High Accurate Local Geoid in Egypt

M. KALOOP, M. RABAH and H. EL-SHMBAKY, Egypt

Keywords: Geoid, Othometric height, Ellipsoidal height, MCS, Geoid Undulation, Geodetic and Cartesian coordinates, GPS.

SUMMARY

According to wide spread use of satellite based positioning techniques, especially GPS (Global Positioning System), a greater attention has started to be paid to precise determination of geoid models with an aim to replace the geometric leveling measurements with GPS measurements during geodetic and surveying works.

This research discusses methodology used in the construction of an Egyptian local geoid with high accuracy, numerical solution of geoid modeling determination applying surface fitting minimum curvature surface (MCS) are presented for deriving the system of linear equations from boundary integral equation.

In addition a comparison study between EGM96, OSU91A and MCS geoid models, mainly it is going to emphasize the applicability of model as a tool for modeling the geoid in a local area precisely using GPS/Leveling data to serve practical applications of geodesy. Also, the obtained results revealed that, the applicability of MCS technique as a tool for determining the precise local geoid in Egypt with law distortion at common points. From this comparison are applying the MCS model on the Naser Lack region.

High Accurate Local Geoid in Egypt

M. KALOOP, M. RABAH and H. EL-SHMBAKY, Egypt

1. INTRODUCTION

The geoid is an equipotent surface of the earth that coincides with the undisturbed mean sea level. Therefore one might say that it describes the actual shape of the earth. The geoid is also the reference surface for most height networks since leveling gives the heights above the geoid. In geodesy, these heights are called orthometric heights (H), but they are the ordinary heights above the sea level [1] The geoid is determined by using several techniques based on a wide variety of using one or more of the different data source such as: Gravimetric method using surface gravity data, Satellite positioning based on measuring both ellipsoidal heights for stations with known orthometric heights, Geopotential models using spherical harmonic coefficients determined from the analysis of satellite orbits, Satellite altimetry using satellite-borne altimetry measurements over the ocean, Astrogeodetic method using stations with measured astronomical and geodetic coordinates, and Oceanographic leveling methods used mainly by the oceanographers to map the geopotential elevation of the mean surface of the ocean relative to a standard level surface [2]. Other methods are the mathematicals models as the same in this paper using MCS method.

The Earth Geopotential Model (EGM96) and the Ohio State University (OSU91A) are examples of the recent global geopotential models representing the earth gravitational potential as spherical harmonic coefficients. Both models are complete to degree and order 360. Therefore, the shortest wavelength of these models is one degree, and their resolution is one-half degree (about 50 km). the calculated geoid over Egypt was calculated by EGM96 model (figure 1.1).

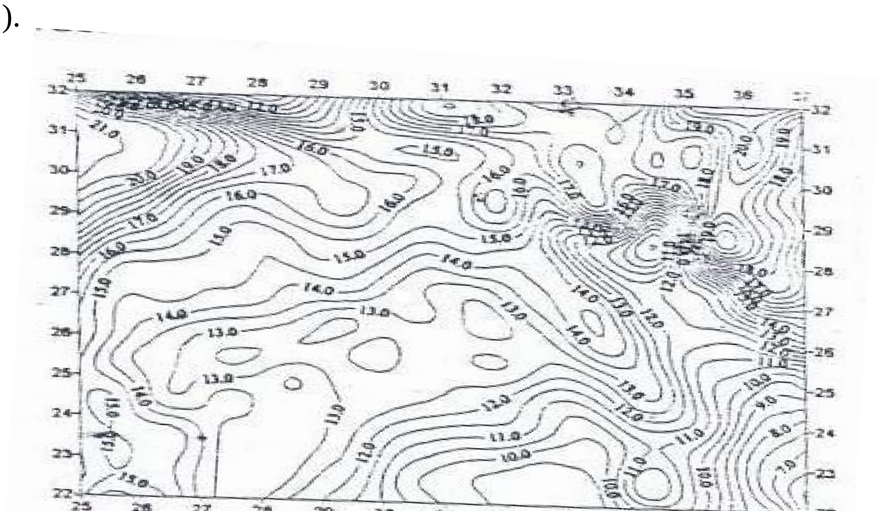


Figure (1.1) the geoid undulation of EGM96, over Egypt
Contour values in meters

Smith and Milbert [1997a] stated that there is an error in the order of one meter in geoid undulation determination using OSU91A model and Geodetic Reference System (GRS80) as the chosen reference field. This effect is due to the difference between the normal potential of GRS80 and the potential of the geoid, and in the case of OSU91A model, it is due to the fact that it is implemented value for the gravitation-mass constant does not equal the corresponding value of the GRS80. In the case of EGM96-based geoid undulations computed by NIMA, a constant bias of 0.41 m was taken into account [3].

