Minimum Detectable Overall Trend Rate in GNSS Time-Series

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Key words: Trend rate, Minimum detectable trend rate, GNSS time-series, Colored noise.

SUMMARY

The overall trend rate, i.e. one-dimensional velocity in a GNSS time-series of a local coordinate component is one of the most important parameters for investigating deformed bodies, such as tectonic regions, landslides, mining fields and engineering buildings. The standard deviation (σ_v) of this trend has been studied in many papers to figure out how precise trend rate can be determined. This standard deviation depends on *i*) the length of time-series (time-span), *ii*) the observation frequency, *iii*) the noise structure in the GNSS data, and *iv*) the type of the regression model if the time-span is shorter than about 2.5 years. Most of these studies, however, consider that only the white noise exists in the data. It has been reported that a GNSS time-series includes not only flicker noise with an amplitude which is 1.5 and 4.0 times bigger than the white noise, but also random walk noise whose amplitude changes depending on the monument type and local effects, as well as some other power-law noises occurring due to the different geophysical processes. Existence of these colored noises means that the time-series is temporally correlated. Hence, omitting them in the analysis of the GNSS time-series leads to very optimistic standard deviation for the trend rate and so, incorrect statistical decisions. This contribution aims to discuss the minimum detectable overall trend rate (MDTR) with the 80% power of the test for one coordinate component in GNSS time-series. While the time-span is longer than one year for daily GNSS data, the MDTR can be given as about $2.8\sigma_v$ from the power function of the non-central χ^2-distribution. This MDTR is studied in GNSS time-series consisting of trend + annual and semi-annual signals for different noise models (different flicker noise and random walk noise models as functions of observing session duration dependent-RMS repeatability), different time-spans between 1 year to 10 years as well as daily and monthly observation frequencies. According to the numerical results, if the flicker noise is dominant over the white noise, it is expected to have about 3-4 times bigger MDTR whereas random walk noise affects badly the trend rate more than the flicker noise does. The MDTR from the 24-hours-daily GNSS time-series with the common noise structure may be less than 1 mm/year if the time-span is longer than three years. This rate increases if the colored noises increase as well. The longer observing session results in smaller MDTR in any noise models as expected. Interestingly, daily and monthly GNSS data provides similar MDTRs if the time-span is more than about 4 years.

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1. INTRODUCTION

Deformation analysis has been one of the main interests of geodesy and surveying community for about five decades. The quality of monitoring of the deformations has been improved since GPS or GNSS measurement technique was involved in deformation studies. The long-term monitoring of the deformed objects with GNSS yields the GNSS time-series for each of the local coordinate components. From this time-series, it is possible to derive overall-trend rate, i.e. one-dimensional velocity, or periodic movements occurring due to the inner or outer forces inducing on the deformed body. The overall-trend rate is used to figure out the movements of the deformed body in the long-term and the change of the deformation of the object in time. It is, therefore, one of the most important parameters in deformation analysis.

Neglecting the short-term jumps or periodicities, a GNSS time-series is usually modelled with the following harmonic regression model:

$$\mathbf{y}-\mathbf{e}=\mathbf{A}\mathbf{x}=\begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & t_i & \cos(2\pi t_i) & \sin(2\pi t_i) & \cos(4\pi t_i) & \sin(4\pi t_i) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} y_0 \\ v \\ c_1 \\ s_1 \\ c_2 \\ s_2 \end{bmatrix} \tag{1}$$

where $\mathbf{y}$ is the $n\times1$ the coordinates vector; $\mathbf{e}$ is the $n\times1$ random error vector of the coordinates, $\mathbf{e}\sim N(\mathbf{0},\mathbf{C})$; $\mathbf{C}$ is the $n\times n$ covariance matrix of the coordinates; $\mathbf{A}$ is the $n\times6$ known coefficient matrix with full column of rank; $\mathbf{x}$ is the 6×1 unknown parameter vector; t_i (i=1,2,…,n) is the observation epoch of time; y_0 is the unknown shift parameter; v is the unknown trend rate; c_1 and s_1 are the unknown cosine and sine amplitudes of the annual signal, c_2 and s_2 are the unknown cosine and sine amplitudes of the semi-annual signal.

Before any realization, if the covariance matrix $\mathbf{C}$ of the coordinates in Eq. (1) can be established based on the previous studies or experiences, one may obtain the expected standard deviation of the trend rate. This standard deviation depends on,

–the observation frequency or the time difference between two sequential epochs; $\Delta T = tj - ti$,
–the length of the time series (time-span), i.e. $T=(n-1)\Delta t$,
–the noise structure in the matrix $\mathbf{C}$,
–the deterministic model in Eq. (1).

