Modeling of Engineering Structures Displacement by using the Euler Method

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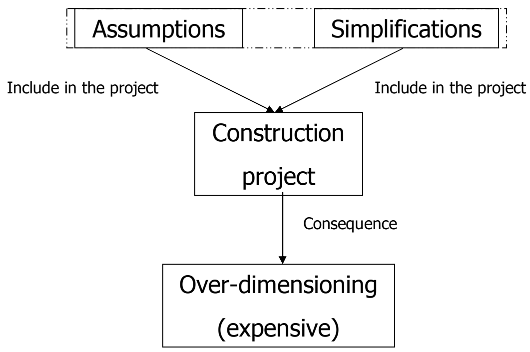
Branko Milovanovic,
Ms. Sci., Teaching Assistant,
University of Belgrade, Faculty of Civil Engineering,
Bulevar kralja Aleksandra 73, milovano@grf.bg.ac.yu

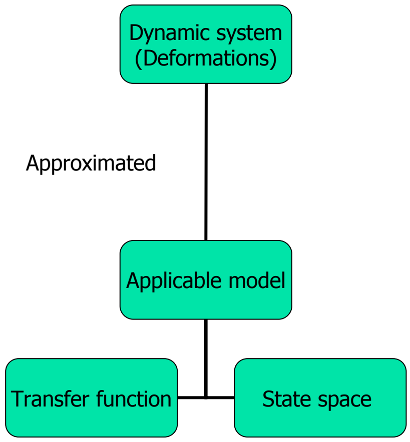
Idea for research



- We must verify the construction project and predict behavior of the engineering structures.
- How can we do that?

Identification of system

- The deformations are result of a process.
- Identification of system
  - Modeling based on observations

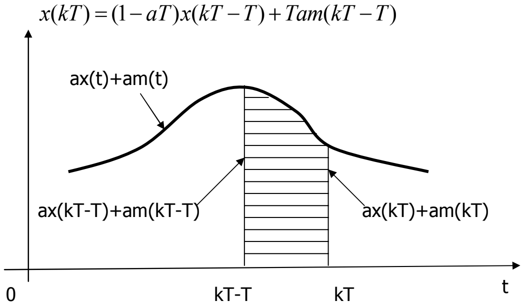


The Euler Method

- It is a first order numerical procedure for solving ordinary differential equations with a given initial value.

$$G(s) = \frac{X(s)}{M(s)} = \frac{a}{s+a}$$ - transfer function of the first order function

$$x(kT) = (1 - aT)x(kT - T) + Tam(kT - T)$$



Subspace method

A linear system can be represent in state space form:

$$\mathbf{x}(kT+T)=\mathbf{A}\mathbf{x}(kT)+\mathbf{B}\mathbf{u}(kT)+\mathbf{w}(t)$$
$$\mathbf{y}(kT)=\mathbf{C}\mathbf{x}(kT)+\mathbf{D}\mathbf{u}(kT)+\mathbf{v}(kT)$$

where:

$\mathbf{x}$ is a $n-$dimensional state vector

$\mathbf{u}$ is a n_u-dimensional input vector

$\mathbf{y}$ is a n_y-dimensional output vector

$\mathbf{v}$ is a n_y-dimensional noise vector

$\mathbf{w}$ is a $n-$dimensional process noise vector

A, B, C and **D** are parameter matrices of the appropriate dimension

$$\mathbf{Y}(kT)=\begin{bmatrix}\mathbf{x}(kT+T)\\ \mathbf{y}(kT)\end{bmatrix}, \quad \Theta=\begin{bmatrix}\mathbf{A} & \mathbf{B}\\ \mathbf{C} & \mathbf{D}\end{bmatrix}$$

$$\Phi(kT)=\begin{bmatrix}\mathbf{x}(kT)\\ \mathbf{u}(kT)\end{bmatrix}, \quad \mathbf{E}(kT)=\begin{bmatrix}\mathbf{w}(kT)\\ \mathbf{v}(kT)\end{bmatrix}$$

$$\mathbf{Y}(kT)=\Theta\Phi(kT)+\mathbf{E}(kT)$$

Structures displacement

- Equation of motion / main equation in Civil Engineering practice

$$M\ddot{x}+C\dot{x}+Kx=F$$

where:
M is the mass,
K is the stiffness,
C is damping coefficient,
x is displacment response of the structure to some extrenal force F, where the dots above the displacment variable are indicative of the derivation order

Euler method

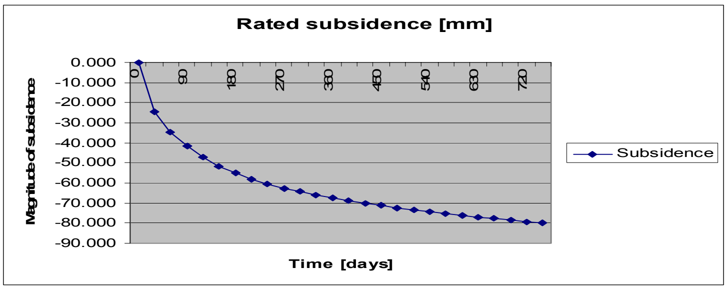
The model is translated into the state-space differential equations.

