Fast Kalman Processing of GPS Carrier-Phases for Mobile Positioning and Atmospheric Tomography

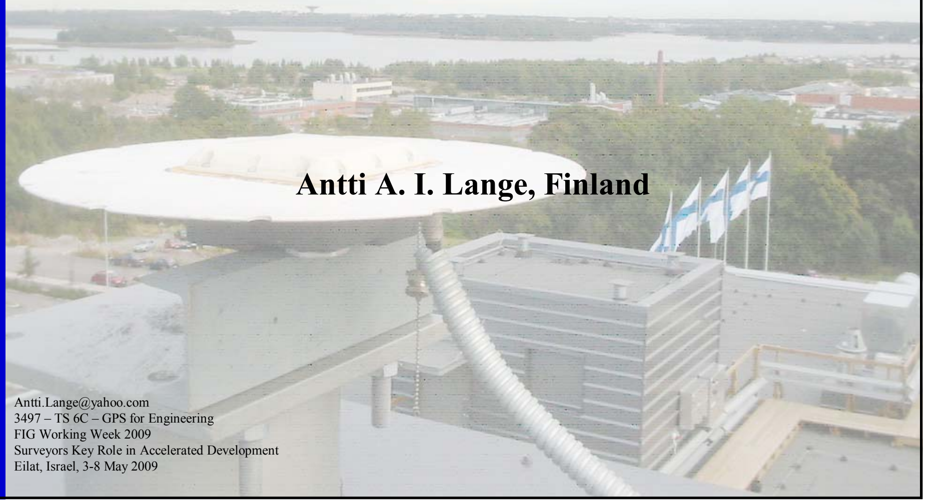

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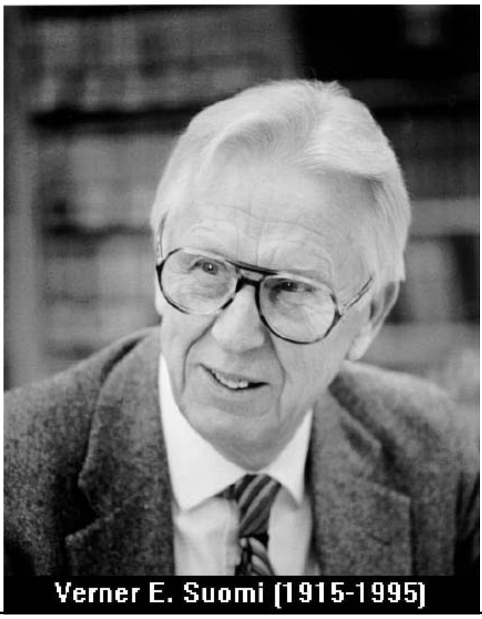
FIG Working Week 2009
Surveyors Key Role in Accelerated Development
Eilat, Israel, 3-8 May 2009

Overview:

- **SuomiNet (USA) and tomography**
- **Helmert-Wolf blocking (HWb)**
- **Fast Kalman processing using HWb**
- **Estimating errors by C.R.Rao’s MINQUE theory using HWb**
- **Concluding remarks**

Professor V. E. Suomi, University of Wisconsin, Madison ”Father of Satellite Meteorology”



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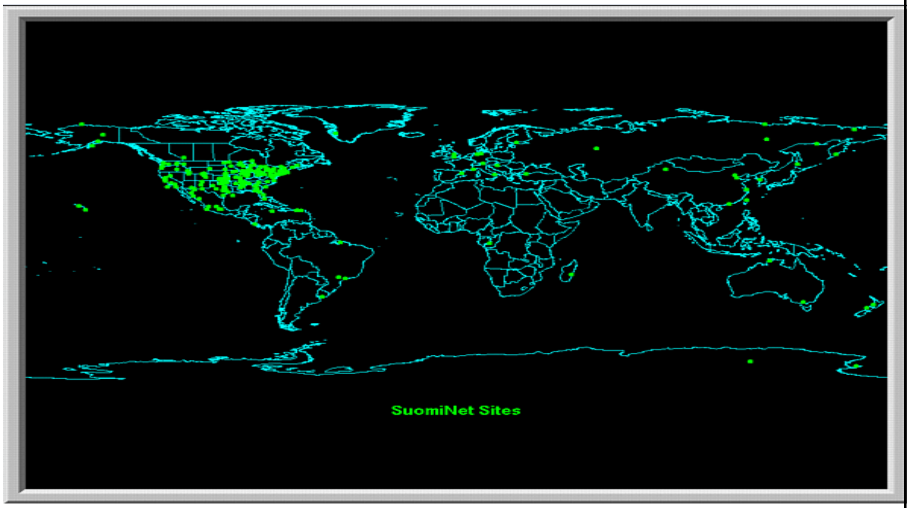
The International SuomiNet GPS sites:



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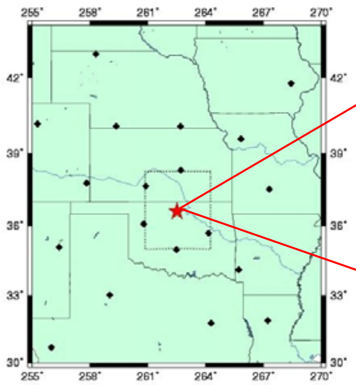
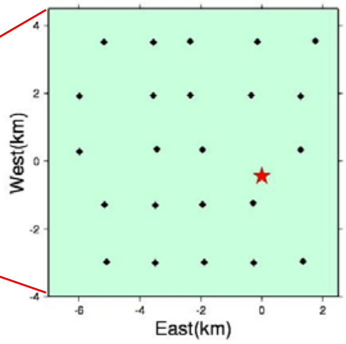
The Local GPS Network in USA, Oklahoma:

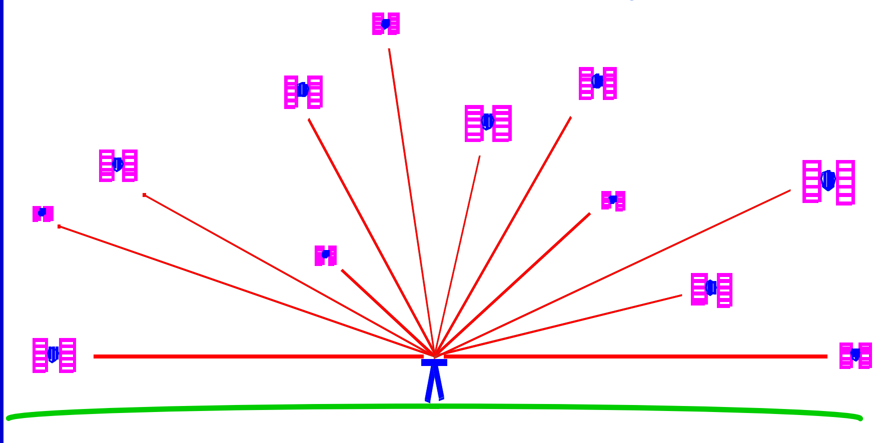


NOAA GPS network with ARM SGP Region

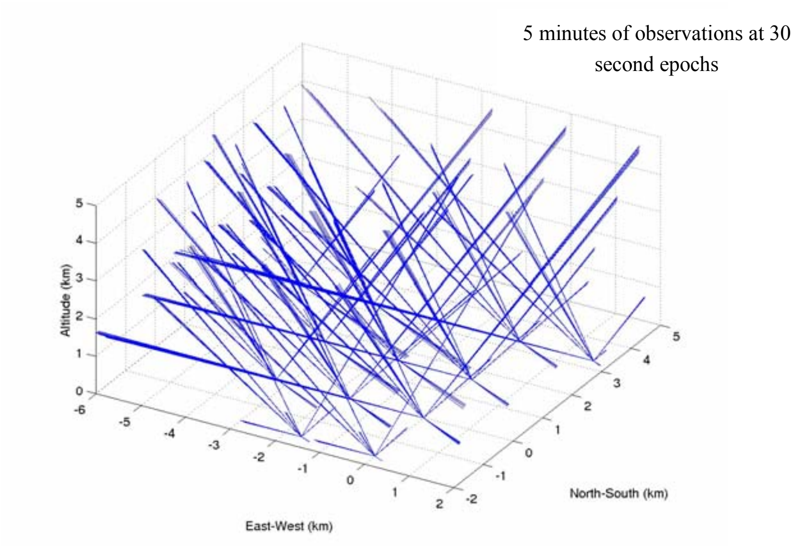


Location of L1 network around ARM SGP Central Facility

GPS slant delays:



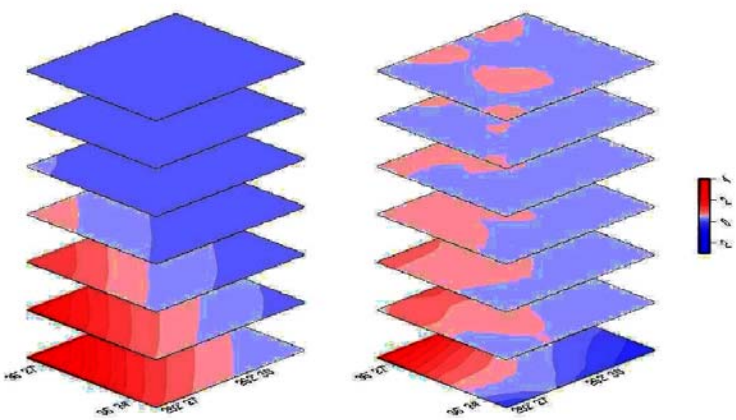
Volume Display of Satellite Ray Paths:



Simulated Tomography Layers:

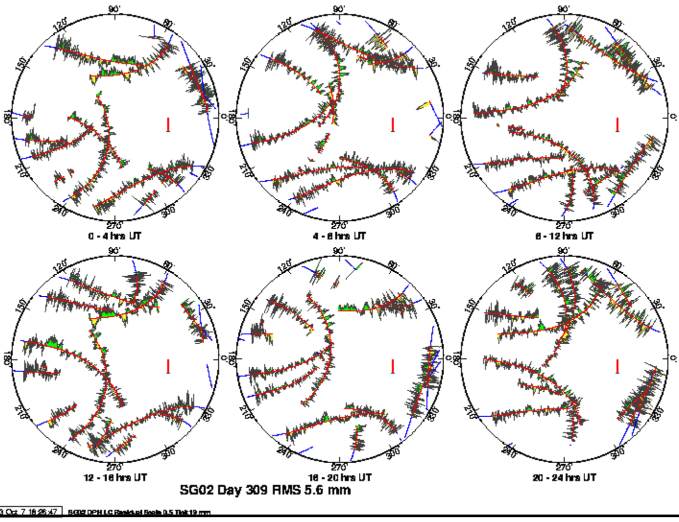
Input Field

Tomography Estimate

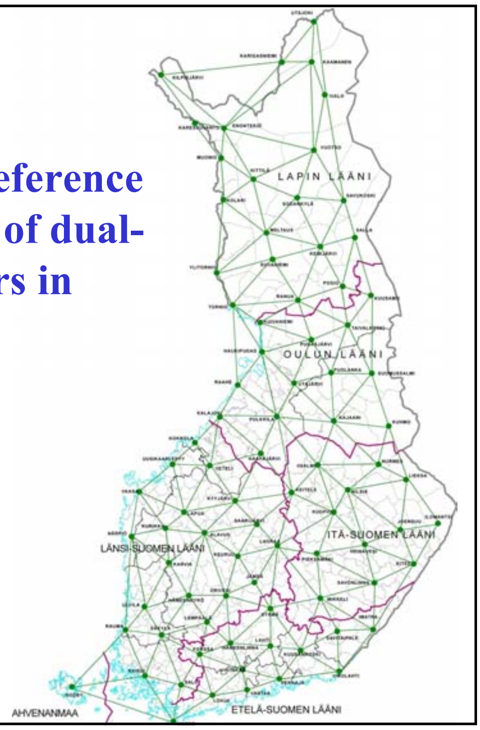


GPS-antenna on the roof-top of FMI:

Skyplot of SuomiNet SG40 GPS-residuals for a day:



The national Virtual Reference Station (VRS) network of dual-frequency GPS receivers in Finland:



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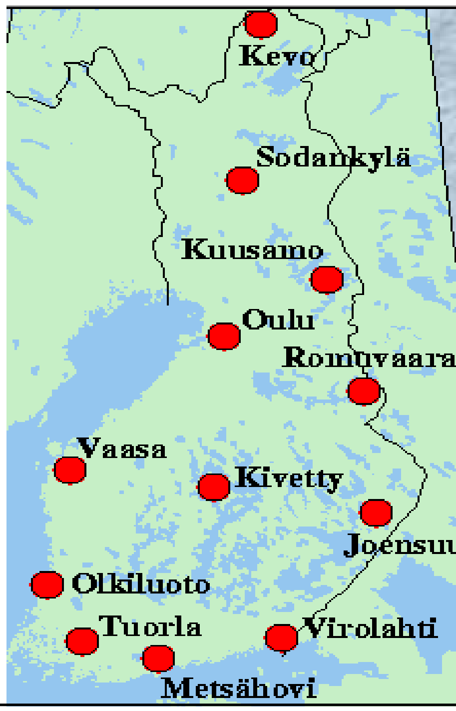
The Finnish permanent GPS network (FinnRef):



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The GPS receiver developed from the Finnish Vaisala radiosonde:

- iTrax03 is the size of a stamp 26x26x4.7mm
- Ultra-low power consumption
- Low cost
- Carrier-phase detection of the GPS L1 signals for precision applications

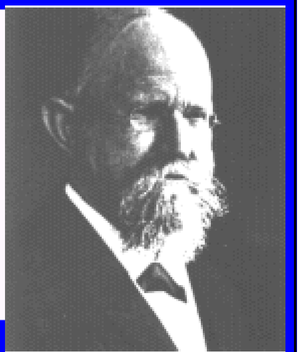
Computing the shape of Earth’s geoid led already in 1880

to Helmert's problem:

The joint regression equation system for all different measurements is written in the Canonical Block-Angular (CBA) form as follows:

$$\begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_K \end{bmatrix} = \begin{bmatrix} X_1 & & & & G_1 \\ & X_2 & & & G_2 \\ & & \ddots & & \vdots \\ & & & X_K & G_K \end{bmatrix} \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \vdots \\ \mathbf{b}_K \\ \mathbf{c} \end{bmatrix} + \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \vdots \\ \mathbf{e}_K \end{bmatrix}$$

This method known as the Helmert-Wolf blocking (HWB)



Helmert

Fastest precise computation was in 1978 given through

Wolf's analytic solution:

$$\hat{\mathbf{c}} = \{\Sigma G'_k R_k G_k\}^{-1} \Sigma G'_k R_k \mathbf{y}_k$$

$$\hat{\mathbf{b}}_k = (X'_k X_k)^{-1} X'_k (\mathbf{y}_k - G_k \hat{\mathbf{c}})$$

where

$\mathbf{c}$ = vector of common adjustments

$\mathbf{b}_k$ = vector of state parameter adjustments for block k

Σ = summation where index k runs over all blocks of observations

G_k = Jacobian matrix for the common adjustments for block k

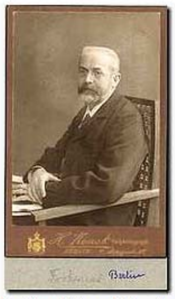
$R_k = I - X_k (X'_k X_k)^{-1} X'_k$ = residual operator for block k

X_k = Jacobian matrix for the state parameters for block k

$\mathbf{y}_k$ = vector of the observations for block k; and,

where observation errors $\mathbf{e}_k$ are orthonormal .