The higher observation frequency and the longer time-span of the time-series lead to the smaller standard deviation of the trend rate. As mentioned in Blewitt and Lavallée (2002), the annual signal (and semi-annual signal) does not cause significant change in the standard deviation of the trend rate if the time-span is longer than about 2.5 years (see also the next section). Hence, the sinusoidal signals in the deterministic model of Eq. (1) become important if the time-span is shorter than 2.5 years.

The noise structure in the matrix **C** is the most complicated part in a GNSS time-series. If the noise structure of the matrix **C** is expressed as $\sigma_W{}^2\mathbf{I}$ (independent and identically distributed white noise with σ_W amplitude), the smaller σ_W yields the smaller standard deviation of the trend rate. In this case, the white noise amplitude can be taken as the observing session duration (τ) dependent-RMS error of repeatability as defined in Eckl et al. (2001). This repeatability can be assumed from GNSS precision studies, for example, as $9\tau^{-0.5}$ [mm×hours$^{-0.5}$] for the North and East components and about $36\tau^{-0.5}$ [mm×hours$^{-0.5}$] for the Up component.

On the other hand, using the covariance matrix $\sigma_W^2\mathbf{I}$ in a GNSS time-series leads to misleading error assessments. Because, it has been shown in many studies that a GNSS time-series contains not only white noise but also colored noises, such as flicker and random walk noises (Zhang et al. 1997; Mao et al. 1999; Williams et al. 2004; Bock et al. 2000; Langbein and Bock 2004; Amiri-Simkooei et al. 2007; Santamaria-Gomez et al. 2011; Wang et al. 2012). Therefore, the covariance matrix of the coordinates in a GNSS time-series should be taken as follows (Williams 2003; Amiri-Simkooei et al. 2007):

$$\mathbf{C} = \sigma_W^2\mathbf{I} + \sigma_F^2\mathbf{Q}_F + \sigma_{RW}^2\mathbf{Q}_{RW} \tag{2}$$

where the indices *F* and *RW* denote the flicker noise and random walk noise, respectively; $\sigma_{(\ldots)}$ and $\mathbf{Q}_{(\ldots)}$ denote the amplitude and the cofactor matrix of the corresponding colored noise. The cofactor matrices of the colored noises can be constructed by using the autoregressive approach of Hosking (1981) as shown in Williams (2003) (see the next section). For $\sigma_F^2 \neq 0$ and $\sigma_{RW}^2 \neq 0$, the noise structure in Eq. (2) results in bigger standard deviations compared to the one with $\sigma_W^2\mathbf{I}$. In other words, neglecting the colored noises in the GNSS time-series yields more optimistic standard deviation for the trend rate.

The covariance matrix **C** in Eq. (2) can be rewritten as follows:

$$\mathbf{C} = \sigma_W^2(\mathbf{I} + \frac{\sigma_F^2}{\sigma_W^2}\mathbf{Q}_F + \frac{\sigma_{RW}^2}{\sigma_W^2}\mathbf{Q}_{RW}) = \sigma_W^2\mathbf{Q} \tag{3}$$

where σ_F/σ_W and σ_{RW}/σ_W denote the flicker noise ratio and random walk noise ratio, respectively. These ratios can be presume according to the knowledge from the former experiences. For instance, the ratio σ_F/σ_W lies between 1.5 and 4.0 [year$^{-0.25}$] in GPS time-series (Zhang et al. 1997; Calais 1999). The random walk noise ratio depends on the monument type and local effects at the GNSS site. The worse monument conditions cause the bigger random walk noise ratio. Henceforth, with these presumed noise ratios, the covariance matrix in a GNSS time-series can be established in order to determine how precise trend rate can be achievable. At this point, the question arisen is how the covariance matrix in Eq. (3) can be written as the function of the observing session duration. For small noise ratios, the amplitude σ_W in Eq. (3) can be taken as the RMS repeatability depending on the observing session duration. However, for big noise ratios, the amplitude σ_W is to be predicted such that the RMS errors in the high frequency (for instance, in 7 days) which are produced by the cofactor matrix **Q** in Eq. (3) are equal to the RMS repeatability. This reconstruction is expressed in the next section. With this reconstructed covariance matrix, one may investigate also the influence of the observing session duration on the standard deviation of the trend rate while the GNSS time-series includes the colored noises.

The next question is which magnitude of trend rate is just detectable with a given power of the test. This magnitude is called the minimum detectable overall trend rate (MDTR) in the sensitivity analysis terminology (e.g. Aydin 2012). As shown in the next section, the MDTR is equal to about 2.8 times the standard deviation of the trend rate if the length of the time-series is longer than or equal to about one year for daily and monthly GNSS data.