2. MODEL AND METHODOLOGY

The mathematical techniques are solving the most problems in the practical methods to compute the geoid and estimated the value of the geoid for low observed data available as notes in Egypt.

From available data in Egypt as shown the (S.Powell, 1997 report) we can compute the geoid undulation in Egypt. As the mathematical techniques are the best solution to compute the empirically or adjusted value of the geoid undulation suppose the few data available. The mathematical methods use the least square techniques to solve the mathematical equations and to obtain from it the parameters of the mathematical equations and the standard deviation.
According to Erol and Celik, (2004) the important factors that affecting the accuracy of GPS/leveling geoid model are [4]:-

- Distribution and number of reference stations (GPS/leveling stations). These points must be distributed homogeneously to the coverage area of the model and have to be chosen to figure out the changes of geoid surface.
- The accuracy of GPS derived ellipsoidal heights (h) and the heights derived from leveling measurements (H).
- Characteristic of the geoid surface area.
- Used method while modeling the geoid

The mathematical method of minimum curvature surface (MCS) is an old and over-popular approach for constructing smooth surface from irregularly spaced data. The surface of minimum curvature corresponding to the minimum of the Laplacian power or, in alternative formulation, satisfies the biharmonic differential equation. Physically, it is model the behavior of an elastic plate. In the one dimensional case, the minimum curvature loads to the natural cubic spline interpolation. In the two-dimensional case, a surface can be interpolated with biharmonic splines or gridded with an iterative finite difference scheme.
In most of the practical cases, the minimum-curvature technique produces a visually pleasing smooth surface. However, in case of large changes in the surface gradient, the method can create strong artificial oscillations in the unconstrained regions. Switching to lower-order methods, such minimizing the power of the gradient, solves the problem of extraneous inflections. On the other side, it also removes the smoothness constraint and leads to gradient discontinuities [5].

The mathematical formula for (MCS) is seeking for a two-dimensional surface $f(x,y)$ in region D, which corresponding to the minimum of the Laplacian power:

$$\iint_D \left|\nabla^2 f(x,y)\right|^2 dxdy \quad ....(1.1)$$

Where ∇^2 denotes the Laplacian operator $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

Alternatively, seeking *f(x,y)* as the solution of the biharmonic differential equation :

$$(\nabla^2)^2 f(x,y) = 0 \quad ....(1.2)$$

Equation (1.1) corresponding to the normal system of equations in the least square optimization problem, [8].
Poisson equation can be expressed as follows:

$$(\nabla^2)^2 f(x,y) = f(x,y) \quad ....(1.3)$$

The solution of this differential equation can be solved as follows:

If ***y=f(x)*** is a function of one variable, then by Taylor theorem:

$$y_1 = y_0 + hy_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + ...$$

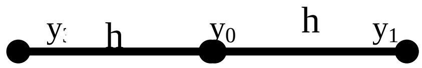


Figure (1.2) The grid arms

$$y_3 = y_0 - hy_0 + \frac{h^2}{2!}y_0'' - \frac{h^3}{3!}y_0''' + ...$$

As shown in figure (1.2).
By adding the two equations and neglecting the higher orders one can get

$$y_1 + y_2 = 2y_0 + h^2 y_0''$$

With an error less than $\left|h^4 y_0^4 / 12\right|$

$$\therefore y_0'' = \frac{1}{h^2}[y_1 + y_3 - 2y_0]$$

Or

$$\frac{d^2 y}{dx^2} = \frac{1}{h^2}[y_1 + y_3 - 2y_0]$$

Similarly for a function of two variables as shown in figure (1.3)

$$\left.\begin{aligned} \frac{d^2\varphi}{dx^2} &= \frac{1}{h^2}[\varphi_1 + \varphi_3 - 2\varphi_0] \\ \frac{d^2\varphi}{dy^2} &= \frac{1}{h^2}[\varphi_2 + \varphi_4 - 2\varphi_0] \end{aligned}\right\} \quad .... (1.4)$$