The Semianalytic Inversion by Frobenius 1845-1917:


Ferdinand Georg Frobenius

The sparse coefficient matrix to be inverted may often have either a **bordered block- or band-diagonal** (BBD) structure. If it is band-diagonal it can be transformed into a block-diagonal form e.g. by means of a generalised Canonical Correlation Analysis (**gCCA**). The large matrix can thus be most effectively inverted in a blockwise manner by using the following **analytic inversion formula**:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} + A^{-1}B(D - CA^{-1}B)^{-1}CA^{-1} & -A^{-1}B(D - CA^{-1}B)^{-1} \\ -(D - CA^{-1}B)^{-1}CA^{-1} & (D - CA^{-1}B)^{-1} \end{bmatrix}$$

of Frobenius where

A = a large **block- or band-diagonal** (BD) matrix to be easily inverted, and,

$(D - CA^{-1}B)$ = a much smaller matrix called the Schur complement of A.

This is the **FKF** method that may make it computationally possible to estimate a much larger number of state and calibration parameters than an ordinary Kalman recursion can do. Their operational accuracies may also be reliably estimated from the theory of Minimum-Norm Quadratic Unbiased Estimation (MINQUE) of C. R. Rao (1920-) and used for controlling the stability of optimal Kalman filtering.

Error covariances of the Helmert-Wolf blocking (HWb) method were in 1982 given through

Lange's Precision Matrix (LPM):

Error variances and covariances of all the estimated parameters and unknowns $\hat{s} = [\hat{b}_1', \hat{b}_2', \ldots, \hat{b}_K', \hat{c}']'$ are given by the following large matrix:

$$\mathrm{Cov}(\hat{s} - E(\hat{s})) = \begin{bmatrix} C_1 + D_1SD_1' & D_1SD_2' & \cdots & D_1SD_K' & -D_1S \\ D_2SD_1' & C_2 + D_2SD_2' & \cdots & D_2SD_K' & -D_2S \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ D_KSD_1' & D_KSD_2' & & C_K + D_KSD_K' & -D_KS \\ -SD_1' & -SD_2' & \cdots & -SD_K' & S \end{bmatrix}$$

where $S = \{\sum_{k=1}^{K} G_k' R_k G_k\}^{-1}$; and,

for $k = 1, 2, \ldots, K$:

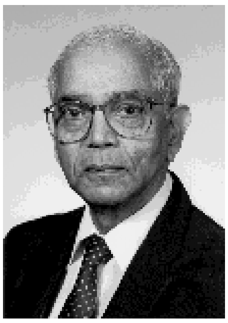
$C_k = (X_k' X_k)^{-1}$

$D_k = (X_k' X_k)^{-1} X_k' G_k$

$R_k = I - X_k (X_k' X_k)^{-1} X_k'$

Prof. Issai Schur
1875-1941

Best Unbiased Estimation of accuracies of correlated observations was solved in 1970 by C.R.Rao's MINQUE theory that can now exploit internal consistencies of the GPS data in optimal fashion:



Calyampudi R. Rao
Professor of Statistics

Fastest possible computation of the Minimum Norm Quadratic Unbiased Estimates (MINQUE) :

vector $\{\sigma^2_i\} = [\sigma^2_1,\ \sigma^2_2, \ldots, \sigma^2_n]^T = \mathbf{F}^{-1}\,\mathbf{q}$

where

n = number of observed carrier-phases

$\mathbf{q}$ = vector $\{q_i\}$ = vector $\{\mathbf{y}^T\,\mathbf{R}\,T_i\,\mathbf{y}\}$

$\mathbf{F}$ = matrix $\{f_{i,j}\}$ = matrix $\{\mathrm{tr}\,\mathbf{R}\,T_i\,\mathbf{R}\,T_j\}$