This contribution aims to the discuss the MDTRs in daily GNSS time-series (partly monthly time-series) for the time-span between 1 year and 10 years, for the flicker noise ratio from 0 to 10 [$\text{year}^{-0.25}$] and the random walk noise ratio from 0 to 10 [$\text{year}^{-0.5}$] as well as observing session duration τ for the deterministic model in Eq. (1).

2. METHODOLOGY

2.1 Cofactor matrices for the flicker and random walk noises

According to the autoregressive approach of Hosking (1981), the cofactor matrix of a colored noise ($\mathbf{Q}_F$ for flicker noise and $\mathbf{Q}_{RW}$ for random walk noise) with spectral index κ ($\kappa=-1$ for flicker noise and $\kappa=-2$ for random walk noise) is obtained by $\mathbf{T}\mathbf{T}^T$, where $\mathbf{T}$ denotes the $n\times n$ transformation matrix defined by (e.g. Williams 2003);

$$\mathbf{T} = \Delta T^{-\kappa/4} \begin{bmatrix} \psi_0 & 0 & 0 & \cdots & 0 \\ \psi_1 & \psi_0 & 0 & \cdots & 0 \\ \psi_2 & \psi_1 & \psi_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \psi_n & \psi_{n-1} & \psi_{n-2} & \cdots & \psi_0 \end{bmatrix} \quad \text{with} \quad \psi_n = \frac{\Gamma(n-\kappa/2)}{n!\,\Gamma(-\kappa/2)} \tag{4}$$

where ΔT is the sampling interval, and Γ is the gamma function. Since it is assumed herein that the GNSS time-series does not have any data gaps, the sampling interval is equal to the time difference between two sequential epochs. For instance, ΔT is 1/365.25 [year] and 1/12 [year] for daily and monthly data, respectively.

2.2 Reconstructed covariance matrix C for observing session duration τ

With the noise ratios of $r_F=\sigma_F/\sigma_W$ and $r_{RW}=\sigma_{RW}/\sigma_W$, the cofactor matrix **Q** in Eq. (3) can be given by

$$\mathbf{Q} = (\mathbf{I} + r_F^2\mathbf{Q}_F + r_{RW}^2\mathbf{Q}_{RW}) \tag{5}$$

Suppose that these noise ratios are known. Now, we will consider the variance factor for this cofactor matrix such that the resulting covariance matrix represents the RMS repeatability in high frequency while the noise ratios stay constants. In practice, the RMS repeatability is obtained from a GNSS time-series by considering the coordinates in w days, for instance, using the coordinates in the form of $\mathbf{y}_w = [y(t_1)\ ...y(t_w)]^T$. The error vector $\tilde{\mathbf{e}}_w = [y(t_1) - \bar{y}\ ...y(t_w) - \bar{y}]^T$ is obtained by using their mean value $\bar{y}$. This error vector can be given also by $\tilde{\mathbf{e}}_w = \mathbf{R}\mathbf{y}_w$, where **R** is the $w\times w$ redundancy matrix, $\mathbf{R} = \mathbf{I} - \mathbf{B}(\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T$, with the $w\times 1$ ones vector **B**. Hence, the RMS repeatability is obtained as follows:

$$\mathrm{RMS}^2 = (\tilde{\mathbf{e}}_w^T\tilde{\mathbf{e}}_w)/w = (\mathbf{y}_w^T\mathbf{R}\mathbf{y}_w)/w \tag{6}$$

If the coordinate vector $\mathbf{y}_w$ is distributed as $\mathbf{y}_w \sim N(\mathbf{B}\bar{y}, \sigma_w^2\mathbf{Q}_{(w)})$, where $\mathbf{Q}_{(w)}$ denotes the $w\times w$ sub-cofactor matrix of **Q** in Eq. (5) for the corresponding w days, the expectation of the quadratic form of $\mathbf{y}_w^T\mathbf{R}\mathbf{y}_w$ can be given by (Koch 1999; p. 134):

$$E(\mathbf{y}_w^T\mathbf{R}\mathbf{y}_w) = \sigma_w^2(tr(\mathbf{R}\mathbf{Q}_{(w)})) \tag{7}$$

The left-hand side of Eq. (7) is also equal to $\mathrm{RMS}^2\times w$ from Eq. (6). Hence, the variance factor σ_w^2 is obtained by

$$\sigma_w^2 = \frac{w\times \mathrm{RMS}^2}{tr(\mathbf{R}\mathbf{Q}_{(w)})} \tag{8}$$

By considering non-overlapped windows with the length of w, from Eq. (8), about n/w variance factors are predicted for whole GNSS time-series. These factors will be very close to each other. Their arithmetic mean, say $\sigma_w'^2$, gives the common variance factor for the cofactor matrix **Q** in order to get

$$\mathbf{C} = \sigma_w'^2(\mathbf{I} + r_F^2\mathbf{Q}_F + r_{RW}^2\mathbf{Q}_{RW}) \tag{9}$$

A generated random error vector according to this covariance matrix results in the desired RMS repeatability in the high frequency while the noise ratios are constants. In Figure 1, for the $\mathrm{RMS}=9\tau^{-0.5}$ [mm×hours$^{-0.5}$], the generated random error vectors with the mvnrnd function of MATLAB for three different noise models are shown.