Where φ_0 is the value of the function *f(x,y)* at the point (x_0 , y_0). It is needed to solve numerically the following partial differential equations:

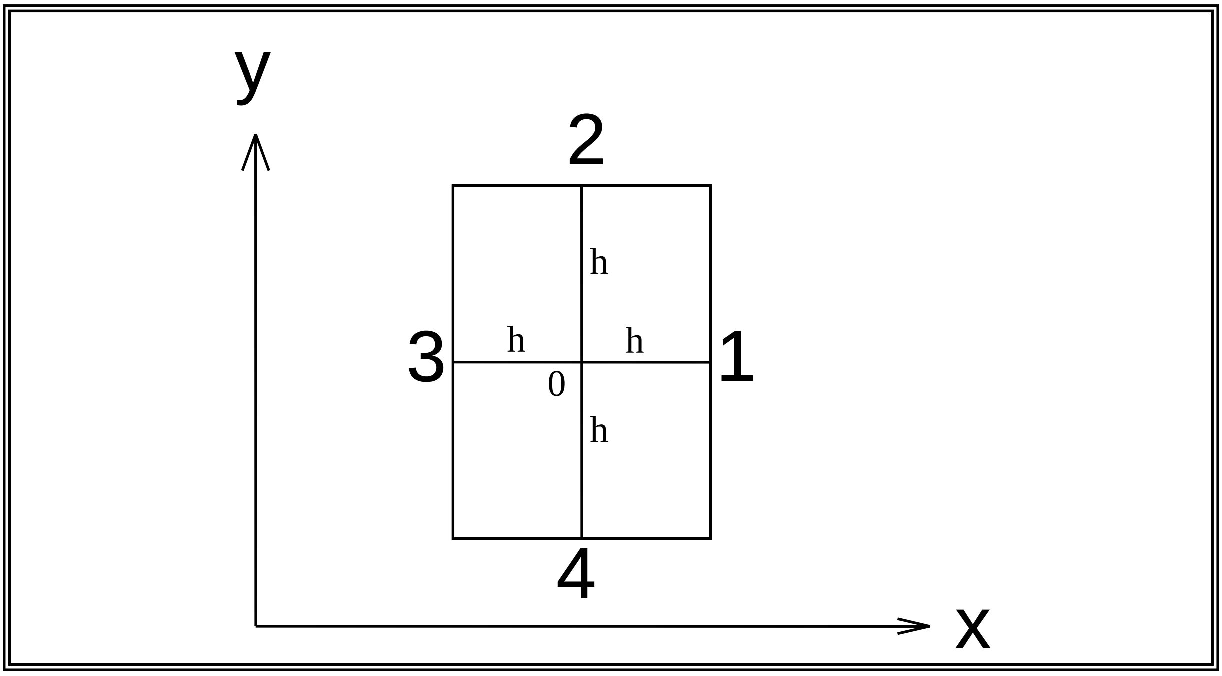


Figure (1.3) The grid for two variables

1- Laplace's equation : $\nabla^2\varphi = 0, i.e. \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = 0$ ……..………..(1.5)

Inside any closed boundary.

2-Poisson's equation $\nabla^2\varphi = f(x, y), i.e. \frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = f(x, y)$ ……..(1.6)

inside any closed boundary.

Replacing $\frac{\partial^2\varphi}{\partial x^2}$ & $\frac{\partial^2\varphi}{\partial y^2}$ by their equivalent expression from (1.3), (1.6) we get the following

difference equations:

** For Laplace's equation :

$$\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4 - 4\varphi_0 = 0 \quad \text{……………..………..(1.7)}$$

** For Poisson's equation :

$$\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4 - 4\varphi_0 = h^2 f(x_0, y_0) \quad \text{..........................(1.8)}$$

Now we can divide the area inside the boundaries into a network or lattice of squares of side (h). The corners of these squares are called nodes of the network. A difference equations must be written (1.7), (1.8) according to the considered problem for each node. These linear equations can then be solved by any method. It must be known the values of *φ(x,y)* at the boundaries to solve the considered problem [6].

