



**UNIVERSITY OF PRISHTINA**  
Faculty of Civil Engineering and Architecture

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**CALCULATION OF THE NATIONAL AREA  
OF THE REPUBLIC OF KOSOVA**



FIG WORKING WEEK 2012  
May 6–10 2012  
Rome, Italy

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"Calculation of the national area of the Republic of Kosova"  
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## Preface

- Area calculation methodology in Kosova:
  - Calculation of areas in flat area (map projection), even in a case of national area
- Area of Kosova in flat area (Kosovaref01): 10905km<sup>2</sup>
- Parameters of Kosovaref01:

Datum	ETRS89
Ellipsoid	GRS80
Map projection	Gaus-Kryger
Prime meridian	Greenwich
Central meridian	21°E
Scale factor	0.9999
False easting	7500000m
Prime parallel	Equator
False northing	0m
Origin of heights	Trieste – Molo Sartorio

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- Research was realized with:
  - ArcGIS software
  - Official data (national boundary produced by Kosova NMO)
  - Three methodologies for calculation of national area
  - Calculation of the value of area in referent ellipsoid GRS80
  - Within surveying department of University of Prishtina

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## Mathematical model for calculation

2.1.1 Calculation of trapezoid areas in reference ellipsoid

$$\overline{AB} = \overline{CD} = M d\varphi$$

$$\overline{AD} = \overline{BC} = r d\lambda A = N \cos \varphi d\lambda$$

$$dS = MN \cos \varphi d\varphi d\lambda = R^2 \cos \varphi d\varphi d\lambda$$

$$S = \int_{\lambda_2}^{\lambda_1} \int_{\varphi_2}^{\varphi_1} MN \cos \varphi d\varphi d\lambda = \int_{\lambda_2}^{\lambda_1} \int_{\varphi_2}^{\varphi_1} \frac{b^2 \cos \varphi}{(1 - e^2 \sin^2 \varphi)^2} d\varphi d\lambda$$

$$S = b^2 \frac{(\lambda_2 - \lambda_1)^2}{2\rho^2} \left[ \left( \frac{\sin \varphi_2}{1 - e^2 \sin^2 \varphi_2} + \frac{1}{2e} \ln \frac{1+e \sin \varphi_2}{1-e \sin \varphi_2} \right) - \left( \frac{\sin \varphi_1}{1 - e^2 \sin^2 \varphi_1} + \frac{1}{2e} \ln \frac{1+e \sin \varphi_1}{1-e \sin \varphi_1} \right) \right]$$

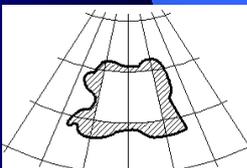
$$S_E = b^2 \frac{(\lambda_2 - \lambda_1)^2}{2\rho^2} \left[ \frac{\sin \varphi}{1 - e^2 \sin^2 \varphi} + \frac{1}{2e} \ln \frac{1+e \sin \varphi}{1-e \sin \varphi} \right]_{\varphi_1}^{\varphi_2} \quad \dots (1)$$

2.1.2 Calculation of incomplete trapezoid areas in reference ellipsoid

$$S = \frac{1}{2} \sum_{i=1}^n x_i * (y_{i+1} + y_{i-1}) \quad \dots (2)$$

$$p = m_0^2 \left( 1 + \frac{\bar{y}^2}{2R^2} + \frac{\bar{y}^4}{24R^4} \right) \quad \dots (3)$$

$$d_p = p - 1 \quad \dots (4)$$

$$S_E = S - S \cdot d_p \quad \dots (5)$$


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## Source data

- National boundary of Kosova within Kosova Global Map dataset
- Free for download in [www.iscgm.org](http://www.iscgm.org)
- Copyright: open and free for research projects
- Last update: August 2011
- Developed by: Kosova Cadastral Agency



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## Creating model for calculation of national area

- First: 3 grids with ribs  $\approx 1\text{km}$ ,  $5\text{km}$  and  $10\text{km}$  were created for whole territory of Kosova
- As model: Trapezoids with ribs  $\Delta\phi=2'30''$  and  $\Delta\lambda=3'30''$  has been selected as final model for calculation



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## Calculation of the national area

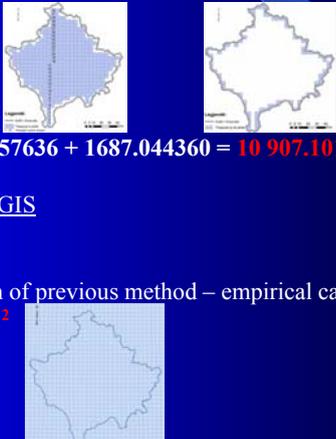
### Trapezoids and area of incomplete trapezoids inside the state border line

Empirical calculation

Complete trapezoids: 416  
Number of rows: 32  
Incomplete trapezoids: 115  
Calculated area:  $S_E = S_{tr} + S_{in.tr} = 9220.057636 + 1687.044360 = 10\,907.101996\text{km}^2$

Semiautomatic calculation by using ArcGIS

Number of divided areas: 574  
Number of rows: 32  
Origin: not in same place with the origin of previous method – empirical calculation  
Calculated area:  $S_V = 10\,907.077975\text{km}^2$

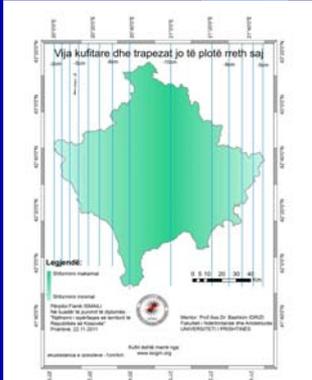


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**Dividing national area under interval of 1cm/km of isograms**

Kosova area divided in: 26 irregular polygons  
Bases of dividing: isograms in interval 1cm/km  
Difference between area values of polygons: from 0.017km<sup>2</sup> to 3665km<sup>2</sup>  
Used: as control method  
Calculated area:  $S_p = 10\,907.088855 \text{ km}^2$



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**National area of the Republic of Kosova**

Method	Value of national area
Empirical calculation	10 907.101996km <sup>2</sup>
Semiautomatic calculation by using ArcGIS	10 907.077975km <sup>2</sup>
Dividing national area under interval of 1cm/km of isograms	10 907.088855 km <sup>2</sup>

**$S_E = 10\,907 \text{ km}^2$**

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## Conclusions

- National area of the Republic of Kosova in GRS80 ellipsoid based on national boundary is 10907km<sup>2</sup>;
- Entire territory of Kosova lies within the area of a secant cylinder;
- State map projection of Kosova has non-proportional distribution of distortions across its territory;
- All distortions values throughout Kosova have negative prefix;
- The territory of Kosova reduces its dimensions in all territory during the process of projecting from ellipsoid into flat surface of map projection;
- Real area of Kosova in referent ellipsoid is larger than calculated area in the flat surface of state map projection for  $\approx 2\text{km}^2$  ( $\approx 0.18\%$ ).
- This comes as a result of utilization the scale factor (0.9999) which doesn't match the characteristics and dimensions of the Kosova territory.
- The calculated area 10907km<sup>2</sup> cannot be taken as final for Kosova, because Kosova has not yet begun the process of demarcation of its border with neighboring countries, except Macedonia (FYRoM).

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