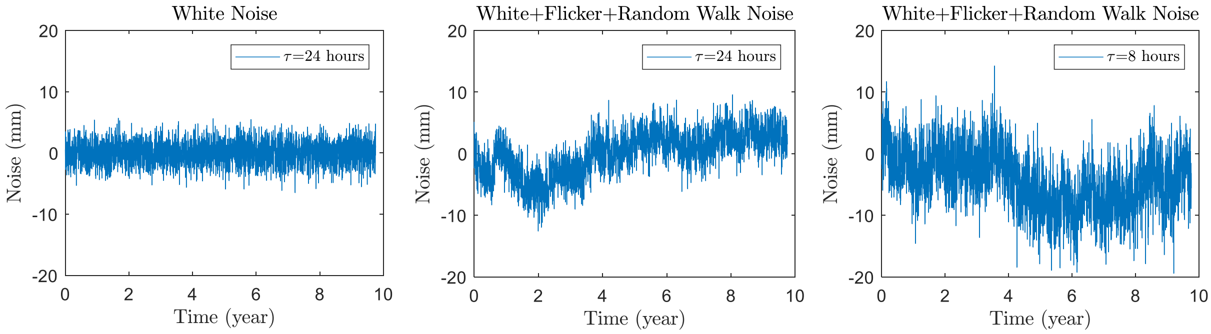

Figure 1. Generated random error vectors for RMS=$9\tau^{-0.5}$ [mm×hours$^{-0.5}$] (left: white noise only, middle: flicker noise ratio is 2 [year$^{-0.25}$], random walk noise ratio is 1 [year$^{-0.5}$], right: same noise ratios as is in the middle panel, but for τ=8 hours])

2.3 Minimum detectable overall trend rate (MDTR)

Suppose that the trend rate and its standard deviation are estimated from the mathematical model in Eq. (1) as $\hat{v}$ and $\hat{\sigma}_v$, respectively. In order to decide whether the trend rate is significant or not, first the following two hypotheses are postulated:

$$H_0 : E(\hat{v}) = 0 \ , \ H_A : E(\hat{v}) = \mu_v \tag{10}$$

If the null hypothesis H_0 is true, the test statistic $(\hat{v}/\hat{\sigma}_v)^2$ follows the central F-distribution, $(\hat{v}/\hat{\sigma}_v)^2 \sim F(1, n-6)$ with the numerator 1 and denominator degrees of freedom $n-6$. In this case, the decision is made by comparing the test statistic with the upper-α percentage value $F_{1,n-6,1-\alpha}$ (threshold value) of the central F-distribution. For instance, if the test statistic exceeds the threshold value, the trend rate $\hat{v}$ is accepted as significant with the α-risk of probability.

Behind the given detection test procedure above, the alternative hypothesis H_A may be true. In this case, the test statistic $(\hat{v}/\hat{\sigma}_v)^2$ follows the non-central F-distribution, $(\hat{v}/\hat{\sigma}_v)^2 \sim F'(1, n-6; \lambda)$ with the non-centrality parameter λ

$$\lambda = \frac{\mu_v^2}{\sigma_v^2} \tag{11}$$

where σ_v^2 is the expected value of $\hat{\sigma}_v^2$. From the known μ_v and σ_v, the probability of correctly accepting the alternative hypothesis, i.e. the power of the test, can be given as follows:

$$\gamma = 1 - cdf(F_{1,n-6,1-\alpha};\ 1,\ n-6,\ \lambda) \tag{12}$$

where *cdf* denotes the cumulative distribution function of the non-central *F*-distribution. Instead of computing the power of the test as in Eq. (12), the non-centrality parameter λ_0 yielding the desired power of the test (for instance, 80%) can be computed (see, Aydin 2012). Hence, if we consider the parameter λ_0 in Eq. (11), the minimum detectable overall trend rate (MDTR) with the given power of the test is obtained by

$$\mu_{v,min} = \lambda_0^{0.5}\sigma_v \tag{13}$$

The standard deviation σ_v can be predicted beforehand using the harmonic regression model in Eq. (1) for different noise structures defined by **C** in Eq. (9). Hence, the MDTRs for these different cases can be obtained by using Eq. (13). These trend rates are discussed in the next section.