3. RESULTS AND DISCUSSIONS

Actually, the full results are shown in M. Sc. thesis [9].MCS technique was a grid transformation technique but in this technique was not depended on a priori variance covariance matrix. After construction the grids above the area of study can be summarized the steps of solution as following:

☞ The differences between two models are computed.

☞ The observation equations can be formed according to laplace model

$$\frac{2\varphi_a}{k_1(k_1+k_2)}+\frac{2\varphi_c}{k_1(k_1+k_2)}+\frac{2\varphi_b}{l_1(l_1+l_2)}+\frac{2\varphi_4}{l_1(l_1+l_2)}-(\frac{2}{(l_1l_2)}+\frac{2}{(k_1k_2)})\phi_0=\begin{cases}0, Laplace\\ h^2 f_0, Poisson\end{cases}$$

☞ Forming the reduced condition equation of the laplace model and applying least square theory with unified technique give the posterior variance.

☞ The variance of used common points is obtained and trials are stopped according to covariance of variance.

☞ Computing the geoid undulation at unknown grids and drawing the contour map. Calculating the distortion at common points, and from that we neglected the points (OZ13, OZ15, OZ19, OZ20, and OZ22) as shown in table (1.1)

Table (1.1) The distortion at common and check points by the MCS program

Point	Distortion (m)
OZ02	0.07721684810177
OZ07	0.00347249793973
OZ08	-0.22918189336117
OZ09	0.05266968934737
OZ10	-0.60297546581746
OZ11	0.77287428229446
OZ12	-0.7017543695959443
OZ13	-1.0794388009004
OZ14	0.3525176513764
OZ15	1.49303195651246
OZ16	0.00570559336130
OZ17	-0.00291306256056
OZ18	0.32937727462008
OZ19	-1.06750166631848
OZ20	-1.83890599444540
OZ21	-0.11787994573827
OZ22	1.083846763956781

Point	Distortion (m)
N7	0.030532957
R5	0.008740674
Y5	0.008160271
P4	0.009515733
A4	0.016913142
E5	0.004675824
B3	0.01980998
S2	0.017392122
A2	0.029476479
L2	0.023046078
F1	0.035859744
Max.distortion	0.035859744
Min.distortion	0.004675824
Average	0.018556637
STDEV	0.010328157
Range	0.03118392

The contour map of the geoid undulation for Egyptian grids by using MCS model is shown in figure (1.4).