T_i = diagonal matrix $(\delta^1{}_i, \delta^2{}_i, \ldots, \delta^N{}_i)$

N = total number of all observations

$$\mathbf{R} = \mathbf{I} - \begin{bmatrix} X_1 & & & G_1 \\ & X_2 & & G_2 \\ & & \ddots & \vdots \\ & & X_K & G_K \end{bmatrix} \begin{bmatrix} C_1 + D_1 S D_1^T & D_1 S D_2^T & \cdots & D_1 S D_K^T & -D_1 S \\ D_2 S D_1^T & C_2 + D_2 S D_2^T & \cdots & D_2 S D_K^T & -D_2 S \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ D_K S D_1^T & D_K S D_2^T & & C_K + D_K S D_K^T & -D_K S \\ -S D_1^T & -S D_2^T & \cdots & -S D_K^T & S \end{bmatrix} \begin{bmatrix} X_1 & & & G_1 \\ & X_2 & & G_2 \\ & & \ddots & \vdots \\ & & X_K & G_K \end{bmatrix}^T$$

In case of uncorrelated measurements of a scalar variable this MINQUE solution would collapse into the simple formula for computing the error variance of a mean: σ^2/n

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Optimal Kalman Filtering:

Rudolf E. Kalman, 1930 -

Observations Equation:

$\mathbf{y}_t = H_t\,\mathbf{s}_t + F^y{}_t\,\mathbf{c}_t + \mathbf{e}_t$ for $t = 1, 2, \ldots$

System Equation:

$\mathbf{s}_t = A_t\,\mathbf{s}_{t-1} + B_t\,\mathbf{u}_{t-1} + F^s{}_t\,\mathbf{c}_t + \mathbf{a}_t$ for $t = 1, 2, \ldots$

where $\mathbf{c}_t$ = the vector representing all those calibration and system errors that are constant over some epochs t.

Stability of Kalman Filtering:

$\mathbf{s}_t$ and $\mathbf{c}_t$ must be **observable**

$\mathbf{u}_t$ must be **controllable**

$\mathbf{e}_t$ and $\mathbf{a}_t$ must neither auto- nor cross-correlate!

- These correlations are factored using the matrices F^y_t and F^s_t with help of Singular Value Decomposition (SVD) or generalized Canonical Correlation Analysis (gCCA).

Decorrelating the error variances of the Observations and the System:

Augmented Model for a moving time volume (length L):

$$
\begin{bmatrix} y_t \\ A\hat{s}_{t-1}+Bu_{t-1} \\ \hline y_{t-1} \\ A\hat{s}_{t-2}+Bu_{t-2} \\ \hline \vdots \\ \hline y_{t-L+1} \\ A\hat{s}_{t-L}+Bu_{t-L} \\ \hline A\hat{c}_{t-1}+Bu_{c_{t-1}} \end{bmatrix}
=
\begin{bmatrix} \begin{matrix} H_t \\ I \end{matrix} & & & \begin{matrix} F^y_t \\ F^s_t \end{matrix} \\ & \begin{matrix} H_{t-1} \\ I \end{matrix} & & \begin{matrix} F^y_{t-1} \\ F^s_{t-1} \end{matrix} \\ & & \ddots & \vdots \\ & & \begin{matrix} H_{t-L+1} \\ I \end{matrix} & \begin{matrix} F^y_{t-L+1} \\ F^s_{t-L+1} \end{matrix} \\ & & & I \end{bmatrix}
\begin{bmatrix} s_t \\ s_{t-1} \\ \vdots \\ s_{t-L+1} \\ \hline c_t \end{bmatrix}
+
\begin{bmatrix} e_t \\ A(\hat{s}_{t-1}-s_{t-1})-a_t \\ \hline e_{t-1} \\ A(\hat{s}_{t-2}-s_{t-2})-a_{t-1} \\ \hline \vdots \\ \hline e_{t-L+1} \\ A(\hat{s}_{t-L}-s_{t-L})-a_{t-L+1} \\ \hline A(\hat{c}_{t-1}-c_{t-1})-a_{c_t} \end{bmatrix}
\tag{18}
$$

i.e. $z_t \quad = \quad Z_t \quad s_t \quad + \quad e_t$

and where vector $\mathbf{c}_t$ represents all those calibration parameters that are constants in the moving volume. As previously, we proceed with

Updating: $$\hat{s}_t = \{Z_t' V_t^{-1} Z_t\}^{-1} Z_t' V_t^{-1} z_t \tag{19}$$

The locally linearized Observations and System Equations with tomography look as follows:

The **Observation Equation** for a moving data-window of length L is obtained for the carrier-phase measurement $\varphi_{i,j,k,t}$ of a receiver as follows:

$$y_{i,j,k,t-l} = \varphi_{i,j,k,t} - \rho_{i,k,t} = \tau_{k,t} + \gamma_{j,t} + \mathbf{g}'_{j,k,t}\mathbf{w}_t + h_{i,j,t}c_t + e_{i,j,k,t} \quad (1t)$$

for $i = 1,2,\dots,m$, $j=1,2,\dots,n$, $k=1,2,\dots,K$, $l=0,1,2,\dots,L-1$ and $t=L, L+1, L+2,\dots,\infty$

where
- y = difference of the total carrier-phases between the j^{th} satellite and the k^{th} receiver
- i = index of the signals (L1, L2, L3,… , G1,… , E1,… , etc.)
- j = index of the satellites (GPS, Glonass and Galileo, etc.)
- k = index of the receivers (or receiver sites)
- l = local index of epochs for a moving data window of length L at epoch t
- t = index of the epoch times (t=1, 2, 3,…)
- φ = total phase of the reconstructed carrier of the i^{th} signal at epoch t
- ρ = propagation distance [phase] in dry air from the j^{th} satellite to the k^{th} receiver at epoch t
- τ = clock correction of the k^{th} receiver at epoch t
- γ = clock correction of the j^{th} satellite at epoch t
- $\mathbf{g}$ = vector of the slant-path 3WV refractivity values of pixel volumes from the j^{th} satellite to the k^{th} receiver at epoch t (see Slant-delay models on pages 39-49 of Kleijer (2004))
- $\mathbf{w}$ = vector of the 3WV values of pixel volumes at epoch t
- h = slant-mapping of the TEC refractivity for the i^{th} signal from the j^{th} satellite to the receiver network(s) at epoch t
- c = the TEC value of the receiver network(s) at epoch t
- e = random measurement error at epoch t; and,
- m, n, K and V = the number of signals, satellites, receivers and pixel volumes, respectively.

There are four **System Equations** as follows:

$$\begin{aligned} \tau_{k,t} &= \tau_{k,t-1} + \zeta_{k,t} \\ \gamma_{j,t} &= \gamma_{j,t-1} + \eta_{j,t} \\ \mathbf{w}_t &= (A_t + dA_t)\mathbf{w}_{t-1} + \mathbf{v}_t \\ c_t &= c_{t-1} + \xi_t \end{aligned} \quad (2t)$$

where
- $\zeta_{k,t}$, $\eta_{j,t}$, $\mathbf{v}_t$ and ξ_t = the random walk terms; respectively
- $\mathbf{w}_t$ = vector $[w_{1,t}, w_{2,t},\dots, w_{V,t}]'$
- $\mathbf{v}_t$ = vector $[v_{1,t}, v_{2,t},\dots, v_{V,t}]'$
- A_t = state transition matrix describing advection of the 3WV values in the air-mass
- dA_t = matrix of the state transition errors to be adjusted by adaptive Kalman Filtering.