The non-centrality parameter λ_0 in Eq. (13) depends on the degrees of freedom (n–6) for the constant α-probability. According to table given in Aydin (2012), while the numerator degrees of freedom is 1 and the power of the test is 80%, the non-centrality parameter becomes 8.4, 8.2, 8.0 and 7.9 for the degrees of freedom 30, 50, 100 and ∞, respectively (the non-centrality parameter is obtained from the non-central χ^2-distribution for the degrees of freedom ∞, see e.g. Aydin and Demirel 2005). For daily GNSS time-series, if the time-span is 1 year, the degrees of freedom is about 360. Hence, for the time-span being longer than or equal to 1 year in daily GNSS time-series, the MDTR is given approximately as follows:

$$\mu_{v,min} \approx 2.8\sigma_v \tag{14}$$

When the time-series includes the trend rate with the magnitude given in Eq. (14), this trend rate will be detected with the power of the test of 80% with the classical test procedure mentioned above if the covariance matrix is precisely known. For a monthly GNSS time-series, the MDTR can be computed also with Eq. (14) when the time-span is about 4 years.

2.4 Minimum time-span to detect the trend in harmonic regression models

The time-span, i.e. the length of the time-series is an important factor while detecting the overall trend. However, this does not mean that the time-span should be at least 1 year or 2.5 years even if the time-series includes annual and semi-annual signals. The only restriction is that the degrees of freedom of the model in Eq. (1) should be bigger than 0 to estimate the trend rate. In this case, of course, the standard deviation and the factor of 2.8 in Eq. (14) gets bigger and so, the MDTR reaches very high value. In other words, the ability of detecting the trend or velocity decreases if the time-span is too short. But it is worth mentioning that this interpretation is valid from the mathematical point of view. In practice, nobody attempts to determine the overall trend rate if the time-span is shorter than 2 years because of many kinds of unexpected errors occurring during the GNSS campaigns. However, why do we insist on 2.5 years? This is mainly due to the common missing interpretation of the paper of Blewitt and Lavallée (2002). They imply that the time-series should be at least 2.5 years in order to estimate the trend rate using a simple regression model with the same precision of the harmonic regression model as in Eq. (1). The ratios between the standard deviations of the trend rates obtained with two kind of models are shown in Figure 2. As seen in this figure, the standard deviation of the trend rate deduced from the simple regression model gets closer to the one of the original harmonic regression model including the annual and semi-annual signals when the time-span is about longer than 2.5 years. Hence, the effect of the corresponding sinusoidal signals is compensated somehow in the regression model after 2.5 years. But it does not mean that the shortest time-span needed is 2.5 years for evaluating the MDTRs in the time-series.

According to the interpretations mentioned above, no time-span restriction exists for investigating the MDTRs except having a positive number for the degrees of freedom in the harmonic regression model in Eq. (1). However, we will consider at least 1 year-time span in GNSS time-series to get rational results.

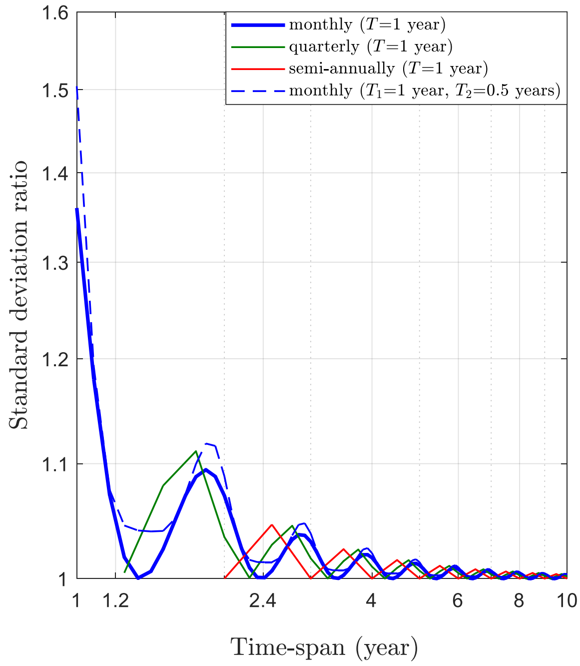

Figure 2. The ratio between the standard deviation of the trend rate obtained from simple regression model and the corresponding harmonic regression model including different sinusoidal signals for monthly, quarterly and semi-annually time-series

3. EXAMPLES AND DISCUSSIONS

In our examples, the time-span is taken from 1 year to 10 years. The covariance matrix **C** is established with Eq. (9) such that the flicker noise and random walk noise ratios lie in the range between 1 and 10 while the RMS repeatability becomes $9\tau^{-0.5}$ [mm×hours$^{-0.5}$].