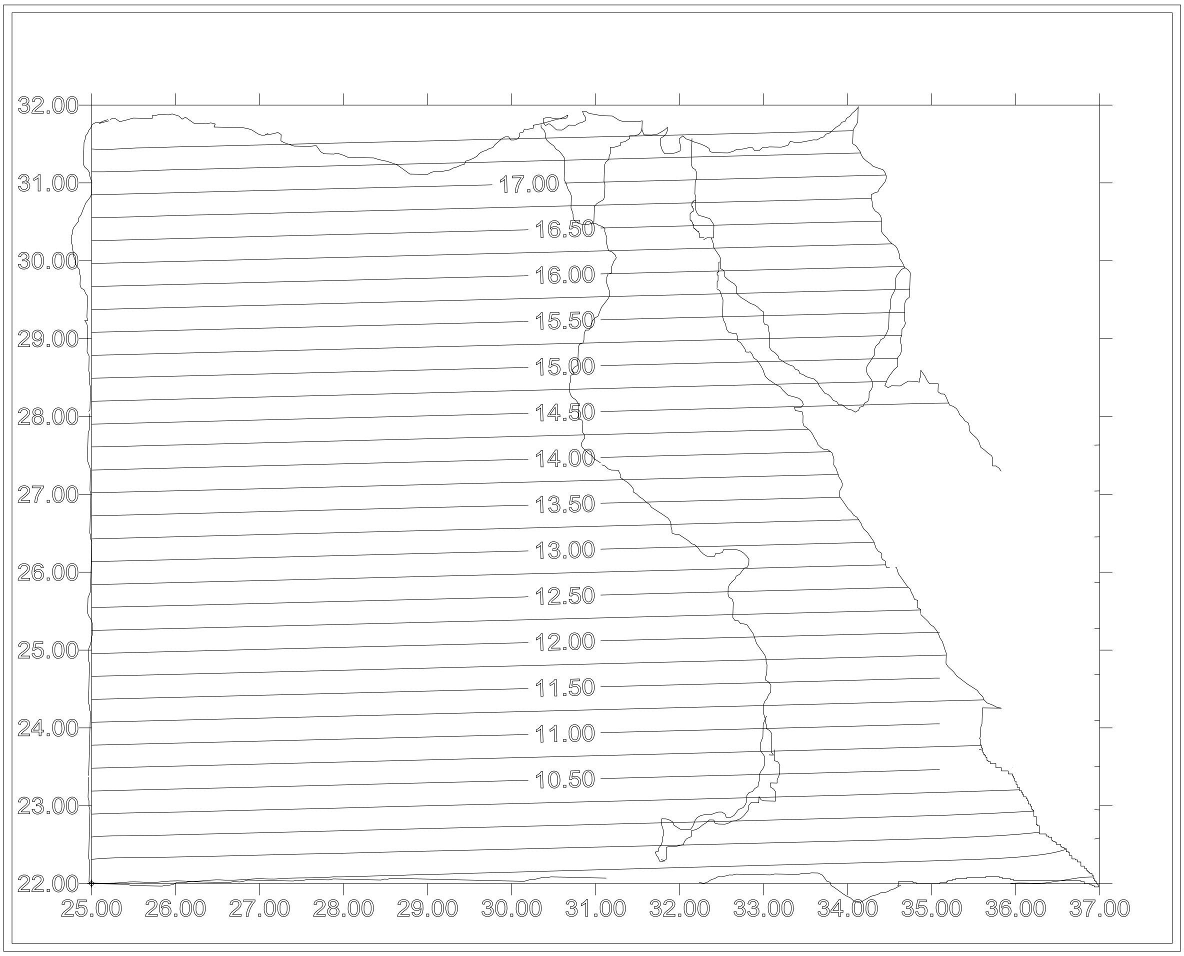


Figure (1.4) The geoid undulation by using MCS model in two direction (geodetic coordinates**)**

**** Comparisons between EGM96, OSU91A and MCS techniques in Egypt**

The known best geoid models in the world are EGM96, as shown in figure (1.1), and OSU91A. In this section, comparison between these models and the MCS model was done:

The value of distortion at check points, maximum and minimum distortion, average distortion and the standard deviation of distortion by using different geoid models are shown in table (1.2).

Determination the value of distortions at check points as follow:

- The value of geoid undulation on OSU-91A and the distortion it from the actual geoid undulation is shown in (S.Powell, 1997).
- Calculated the value of the geoid undulation on the EGM96 at check points uses the Geomatics program, determining the distortion between it and the actual geoid undulation from S.Powell report.
- Calculated the distortion at check points uses the MCS program .

Table (1.2) The comparison between different geoid model at check points

point	OSU-91A	EGM-96	MCS
N7	2.21	0.827	0.030532957
R5	0.549	0.376	0.008740674
Y5	0.669	0.561	0.008160271
P4	0.708	0.928	0.009515733
A4	0.387	0.581	0.016913142
E5	0.701	0.149	0.004675824
B3	0.453	0.05	0.01980998
S2	0.226	0.322	0.017392122
A2	0.456	0.024	0.029476479
L2	0.835	0.278	0.023046078
F1	1.068	0.163	0.035859744
max	2.21	0.928	0.035859744
min	0.226	0.024	0.004675824
S.D.	0.535915377	0.3037	0.010328157
average	0.751090909	0.3872	0.018556637

The distortion at the check points as shown in figure (1.5), the average distortion and the standard deviation of the distortion as shown in figure (1.6).

