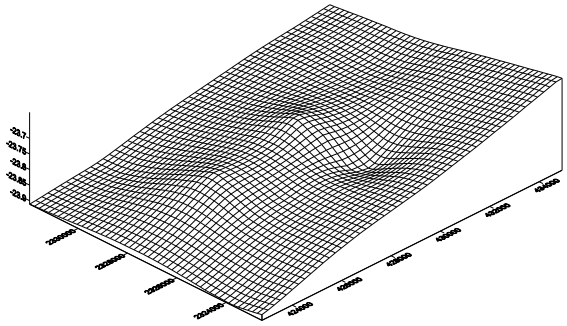


Figure 5b. Chart 3D grid corrected

It could be shown that the geoid model after adjustment at the grid points becoming smoother.

3.3 Building local geoid model

Base on geoid undulation values after adjusting geoid undulation of the grid points to build a local model for Cam Pha – Mong Duong area in the format of geoid models in GPS data processing software (GPSurvey, Trimble Geomatics Office, Trimble Total Control...). The built model is called GeoCP_MD.GGF.

Using this model to determine geoid undulation values of 9 points GPS/leveling.

Table 4. Results of geoid undulation is determined from the model GeoCP_MD.GGF

Name of points	Geoid undulation from the global geoid models (m)	Geoid undulation derived from model GeoCP_MD.GGF (m)
IV-01	-23.879	-23.884
IV-06	-23.802	-23.787
IV-02	-23.719	-23.749
IV-18	-23.711	-23.731
107406	-23.875	-23.904
IV-14	-23.810	-23.812
IV-16	-23.747	-23.824
IV-12	-23.792	-23.721
IV-09	-23.850	-23.769

Performing comparison between leveling height determined from GPS leveling used the global geoid model (h^{GPS}) and leveling height determined from geoid model GeoCP_MD.GGF (h^{GeoCP_MD}) with leveling height determined from leveling after adjustment (h^{TC}).

Table 5. Comparison of results using geoid model GeoCP_MD.GGF

Name of points	h^{TC} (m)	h^{GPS} (m)	h^{GeoCP_MD} (m)	$\lvert h^{TC} - h^{GPS} \rvert$ (m)	$\lvert h^{TC} - h^{GeoCP\_MD} \rvert$ (m)
IV-01	29.906	29.924	29.903	0.018	0.003
IV-06	22.985	23.023	22.986	0.038	0.001
IV-02	21.945	21.939	21.943	0.007	0.002

IV-18	3.234	3.237	3.235	0.004	0.001
107406	2.451	2.445	2.447	0.006	0.003
IV-14	47.966	47.987	47.965	0.021	0.000
IV-16	103.096	103.041	103.069	0.055	0.027
IV-12	141.549	141.642	141.564	0.093	0.015
IV-09	102.621	102.725	102.634	0.104	0.013

After researching theory and conducting experiments calculations on practical data, we give some conclusions and proposals as follow:

4. CONCLUSIONS AND PROPOSALS

- Data is used to calculate is the height difference (ellipsoidal height difference, leveling height difference and geoid height difference determined from the global geoid model). Geoid height differences adjustment of any pair of GPS/leveling point performance right in geoid height difference determined by GPS-leveling method.
- The local geoid model GeoCP_MD.GGF has been set up to identify leveling in the area Cam Pha – Mong Duong with accuracy is higher than leveling values determined by the global geoid model EGM2008. Thereby, it enhances the ability to leveling in GPS measurements in the region in the next stage.
- Accuracy of the method depends on the accuracy of the determination of the ellipsoidal height by GPS method and leveling height by leveling method at the GPS/leveling points.
- By applying the adjustment method of free network, the total squared of the geoid undulation correction to satisfy the condition [dNdN] = min, this means that the GPS and leveling measurements least change the existing geoid model.
- This is just an experiment in small areas. To be more precise conclusions should apply this method to establish the geoid model for other areas of experimental wider sizes.
- If there is data measured GPS-leveling we can apply this method to establish the local geoid model accuracy for Vietnam. Thereby, it helps to improve the applicability of the method of high GPS in Vietnam.

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