Firstly, the daily GNSS time-series is investigated. For the observing session duration τ=24 hours, the MDTRs are given in Figure 3. The left panel of Figure 3 shows the changing of the

MDTR against changing of time-span and flicker noise ratio whereas the right panel denotes the changing of the MDTR against changing of time-span and random walk noise ratio while the flicker noise ratio is fixed to 2 [year$^{-0.25}$].

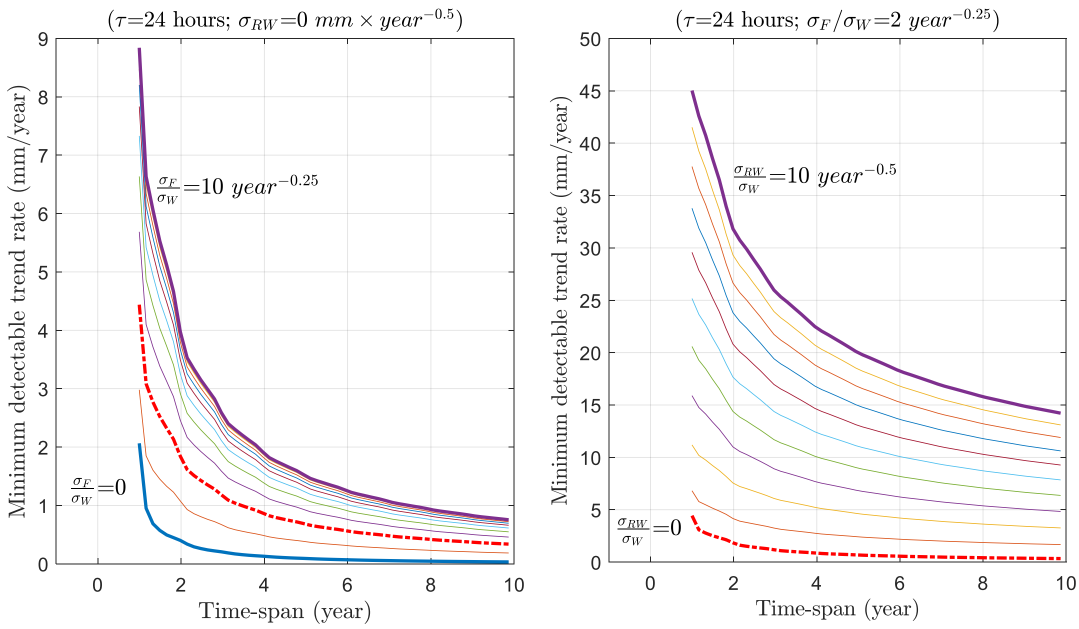

Figure 3. MDTRs for different flicker and random walk noise ratios while the RMS repeatability ($9\tau^{-0.5}$) is about 1.8 mm (τ=24 hours) with the time-span from 1 year to 10 years (left: changing of flicker noise ratio, right: changing of random walk noise ratio) (notice that the scale of the y axes are different)

According to Figure 3-left panel, it is seen that the MDTR gets bigger while increasing flicker noise ratio. However, the change of the MDTR is not proportional to the change in the flicker noise ratio if the time-span is smaller than about 7 years. The blue line in this panel denotes the case in which the time-series includes only white noise. According to this case, the MDTR becomes about 0.5 mm/year if the time span is shorter than or equal to 2 years. But this is not a realistic case encountered in practice. The dashed-red line denotes the flicker noise ratio of 2 [year$^{-0.25}$]. If a GNSS time-series has such a flicker noise, the same ability in the white noise only-case is obtained nearly in 10 years. The MDTR becomes 1 mm/year when the time-span is about 3 years for this flicker noise ratio and RMS repeatability of about 1.8 mm. From Figure 3-right panel, it is seen that the increment in the random walk noise causes bigger MDTRs than the flicker noise does. This implies how the monuments and locations of the GNSS sites are important in deformation studies. If the random walk noise occurs in GNSS time-series, it is impossible to detect the trend rates of 1 mm/year according to the measurement conditions specified in this example.

The same MDTRs in Figure 3 are computed for the observing session duration of τ=8 hours (the RMS repeatability is around 3.2 mm) and given in Figure 4. When we compare the panels with those in Figure 3, it is seen that the session duration is an important parameter for MDTRs in GNSS time-series. The MDTRs in Figure 4 are about $(24/8)^{0.5}$ times bigger than those in Figure 3. The achievable MDTR of 1 mm/year in 3 years in Figure 3 for τ=24 hours can be succeeded in about 6 years if the observing session duration decreases to τ=8 hours.