$$\dot{x}_1(t)=x_2(t)$$
$$\dot{x}_2(t)=-\frac{C}{M}x_2(t)-\frac{K}{M}x_1(t)+\frac{F}{M}=-ax_2(t)-bx_1(t)+m\,.$$
$$\begin{bmatrix}x_1[(k+1)T]\\ x_2[(k+1)T]\end{bmatrix}=\begin{bmatrix}1 & T\\ -bT & 1-aT\end{bmatrix}\begin{bmatrix}x_1(kT)\\ x_2(kT)\end{bmatrix}+\begin{bmatrix}T\\ 0\end{bmatrix}m,$$

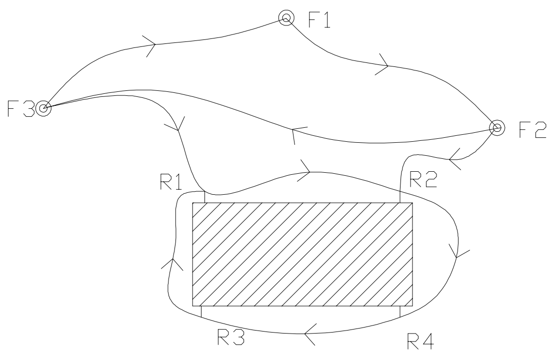
Example

A wide area is loaded with 0.1 MN/m^2 *for a short period of time. The terrain profile is a mediaeval plastic clay, with the thickness of* 5.5 m, *horizontally intersected by a sand layer of* 0.5 m *thickness. Underneath the clay layer is an incompressible and impermeable substrate. The clay's module of compressibility is* 5.0 MN/m^2 . *The coefficient of consolidation is* 5*10^{-4} cm^2/s.

- 26 epoch of observation – simulated
  - 0.5 mm (per station)
  - T=0.0822 - monthly



Example



$$\Delta_i^k = \hat{H}_i^{k+1} - \hat{H}_i^k$$
$$\Delta_i^{2k} = \Delta_i^{k+1} - \Delta_i^k$$

Example

Calculation of the first derivative

$$\Delta_i^k = \hat{H}_i^{k+1} - \hat{H}_i^k$$
$$\Delta_i^{2k} = \Delta_i^{k+1} - \Delta_i^k$$
$$f_i' = \frac{1}{T}(\Delta_i^k - \frac{1}{2}\Delta_i^{2k})$$

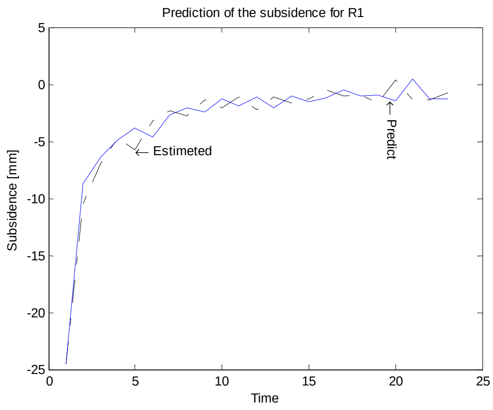
Estimated parameters

Label	$\hat{a}$	$\hat{b}$
R1	-137.52	6.08
R2	-128.68	5.58
R3	-496.96	34.32
R4	-142.75	6.23

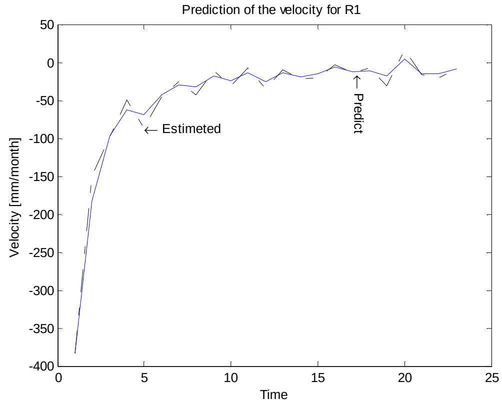
Standard deviations of prediction:

Label	Subsidence [mm]	Velocity [mm/month]
R1	1.0010	9.9049
R2	1.0489	8.7808
R3	1.2363	60.6044
R4	0.7517	8.9279

Example



Example



Conclusion

- Deformation process – non-linear
- We can use successfully linear method – subspace method for subsidence approximation
- System's ill-conditioning
- For prediction is enough only five epoch of measurements

Same results I got when I use transfer function models

- First order dynamical system- three epoch need for prediction
- AR model – where the order of model is decided by autocorrelation function - need four epoch

- Thank you for your attention!