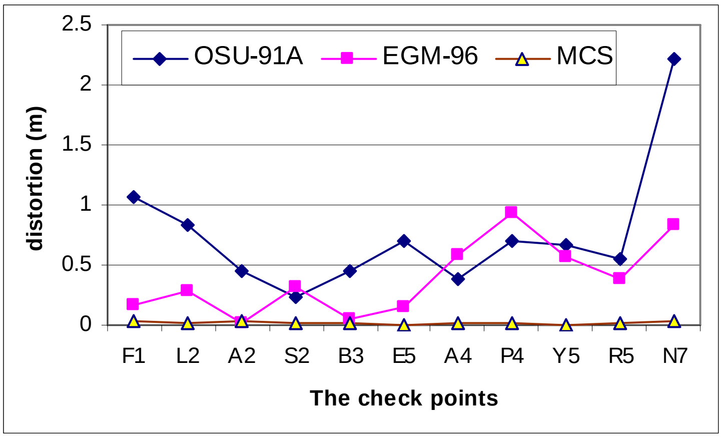
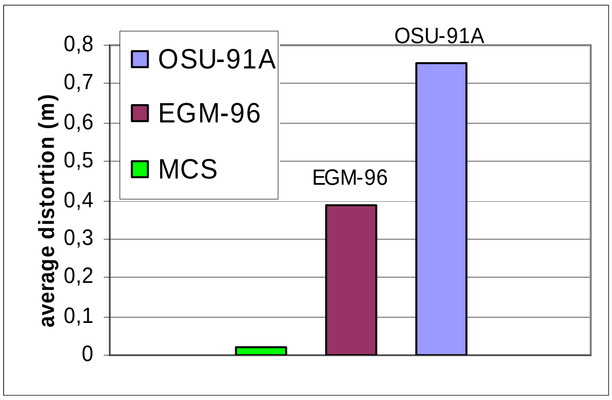


Figure (1.5) The distortion at check points by different geoid models



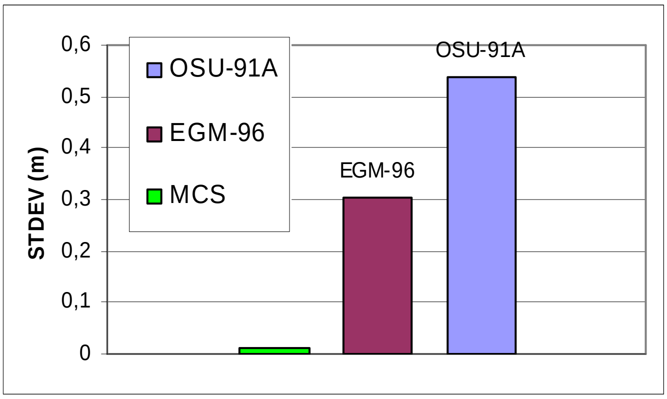


Figure (1.6) The average and standard deviation distortion by different geoid models

From table (5.15) and those figures, it is obvious that, the MCS technique is the best model for Egypt and gives the geoid undulation by high accurate and less time and costs.

*** Case study by using MCS model**

A case study is at the region "Nasser lack". To verify the MCS model was chosen. The data of this region is shown in table (1.3), and available from the NRI, with a comparison of the observed geoid undulation (N_{obs}) and calculated geoid undulation by the MCS technique (N_{MCS}).

Table (1.3) The comparison between the geoid undulation (N_{obs} and $N_{MCS.}$)