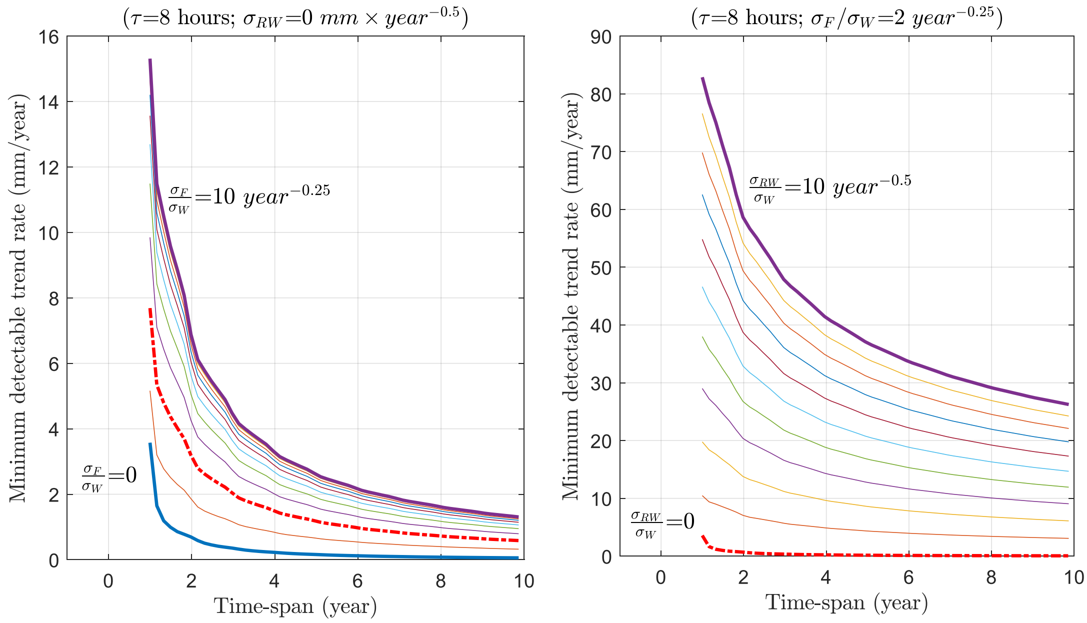

Figure 4. MDTRs for different flicker and random walk noise ratios while the RMS repeatability ($9\tau^{-0.5}$) is about 3.2 mm (τ=8 hours) with the time-span from 1 year to 10 years (left: changing of flicker noise ratio, right: changing of random walk noise ratio) (notice that the scale of the y axes are different)

How does the observation frequency affect the MDTRs? To reply this question, the MDTRs are recomputed for the same conditions above but for monthly data. For different observing session duration (τ=8, 12 and 24 hours) while the monthly GNSS data includes the flicker noise with the ratio of 2 [year$^{-0.25}$], the MDTRs are given in Figure 5. For comparison purposes, the MDTR in the daily GNSS data is also shown as thick-grey line in Figure 5 (this line is the same with the dashed-red line in Figure 3). As seen in Figure 5, the MDTR in monthly GNSS data is nearly the same with the one in daily GNSS data when the time-span is 4 years if the observing session duration is 24 hours. Moreover, after 6 years, all cases considered in Figure 5 results in very similar MDTR results. Consequently, it is seen that the long time-series may compensate the effects of observing session duration and observation frequency in detecting the trend rates for the example considered here.

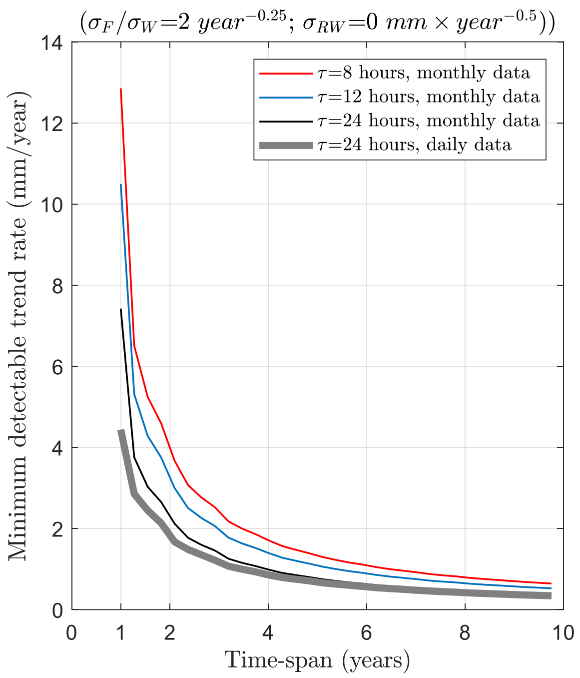

Figure 5. Comparison of MDTRs in monthly GNSS time-series with daily GNSS time-series (flicker noise ratio is 2 [year$^{-0.25}$] while no random walk exists)