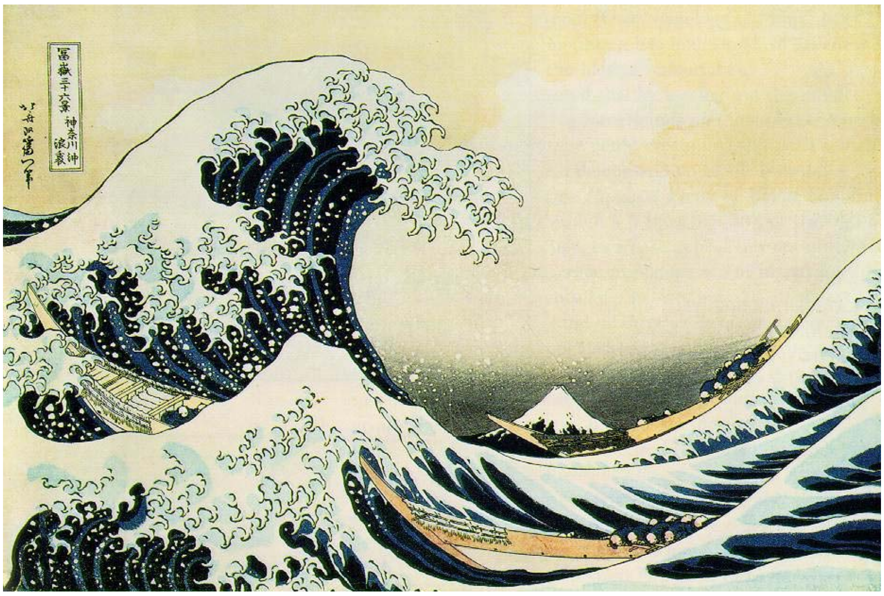
The dual-polarized weather radar and the GPS antenna of SuomiNet station SG40 at FMI, Helsinki, Finland:

The mass pile-up of 17 March 2005 in Helsinki

air -6 ^{o}C, 300 cars wrecked, 70 injuries and 3 deaths:

”The Great Wave off Kanagawa”

by Katsushika Hokusai (1760-1849):



Concluding remarks:

- GPS tomography detects water vapor unlike weather radar
- The Fast Kalman Processing using the HWb method applies to real-time precision GPS engineering
- Reliable accuracy estimates of each GPS signal are now operationally computable from the MINQUE theory by making use of the HWb method (pat. pend. PCT/FI2007/00052)
- Early warning systems for tsunamis, earth quakes, shaking buildings, etc. can operate using low-cost GPS/Glonass/Galileo/Beidou receivers
- A EUREKA project is proposed under the title of VRS2MOBILE for precision piloting and navigation (crf. The NASA Global Differential GPS service)

Thank you for your attention!

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Meteorological R&D on the use of ground-based GPS signals in Europe:

- **EU COST Action 716:** project ended March 2004
- **FP5 TOUGH: T**owards **O**ptimal **U**se of **G**PS data for **H**umidity measurements, continues
- **E-GVAP (EUCOS GPS water VAPour):** e.g. Forward modeling of the GPS signal delays for NWP by FMI
- **GPS /GALILEO water vapour tomography:** raw GPS data from dense Virtual Reference Station (VRS) landsurvey networks etc.

5.2.02

The linearized Observations Equation:

$$d\underline{y} = \mathrm{Jacobian}(A{+}S,\underline{b})\ d\underline{b} + \mathrm{Jacobian}(S,\underline{c})\ d\underline{c} + d\underline{e}$$

$$\begin{bmatrix} d\underline{y}_1 \\ d\underline{y}_2 \\ d\underline{y}_3 \\ \cdot \\ \cdot \\ \cdot \\ d\underline{y}_K \end{bmatrix} = \left[\begin{array}{ccccc|c} X_1 & 0 & 0 & \cdots & 0 & G_1 \\ 0 & X_2 & 0 & \cdots & 0 & G_2 \\ 0 & 0 & X_3 & & 0 & G_3 \\ \cdot & \cdot & & \cdot & & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ \cdot & \cdot & & & \cdot & \cdot \\ 0 & 0 & & & X_K & G_K \end{array}\right] \cdot \begin{bmatrix} d\underline{b}_1 \\ d\underline{b}_2 \\ d\underline{b}_3 \\ \cdot \\ \cdot \\ \cdot \\ d\underline{b}_K \\ d\underline{c} \end{bmatrix} + \begin{bmatrix} d\underline{e}_1 \\ d\underline{e}_2 \\ d\underline{e}_3 \\ \cdot \\ \cdot \\ \cdot \\ d\underline{e}_K \end{bmatrix}$$

X_k : the Jacobian matrix of partial derivatives of A + S functions with respect to the atmospheric state parameters

G_k : the Jacobian matrix of partial derivatives of S functions with respect to the calibration parameters

The Semianalytic Inversion by Frobenius 1845-1917:


Ferdinand Georg Frobenius

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