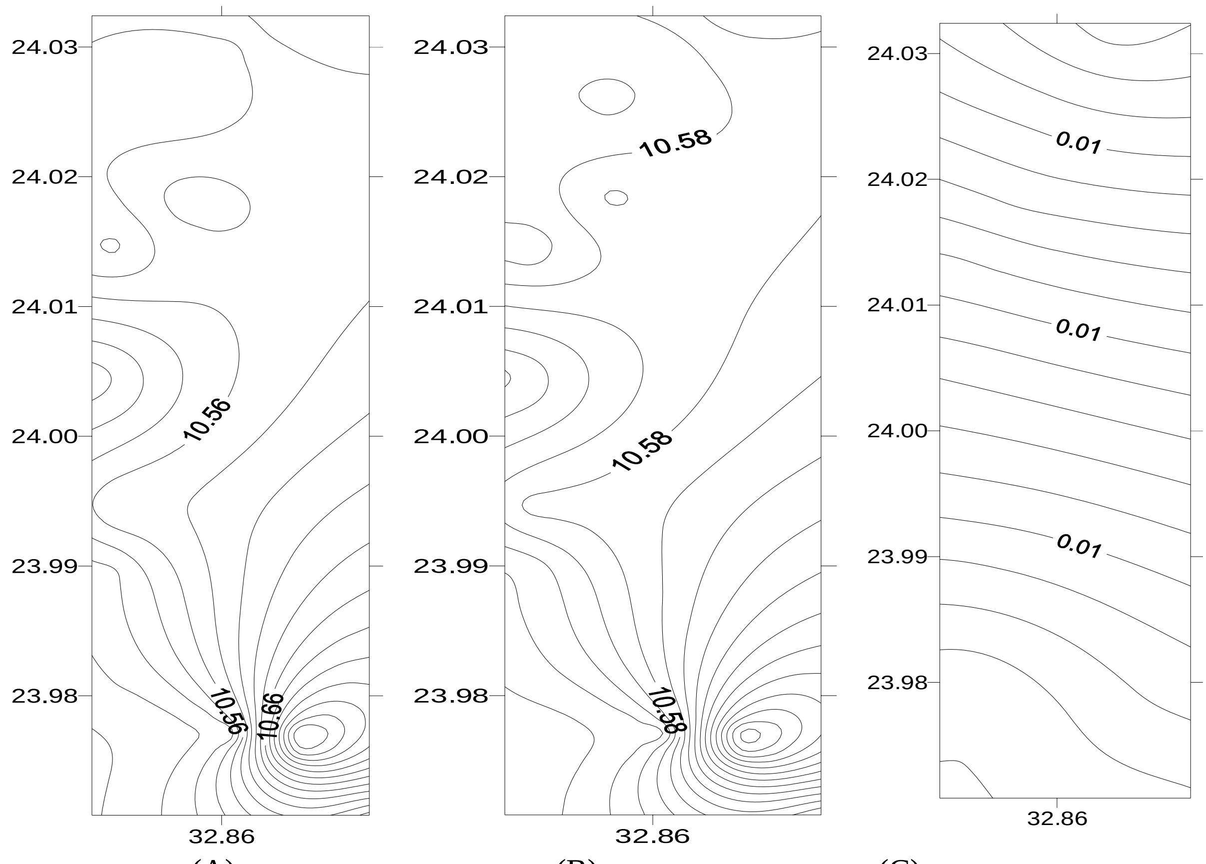
point	latitu.	longitu.	h	H	N_{obs} (m)	N_{MCS} (m)
dam13	24.032368	32.863689	131.84	121.2986	10.5414	10.54973253
dam12	24.030022	32.860706	153.51	142.9274	10.5826	10.59090545
dam11	24.025503	32.858181	151.8	141.204	10.596	10.60426787
dam10	24.018234	32.857872	178.13	167.5824	10.5476	10.5558244
dam09	24.014378	32.853733	184.23	173.6234	10.6066	10.61478336
dam08	24.004421	32.852749	186.38	175.8943	10.4857	10.49382128
dam07	23.994453	32.854034	192.19	181.6111	10.5789	10.5869694
dam06	23.989676	32.853607	196.69	186.1762	10.5138	10.52183982
dam04	23.981094	32.853863	200.32	189.8198	10.5002	10.50819131
dam03	23.978214	32.855679	196.4	185.91	10.49	10.49798286
dam02	23.977167	32.860516	197.81	187.2999	10.5101	10.51809869
dam01	23.976591	32.864426	202.73	191.9123	10.8177	10.82571305
dam00	23.970802	32.868248	200.76	190.209	10.551	10.55899684

As shown table (1.4) the different in the geoid undulation at this zone varied between maximum different (0.833 cm) and minimum different (0.798 cm). The contour maps of the observed value and calculated value by MCS for the geoid undulation and the different between two geoid values are shown in figure (1.7). These are drawing by SURFER program with kriging gridding method.

Table (1.4) The different at points of case study

Point	Dam 13	Dam 12	Dam 11	Dam 10	Dam 09	Dam 08	Dam 07	Dam 06	Dam 04	Dam 03	Dam 02	Dam 01	Dam 00	Max. dist	Min. dist.	average	S.D.
Distortion (m)	0.008332526	0.00830545	0.00826787	0.008224402	0.008183355	0.008121276	0.0080694	0.008039821	0.007991311	0.00798286	0.007998689	0.008013049	0.007996841	0.008332526	0.00798286	0.00811745	0.000129723

From figure (1.8), it is obvious that, there is a high correlation between geoid undulation (N_{obs}) and (N_{MCS}) where R^2=1.00.