4. CONCLUSIONS

The overall trend rate in a GNSS position time-series is an important parameter for investigating deformed bodies, such as tectonic regions, landslides, mining fields and engineering buildings. The standard deviation of this trend has been studied in many papers to figure out how precise trend rate can be determined. This standard deviation depends on the length of time-series (time-span), the observation frequency, the noise structure in the GNSS data, and the type of the regression model if the time-span is shorter than about 2.5 years. Generally, only the white noise is assumed in the data. However, it has been reported that a GNSS time-series includes also flicker noise, random walk noise as well as some other power-law noises occurring due to the different geophysical processes. Existence of these colored noises means that the time-series is temporally correlated. Hence, omitting them in the analysis of the GNSS time-series leads to very optimistic standard deviation for the trend rate and so, incorrect statistical decisions. This contribution aims to discuss the minimum detectable overall trend rate (MDTR) with the 80% power of the test in GNSS time-series sets consisting of trend, annual and semi-annual signals for different noise models (different flicker noise and random walk noise models as functions of observing session duration dependent-RMS repeatability), different time-spans between 1 year to 10 years as well as daily and monthly observation frequencies. According to the numerical results, if the flicker noise is dominant over the white noise, it is expected to have about 3-4 times bigger MDTR whereas random walk noise affects badly the trend rate more than the flicker noise does. The MDTR from the 24-hours-daily GNSS time-series with the common noise structure may be less than 1 mm/year if the time-span is longer than three years.

This rate increases if the colored noises increase as well. The longer observing session results in smaller MDTR in any noise models as expected. Furthermore, daily and monthly GNSS data provides similar MDTRs if the time-span is more than about 4 years.

REFERENCES

Amiri-Simkooei, A. R., Tiberius, C. C. J. M. and Teunissen, S. P. (2007). Assessment of noise in GPS coordinate time series: methodology and results. Journal of Geophysical Research: Solid Earth, 112(B7).

Aydin, C. (2012). Power of global test in deformation analysis. Journal of Surveying engineering, 138(2), 51-56.

Aydin, C., and Demirel, H. (2005). Computation of Baarda's lower bound of the non-centrality parameter. Journal of geodesy, 78(7-8), 437-441.

Blewitt, G. and Lavallée, D. (2002). Effect of annual signals on geodetic velocity. Journal of Geophysical Research: Solid Earth, 107(B7), ETG-9.

Bock, Y., Nikolaidis, R. M., de Jonge, P. J. and Bevis, M. (2000). Instantaneous geodetic positioning at medium distances with the Global Positioning System. Journal of Geophysical Research: Solid Earth, 105(B12), 28223-28253.

Calais, E. (1999). Continuous GPS measurements across the Western Alps, 1996–1998. Geophysical Journal International, 138(1), 221-230.

Eckl, M. C., Snay, R. A., Soler, T., Cline, M. W. and Mader, G. L. (2001). Accuracy of GPS-derived relative positions as a function of interstation distance and observing-session duration. Journal of geodesy, 75(12), 633-640.

Hosking, J. R. M. (1981). Fractional differencing. Biometrika 68 165–176. Mathematical Reviews (MathSciNet): MR614953 Zentralblatt MATH, 464.

Koch KR (1999). Parameter estimation and hypothesis testing in linear models. Springer-Verlag, Berlin/Heidelberg

Langbein, J. and Bock, Y. (2004). High-rate real-time GPS network at Parkfield: Utility for detecting fault slip and seismic displacements. Geophysical Research Letters, 31(15).

Mao, A., Harrison, C. G. and Dixon, T. H. (1999). Noise in GPS coordinate time series. Journal of Geophysical Research: Solid Earth, 104(B2), 2797-2816.

Santamaría-Gómez, A., Bouin, M. N., Collilieux, X. and Wöppelmann, G. (2011). Correlated errors in GPS position time series: Implications for velocity estimates. Journal of Geophysical Research: Solid Earth, 116(B1).

Wang, W., Zhao, B., Wang, Q. and Yang, S. (2012). Noise analysis of continuous GPS coordinate time series for CMONOC. Advances in space research, 49(5), 943-956.

Williams, S. D. P. (2003). The effect of coloured noise on the uncertainties of rates estimated from geodetic time series. Journal of Geodesy, 76(9-10), 483-494.

Williams, S. D., Bock, Y., Fang, P., Jamason, P., Nikolaidis, R. M., Prawirodirdjo, L., Miller, M., and Johnson, D. J. (2004). Error analysis of continuous GPS position time series. Journal of Geophysical Research: Solid Earth, 109(B3).

Zhang, J., Bock, Y., Johnson, H., Fang, P., Williams, S., Genrich, J., Wdowinski, S.,and Behr, J. (1997). Southern California Permanent GPS Geodetic Array: Error analysis of daily position estimates and site velocities. Journal of geophysical research: solid earth, 102(B8), 18035-18055.

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