(A) (B) (C)

Figure (1.7) A) a contour map of observed geoid, B) a contour map of MCS geoid and C) the difference between A and B.

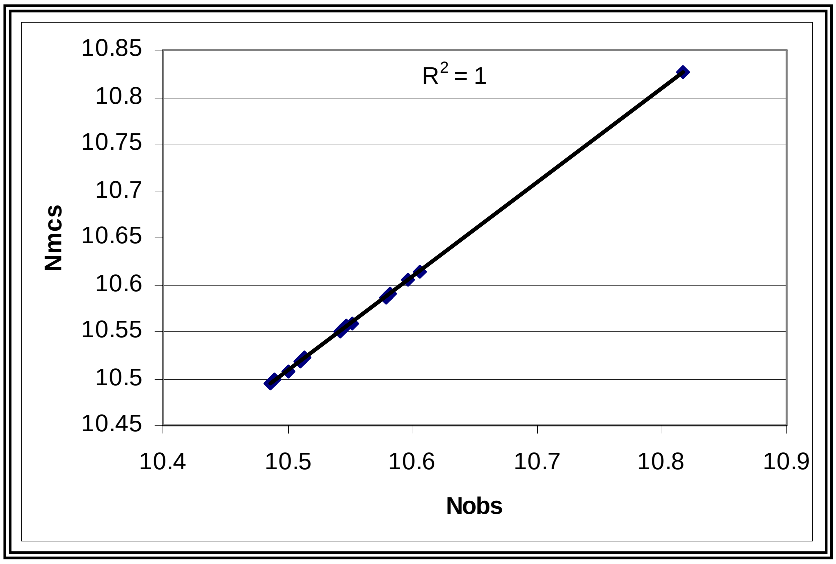


Figure (1.8) The correlation between the observed (N_{obs}) and the calculated (N_{MCS}) geoid undulation

4. CONCLUSION

Based on the previous analysis and numerical results obtained in this paper, the following conclusions can be summarized as follows:
*Among available data and techniques, GPS/Leveling with MCS technique might be the most appropriate combination for geoid model precise outputs in Egypt.
* Geoid undulation average distortion difference between engineering method and MCS technique was 1.86 cm (acceptable range: 0.50 – 3.60 cm) with 1.033 cm standard deviation.
* The MCS technique gives a best geoid undulation beside the EGM96 and OSU91 geoid models in Egypt. Recommend to use MCS technique to compute the geoid undulation in Egypt.

REFERENCES

[1] Lars Harrie, 1993 " Some specific problems in geoid determination" Department of Geodesy and Photogrammetry Royal Institute of Technology, Sweden
[2] A.A.Saad, G.M.Dawood, 2002 " A precise integrated GPS/Gravity geoid model for Egypt" Shoubra Faculty of Engineering, Egypt.
[3] Smith and Milbert, 1997b"Evaluation of EGM96 model of the geopotential in the United States" International Geoid Service Bulletin.
[4] Erol and Celik, 2004. "Precise local geoid determination to make GPS technique more effective in practical applications of geodesy" FIG Work 2004, 22-27 May, Athens, Greece.Turkey.
[5] H.EL-Shmbaky, 2004 "Development and Improvement the transformation parameters for Egyptian coordinates", Public work Dep., Faculty of Engineering, EL-Mansoura UN. Egypt.
[6] A.Sedeek,1992 "Engineering mathematics literature notes" Faculty of Engineering. Mansoura University.
[7] S.Powell, 1997 "Results of the final adjustment of the new national geodetic network", The Egyptian survey authority.
[8] Nikos Drakos, 1997 "Regularizing smooth data with splines in tension." Computer Based Learning Unit, University of Leads.
[9] M.Kaloop, 2006 "Precise leveling by using global positioning system", M.Sc. thesis, Public Work Department, Faculty of Engineering, Mansoura University, Egypt.

CONTACTS

Mosbeh Rashed Mosbeh Kaloop
B.App.Sc.(Civil)(Eg.); M.Sc (Sur.)(Eg.);Phd-Student (China)
Association Lecturer, Mansoura Un.
EGYPT
Email: razankaloop@yahoo.com